Maximum Relative Strangeness Content in Heavy Ion Collisions Around 30 A·GeV

P. Braun-Munzinger\textsuperscript{a}, J. Cleymans\textsuperscript{b}, H. Oeschler\textsuperscript{c}, and K. Redlich\textsuperscript{a,d}

\textsuperscript{a} Gesellschaft für Schwerionenforschung, D-64291 Darmstadt, Germany
\textsuperscript{b} Department of Physics, University of Cape Town, Rondebosch 7701, Cape Town, South Africa
\textsuperscript{c} Institut für Kernphysik, Darmstadt University of Technology, D-64289 Darmstadt, Germany
\textsuperscript{d} Institute of Theoretical Physics, University of Wroclaw, PL-50204 Wroclaw, Poland.

Abstract

It is shown that the ratio of strange to non-strange particle production in relativistic heavy ion collisions is expected to reach a maximum at beam energies around 30 A·GeV in the lab frame. This maximum is unique to heavy ion collisions, and has no equivalent in elementary particle collisions. The appearance of the maximum is due to the energy dependence of the chemical freeze-out parameters and is clearly seen as a pronounced peak in the Wroblewski factor as a function of the incident energy as well as in the behavior of the $K^+/\pi^+$ ratio. Below 30 A·GeV strange baryons contribute strongly because of the very large value of the chemical potential $\mu_B$. As the energy increases, the decrease of the baryon chemical potential coupled with only moderate increases in the associated temperature causes a decline in the relative number of strange baryons above energies of about 30 A·GeV leading to very pronounced maxima in the $\Lambda/\pi^+$ and $\Xi^-/\pi^+$ ratios.
1 Introduction

The experimental data from heavy ion collisions show that the $K^+/\pi^+$ ratio is larger at BNL-AGS energies than at the highest CERN-SPS energies [1-6] and even at RHIC [7]. This behavior is of particular interest as it could signal the appearance of new dynamics for strangeness production in high energy collisions. It was even conjectured [8] that this property could indicate an energy threshold for quark-gluon plasma formation in relativistic heavy ion collisions. In this paper we analyze the energy dependence of strange to non-strange particle ratios in the framework of a hadronic statistical model. The statistical approach has been very successful in describing particle yields from low energies starting with SIS and AGS all the way up to SPS [9-16] and RHIC [6, 17, 18] energies. In particular it was found that, in the whole energy range, the hadronic yields observed in heavy ion collisions resemble those of a population in chemical equilibrium along a unified freeze-out curve determined by the condition of fixed energy/particle $\simeq 1 \text{ GeV}$ [13]. The chemical freeze-out curve provides a relation between the temperature $T$ and the baryon chemical potential $\mu_B$ when the particle composition of the system is frozen in.

In this paper we use the energy dependence of the thermal parameters at chemical freeze-out as determined from the analysis of [10-16] to show that the ratio of strange to non-strange particle multiplicities reaches a maximum at approximately 30 A·GeV lab energy. Compared to other $q\bar{q}$ pairs, the relative strangeness content actually declines by almost a third between 30 A·GeV and RHIC energies. We emphasize that this maximum is unique to heavy ion collisions and does not occur in the collisions of elementary particles like p-p, p-\bar{p} or e$^+$-e$^-$. The values of the baryon chemical potentials and temperatures at different collision energies are shown in Fig. 1. The values of $\mu_B$ at SIS, AGS and SPS energies were taken from a detailed analysis [14, 15] of the data on particle multiplicities extrapolated to full phase space while the value of $\mu_B$ at RHIC was obtained from data taken at mid-rapidity [18]. The energy dependence can be parametrized phenomenologically as

$$\mu_B(s) \simeq \frac{a}{1 + \sqrt{s}/b}$$

where $a \simeq 1.27 \text{ GeV}$ and $b \simeq 4.3 \text{ GeV}$. The results of this parametrization are shown by the full line in the upper part of Fig. 1. At freeze-out the chemical potential is related to the temperature via the phenomenological condition of fixed energy per hadron [13], namely

$$\langle E \rangle / \langle N \rangle \simeq 1 \text{ GeV}.$$  

Applying Eq. (1) together with Eq. (2) leads to the energy dependence of the temperature shown by the full line in lower part of Fig. 1 together with the values obtained in [15, 18].

Both the temperature and the chemical potential exhibit a strong variation with energy. The GSI/SIS results have the lowest freeze-out temperatures and the highest baryon chemical potentials. As the beam energy is increased a clear shift towards higher $T$ and lower $\mu_B$ occurs. Above AGS energies, the temperature exhibits only a moderate change and converges to its maximal value in the range of 160 to 180 MeV. This value is remarkably close to the critical temperature $T_c \sim 170 \pm 8 \text{ MeV}$ extracted from the lattice calculations of QCD [19].

In the statistical approach, the energy dependence of the basic thermal parameters determines the energy dependence of many relevant observables, for instance particle yields. For strange particle production this determines the ratios of strange to non-strange particle multiplicities as well as the Wroblewski factor [20] defined as

$$\lambda_s \equiv \frac{2\langle ss \rangle}{\langle uu \rangle + \langle dd \rangle}$$

2
where the quantities in angular brackets refer to the number of newly formed quark-antiquark pairs, i.e., it excludes all quarks that were present in the target and projectile.

Applying the statistical model to particle production in heavy ion collisions calls for the use of the canonical ensemble to treat the number of strange particles particularly for data in the energy range from SIS up to AGS [14, 21]. For these energies, the number of strange particles per event is so small that the description of strangeness conservation on the average, as contained in the grand canonical approach is not adequate. The exact conservation of quantum numbers in relativistic statistical mechanics has been well established for some time now [22, 23]. In the following we present a new analytical expression of the canonical partition function and the corresponding multi-strange particle multiplicities. This is quantitatively equivalent to previous results [23, 24], however, the result can now be presented in a compact format well suited for numerical evaluation.

2 Canonical strange particle multiplicities

The canonical partition function of a hadronic resonance gas constrained by the strangeness neutrality condition reads [23],

\[
Z_{S=0}^{C} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \exp \left( \sum_{n=-3}^{3} S_n e^{in\phi} \right),
\]

(4)

where \( S_n = V \sum_k Z_k^l \) with \( V \) being the volume and the sum is over all particles and resonances carrying strangeness \( n \). For a particle of mass \( m_k \), with spin-isospin degeneracy factor \( g_k \), carrying baryon number \( B_k \) and charge \( Q_k \), with the corresponding chemical potentials \( \mu_B \) and \( \mu_Q \), the one-particle partition function is given, in Boltzmann approximation, by

\[
Z_1^k = \frac{g_k}{2\pi^2 m_k^2 T} e^{m_k T} \exp(B_k \mu_B + Q_k \mu_Q).
\]

(5)

The integral representation of the partition function in Eq. (4) is not convenient for a numerical analysis as the integrand is a strongly oscillating function. However, the \( \phi \) integration in Eq. (4) can be done exactly. Indeed, rewriting Eq. (4) as

\[
Z_{S=0}^{C} = \frac{1}{2\pi} e^{S_0} \int_{-\pi}^{\pi} d\phi \prod_{n=1}^{3} \exp \left[ \frac{x_n}{2} \left( a_n e^{in\phi} + a_n^{-1} e^{-in\phi} \right) \right],
\]

(6)

and using the relation \( e^{\frac{x}{2}(+\frac{1}{2})} = \sum_{m=-\infty}^{+\infty} t^m I_m(\rho) \) one obtains, after integrating, the partition function in a form free of oscillating terms

\[
Z_{S=0}^{C} = e^{S_0} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} a_3^p a_2^n a_1^{-2n-3p} I_n(x_2) I_p(x_3) I_{-2n-3p}(x_1),
\]

(7)

where

\[
a_i = \sqrt{S_i/S_{-i}}
\]

(8)

\[
x_i = 2\sqrt{S_i S_{-i}}
\]

(9)

and \( I_i \) are modified Bessel functions. The contribution of thermal pions to \( S_0 \) is calculated using Bose-Einstein statistics.
The expression for the particle density, \( n_i \), can be obtained from the partition function Eq. (3) by following the standard method [23]. For a particle \( i \) having strangeness \( s \) the result is

\[
  n_i = \frac{Z_i^1}{Z_{\text{C}}^{S=0}} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} a_3^p a_2^n a_1^{-2n-3p-s} I_n(x_2) I_p(x_3) I_{-2n-3p-s}(x_1). \tag{10}
\]

The leading term in \( x_3 \) corresponds to the approximate form used in [24] to study the centrality dependence of strangeness production in Pb-Pb collisions at SPS energies.

It can be verified that, in the limit of large \( x_i \), the above formulae coincide with the grand canonical result. In the opposite limit, that of small \( x_i \), the equilibrium density of (multi)strange particles is strongly suppressed relative to its grand canonical value. The above equation can therefore be applied both at SIS energies where the number of strange particles produced per event is as small as \( 10^{-2} \), and at SPS where over a hundred strange particles are produced on average in each event.

### 3 Results

The theoretical results of the previous section combined with the energy dependence of the thermal parameters shown in Fig. 1 provide a basis for the study of the energy dependence of strangeness production in heavy ion collisions. Of particular interest are the ratios of strange to non-strange particle multiplicities as well as the relative strangeness content of the system as expressed by the Wroblewski factor [20]. The thermal model calculations for Au-Au and Pb-Pb collisions are performed using a canonical correlation volume given by a radius of \( \sim 7 \) fm. This radius could vary with energy but for simplicity this was not taken into account in our calculations. At a later stage, when more data become available, this approximation will of course have to be reconsidered. Furthermore, we assume strangeness to be in complete equilibrium, that is, the strangeness saturation factor is taken as \( \gamma_s = 1 \). When more data will become available, this point might be refined.

We turn our attention first to the energy dependence of the Wroblewski ratio. The quark content used in this ratio is determined at the moment of chemical freeze-out, i.e. from the hadrons and especially, hadronic resonances, before they decay. This ratio is thus not an easily measurable observable unless one can reconstruct all resonances from the final-state particles. The results are shown in Fig. 2 as a function of invariant energy \( \sqrt{s} \). The values calculated from the experimental data at chemical freeze-out in central A-A collisions have been taken from reference [15]\(^1\). The values of \( \lambda_s \) were extracted from fully integrated \( 4\pi \) data with the exception of the result form RHIC where particle ratios measured at mid-rapidity [7] were used. The solid line in Fig. 2 describes the statistical model calculations in complete equilibrium along the unified freeze-out curve [13] and with the energy dependent thermal parameters presented here. From Fig. 2 we conclude that around 30 \( A \)-GeV lab energy the relative strangeness content in heavy ion collisions reaches a clear and well pronounced maximum. The Wroblewski factor decreases towards higher incident energies and reaches a limiting value of about 0.43.

The appearance of the maximum can be traced to the specific dependence of \( \mu_B \) on the beam energy. In Fig. 2 we also show the results for \( \lambda_s \) calculated under the assumption that only the temperature varies with collision energy but the baryon chemical potential is kept fixed at zero. In this case the Wroblewski factor is indeed seen to be a monotonic function of

\(^1\)There the statistical model was fitted with an extra parameter \( \gamma_s \) to account for possible chemical under-saturation of strangeness. At the SPS, \( \gamma_s \approx 0.7 \) give the best agreement with \( 4\pi \) data.
energy. The assumption of vanishing net baryon density is close to the prevailing situation in e.g. p-p and e+e− collisions. In Fig. 2 the results for λs extracted from the data in p-p, p-¯p and e+e− are also included [15]. The dashed line represents results with μB = 0 and a radius of 1.2 fm. There are two important differences in the behavior of λs in elementary compared to heavy ion collisions. Firstly, the strangeness content is smaller by a factor of two. In elementary collisions particle multiplicities follow the values given by the canonical ensemble with radius 1.2 fm whereas in A-A collisions there is a transition from canonical to grand canonical behavior. Secondly, there is no significant maximum in the behavior of λs in elementary collisions due to the vanishingly small baryon density in the p-¯p and e+e− systems.

The position of the maximum is further clarified in Fig. 3 which shows values of constant λs in the T − μB plane. As expected λs rises with increasing T for fixed μB. Following the chemical freeze-out curve, shown as a thick full line in Fig. 3, one can see that λs rises quickly from SIS to AGS energies, then reaches a maximum around μB ≈ 500 MeV and T ≈ 130 MeV. These freeze-out parameters correspond to 30 GeV lab energy. At higher incident energies the increase in T becomes negligible but μB keeps on decreasing and as a consequence λs also decreases.

The importance of finite baryon density on the behavior of λs is demonstrated in Fig. 4 showing separately the contributions to ⟨s¯s⟩ coming from strange baryons, from strange mesons and from hidden strangeness, i.e., from hadrons like φ and η. As can be seen in Fig. 4, the origin of the maximum in the Wroblewski ratio can be traced to the contribution of strange baryons. Even strange mesons exhibit a broad maximum. This is due to the presence of associated production of e.g. kaons together with hyperons. This channel dominates at low √s and loses importance at high incident energies.

Next we study how the behavior of the Wroblewski ratio is reflected in specific particle yields. Of particular interest is the K yield which at high energies is responsible for almost 80% of the total strangeness production while at lower energies this contribution decreases to 50%, due to the associated production with hyperons.

The energy dependence of the K+/π+ ratio measured at midrapidity is shown in Fig. 5. The model gives an excellent description of the data, showing a broad maximum at the same energy as the one seen in the Wroblewski factor. In general, of course, statistical-model calculations should be compared with 4π-integrated results since strangeness does not have to be conserved in a limited portion of phase space. A drop in this ratio for 4π yields has been reported from preliminary results of the NA49 collaboration at 158 AGeV [3]. This decrease is, however, not reproduced by the statistical model without further modifications, e.g. by introducing an additional parameter γs ∼ 0.7 [15]. This point might be clearer when final data, also at other beam energies will become available.

The appearance of the maximum in the relative strangeness contribution becomes also obvious when considering ratios which are more sensitive to the baryon chemical potential. Figure 6 shows the energy dependence of the Λ/π+, Ξ−/π+ and the Ω/π+ ratios. As can be seen from the figure there is a very clear pronounced maximum especially in the Λ/π+ ratio. The relative enhancement of Λ is stronger then that of Ξ− or Ω−. There is also a shift of the maximum to higher energies for particles with increasing strangeness quantum number. These differences appear as a consequence of the enhanced strangeness content of the particles which suppresses the dependence of the corresponding ratio on μB.
4 Conclusions

In conclusion, using the energy dependence of thermal parameters determined from an analysis of available data, we have shown that the statistical model description of relativistic heavy ion collisions predicts that the yields of strange to nonstrange particles reaches a well defined maximum near 30 GeV lab energy. It is demonstrated that this maximum is due to the specific shape of the freeze-out curve in the $T - \mu_B$ plane. In particular, a very steep decrease of the baryon chemical potential with increasing energy causes a corresponding decline of relative strangeness content in thermal systems created in heavy ion collisions above lab energies of 30 GeV. The saturation in $T$, necessary for this result, might be connected to the fact that hadronic temperatures cannot exceed the critical temperature $T_c \approx 170$ MeV for the phase transition to the QGP as found in solutions of QCD on the lattice.

The maximum in the relative strangeness content is unique to heavy ion collisions: there is no equivalent behavior in the collisions of elementary particles. It has already been observed in the energy dependence of the $K^+/\pi^+$ ratio, and is predicted to be even more pronounced in the $\Lambda/\pi^+$ ratio. The maxima in the $\Xi^-/\pi^+$ and $\Omega^-/\pi^+$ ratios are expected to occur at a slightly higher beam energy and to be less pronounced.

It is thus clear that in order to study collective effects related to strangeness production, one interesting area is the energy region around 30 A-GeV. Such a program is being planned for the near future at GSI.

Acknowledgments

One of us (K.R.) acknowledges a stimulating discussions with M. Gaździcki and B. Friman as well as the partial support of the Polish Committee for Scientific Research (KBN-2P03B 03018). J.C. acknowledges partial financial support by the URC of the University of Cape Town. We also acknowledge stimulating discussions with the members of the GSI theory division.

References


Figure 1: Behavior of the freeze-out baryon chemical potential $\mu_B$ (upper curve) and the temperature $T$ (lower curve) as a function of energy. The temperature $T$ as a function of beam energy is determined from the unified freezeout condition $\langle E \rangle / \langle N \rangle = 1 \text{ GeV} [13]$. 
Figure 2: The Wroblewski ratio $\lambda_s$ (for definition see text) as a function of $\sqrt{s}$. The thick solid line has been calculated using the freeze-out values of the temperature and the baryon chemical potential. The dotted line has been calculated using $\mu_B = 0$ and only varying $T$. The dashed line has been calculated using a radius of 1.2 fm, keeping $\mu_B = 0$ and taking the energy dependence of the temperature as determined previously. All calculations are performed using strangeness saturation $\gamma_s = 1$. 
Figure 3: Lines of constant Wroblewski factor $\lambda_s$ (for definition see text) in the $T - \mu_B$ plane (thin solid lines) together with the freeze-out curve (thick solid line) [13].
Figure 4: Contributions to the Wroblewski factor (for definition see text) from strange baryons (dotted line), strange mesons (dashed line) and mesons with hidden strangeness (dash-dotted line). The sum of all contributions is given by the full line.
Figure 5: $K^+ / \pi^+$ ratio obtained around midrapidity as a function of $\sqrt{s}$ from the various experiments. For the references for all data points see [3, 6]. The full line shows the results of the statistical model in complete equilibrium. The value at RHIC was estimated using results from the STAR collaboration on the $K^-/\pi^-$ and $K^+/K^-$ ratios, assuming $\pi^-/\pi^+ = 1.007$. 
Figure 6: Prediction for the $\Lambda/\pi^+$ (note the factor 5), $\Xi^-/\pi^+$ and the $\Omega^-/\pi^+$ ratios as a function of $\sqrt{s}$. For compilation of data see [15].