Closing the Window on Warm Dark Matter

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Sterile neutrinos may be one of the best Warm Dark Matter candidates we have today. Both lower and upper bounds on the mass of the sterile neutrino come from astronomical observations. We show that the proper inclusion of the neutrino momentum distribution and the solution of the kinetic equations with the correct coherence breaking terms lead to the near exclusion of the sterile neutrino as a dark matter candidate.

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\section{I. INTRODUCTION}

Astrophysics provides an increasing amount of independent indications that the dark matter of the universe is warm, so that the small-scale fluctuations are damped out by free streaming. This is most easily achieved by giving a keV mass to the DM particle, in which case the preferred candidate is the sterile neutrino. Support for warm dark matter (WDM) comes from simulations of the number of satellite galaxies [1] and of disk galaxy formation without the need for stellar feedback [2], which both find that a DM particle mass of about 1 keV is optimal: a significantly larger mass has little impact on galaxy formation, and a significantly smaller mass would lead to the well known difficulties faced by hot dark matter. A quantitative lower limit on the candidate WDM particle mass is inferred from the existence of a massive black hole at large redshift [3] and the requirement of sufficiently early galaxy formation to account for reionization of the universe and the observed Ly-\alpha forest properties [4], constraining the DM mass to be larger than 0.75 keV. A recent discussion of x-ray emission from decays of sterile neutrinos [5] has imposed an upper limit of about 5 keV on the neutrino mass. Here we discuss a reinterpretation of these bounds on neutrino mass, and demonstrate that solution of the kinetic equations with the correct coherence breaking terms lead to the virtual exclusion of the sterile neutrino as a dark matter candidate.

\section{II. WDM PARTICLE DECOUPLING}

All of these studies [1–4] are based on the mass-dependent cut-off on small scales, produced by free-streaming. In the previously cited studies, a “conventional” WDM model was considered for the underlying particle physics (see refs. [2,6] for recent overviews of such particle models). In such WDM models, the particles decouple in the early Universe at higher temperatures than do massless neutrinos. Therefore they do not share the entropy release from the successive particle annihilations. Since they were relativistic at decoupling, their distribution function in momentum space is subsequently that of a massless fermion, but with a temperature, $T_W$, which is given today by

$$T_W = T_{v_0} \left(\Omega_W h^2 \frac{94 \text{ eV}}{m_W}\right)^{1/3},$$

where $T_{v_0} \approx 1.946$ K, $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_W$ is the present energy density of WDM in units of the critical density, and $m_W$ is the WDM mass. The needed entropy release in these conventional models is much bigger than allowed in the standard model, and such WDM candidates should therefore have decoupled before a larger gauge group breaks down.

Now a very natural candidate for the WDM particle is a massive sterile neutrino mixed with an ordinary neutrino [7–11]. Since the mixing angle is temperature dependent [12] (and for small vacuum mixing angle, $\sin^2 2\theta \sim 10^{-7}$), a small amount of these heavy neutrinos, relative to ordinary active neutrinos, can be produced at high temperatures. The distribution function of sterile neutrinos is, to a fair approximation, characterized by the temperature of massless neutrinos, but smaller by a factor $\chi$. For a specific choice of $m_W$ and $\Omega_W$ today the value of $\chi$ can be found from

$$\chi = \Omega_W h^2 \left(\frac{94 \text{ eV}}{m_W}\right) \sim 10^{-2},$$

therefore the two models produce the same contribution to $\rho_{tot}$ of WDM particles today if

$$\chi = \left(\frac{T_W}{T_{v_0}}\right)^3.$$

However, WDM particles with $m_W$ have a different distribution function in these two models, and their free-
III. LOWER BOUNDS

The two neutrino models are easily included in a Boltzmann code, in order to compute the present matter power spectrum $P(k)$. Using the code cmbfast [13], we have found analytical fits for the transfer functions, $T(k)$, relating the power spectrum in the WDM to the CDM scenario

$$T^2(k) = \frac{P^W_k}{P^\text{CDM}_k} \quad \text{for} \quad \Omega_W = \Omega_{\text{CDM}},$$

where $P^W$ is the power spectrum for cWDM, and a similar expression with $P^W(k)$ for the sWDM model. These transfer functions, which essentially reflect the free streaming cut-off, have the form

$$T(k) = \left[1 + (\alpha k)^2\right]^{-5/3},$$

where $k$ is the wavenumber in units $h\text{Mpc}^{-1}$, $\nu = 1.12$, and $\alpha$ depends on the cosmological parameters as

$$\alpha = A \left(\frac{\Omega_W}{0.3}\right)^b \left(\frac{h}{0.65}\right)^c \left(\frac{m_W}{500 \text{eV}}\right)^d.$$

Numerically we find for cWDM, $A = 1.07$, $b = 0.11$, $c = 1.20$ and $d = -1.11$ in good agreement with [6]. For the sWDM one can derive similar numbers by noting that the mass in the cWDM case differs by $(T_W/T_v)$. This means that if we have a dependence $\Omega_{W}^{b} h^{c} m_W^{d}$ for the sWDM case, and a dependence $\Omega_{W}^{b'} h^{c'} m_W^{d'}$ for the cWDM case, then one finds

$$\Omega_{W}^{b} h^{c} m_W^{d} = \Omega_{W}^{b'} h^{c'-2d'/3} m_W^{d'+d'/3},$$

which is solved by $b' = b + d/4$, $d' = 3d/4$ and $c' = c + d/2$, in good agreement with what we found numerically, by explicitly changing the massive neutrino phase-space distribution function like in ref. [14]. To be very explicit, this means that for a given cut-off scale of the power spectrum one can find the corresponding mass of the elementary particle, and the mass will differ in the two cases. E.g. if for cWDM one finds $m_W = 1 \text{ keV}$, then this corresponds in the sWDM case to $m_W = (\Omega_W h^2 94/1000)^{-1/3} \text{keV} \approx 3.8 \text{ keV}$, when using $h = 0.7$ and $\Omega_W = 0.4$. In other words, if one believes that sterile neutrinos indeed constitute the dark matter, and they are produced, as described in refs. [7–11], then the bounds obtained in refs. [3,4], $m_W > 0.75 \text{keV}$, should really be multiplied by a factor 3.4, and the lower bound on sterile neutrinos as dark matter is thus about 2.6 keV.

IV. UPPER BOUNDS

Sterile neutrinos in the keV mass range have a decay time that is of cosmological interest. Recently a very interesting paper appeared [5], where the signature from decaying sterile neutrinos in galaxies and clusters of galaxies was studied in detail *. A bound $m < 5 \text{ keV}$ was derived, using the relation between mass and mixing angle obtained in [10]. It is now very interesting to note that as shown in ref. [11] the required mixing angle in reality should be about a factor two larger to generate the same cosmological energy density. This factor of 2 was obtained by considering the exact form of the coherence breaking terms in the evolution equations for the neutrino density matrix. Including this effect would further strengthen the bound on $m < 5 \text{ keV}$ by approximately this factor of 2. Thus we see that the excluded region from above [3,4] and from below [5,11] overlap, almost completely excluding the sterile neutrino as a dark matter candidate.

However, the exclusion of sterile neutrinos as a dark matter candidate cannot be said to be complete yet, for the following reasons. First, the temperature of the sterile neutrinos is really slightly lower than the active neutrino temperature, since the sterile neutrinos are being produced while the muons are still present in the Universe, $T \approx 130 \text{ MeV}$ [16]. Furthermore, the factor of 3-4 found above depends on the specific values of $\Omega_W$ and $h$, and can therefore change slightly. Lastly, and probably most importantly, the rather simplistic combination of the results of ref. [10] with the factor 2 of ref. [11] deserves a more careful investigation. Such an analysis is unfortunately not completely trivial, since it demands the numerical solution of the non-linear equations describing the density matrix, taking into account the exact form of the coherence breaking terms. It is therefore crucial to first reach a better understanding of the neutrino production mechanism.

*One could imagine a slight change of strategy in the analysis of ref. [5], namely to consider regions with large concentration of dark matter, but with little baryonic matter. Such “dark blobs” may have been observed by inverting the matter distribution in clusters of galaxies from weak lensing, and comparing with the baryonic matter inferred from optical observations of the cluster [15], but the significance of such blobs is far from being established.
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