ABSTRACT

We study the relation between the gravitational lensing observables and the cosmological parameters. We estimate the sensitivity of the lensing degeneracy to cosmological parameters. We demonstrate the lens model degeneracy by showing that the lensing observables are primarily dependent on the lens model, while the dependence on cosmological parameters is weak. We study the relation between the gravitational lensing observables and the cosmological parameters. We estimate the sensitivity of the lensing degeneracy to cosmological parameters.

KEY WORDS: gravitational lensing - dark matter - cosmology: theory

2 OBSERVABLES

We shall derive lens equation and define critical curves for analytic numerical analyses.

2.1 LENS EQUATION AND CRITICAL CURVES

Let \( \frac{x}{|x|} = \theta \) be the source position, the image position in the lens plane, respectively. \( \theta = \frac{\theta}{\theta_0} \) is the angular diameter distance to the lens, source, respectively. \( \theta_0 \) is the angular diameter distance to the lens, source, respectively. Here and \( \theta = \frac{\theta}{\theta_0} \) is the angular diameter distance to the lens, source, respectively.
where
\[
m(x) = 2 \int_0^x \frac{\Sigma(x')}{\Sigma_{cr}} x' dx',
\]
\[
\Sigma_{cr} = \frac{D_S}{4\pi G D_L D_{LS}}.
\]
Here $D_{LS}$ is the angular diameter distance between the lens and the source.

The determinant of the Jacobian $A \equiv \partial y/\partial x$ of the mapping Eq. (1) is calculated as
\[
\det A = \left(1 - \frac{m}{x}ight) \left(1 - \frac{d}{dx} \left(\frac{m}{x}\right)\right).
\]
The critical curves for the axisymmetric lenses are where $\det A = 0$. Circles where $m/x = 2$ are called tangential critical curves, while those where $d(m/x)/dx = 1$ are called radial curves.

### 2.2 Isothermal model

For an isothermal model with a core, the mass profile is given by
\[
\rho(r) = \frac{\sigma^2}{2\pi G r^3 + r_c^3},
\]
where $\sigma$ is the one-dimensional velocity dispersion and $r_c$ is the core radius. Take $\xi_0 = 4\pi\sigma^2 D_L D_{LS}/D_S$, then the lens equation becomes
\[
y = x - \frac{1}{2} \left(\sqrt{x^2 + x_c^2} - x_c\right),
\]
where $x_c$ is the dimensionless quantity corresponding to $r_c$. The radius of the tangential critical curve, $\theta_c$, is then given by
\[
\sqrt{\theta_c^2 + r_c^2} + \theta_c = 4\pi \sigma^2 \frac{D_{LS}}{D_S},
\]
where $\theta_c = r_c/D_L$. The circular velocity at radius $r$ is
\[
v^2(r) = \frac{GM(\leq r)}{r} = 2\sigma^2 \left(1 - \frac{r_c}{r} \tan^{-1} \frac{r}{r_c}\right).
\]

### 2.3 NFW model

For the Navarro-Frenk-White (NFW) model (Navarro et al. 1997), the mass profile is given by
\[
\rho(r) = \frac{\rho_s}{(r/r_s)(\ln r/r_s + 1)^2}.
\]
Take $\xi_0 = r_s$, then the lens equation is
\[
y = x - \frac{4\rho_s r_s g(x)}{\Sigma_{cr} x},
\]
where $g(x)$ for $x < 1$ is defined by
\[
g(x) = \ln \frac{x}{2} + \frac{2}{\sqrt{1-x^2}} \tanh^{-1} \sqrt{\frac{1-x}{1+x}}.
\]
while for $x > 1$
\[
g(x) = \ln \frac{x}{2} + \frac{2}{\sqrt{x^2-1}} \tan^{-1} \sqrt{\frac{x-1}{x+1}}.
\]
The circular velocity is
\[
v^2(r) = \frac{4\pi G \rho_s r_s^3}{r} \left(\ln(1+r/r_s) - \frac{r/r_s}{1 + r/r_s}\right).
\]

### 2.4 Truncated isothermal model

The truncated isothermal sphere (TIS) model is a particular solution of the Lane-Emden equation that results from the collapse and virialization of a top-hat density perturbation (Shapiro et al. 1999). The mass profile is well fitted by

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The velocity dispersion within the lens equation is determined spectroscopically, Five arcs are clearly seen in the important observable but we will not consider it here. Or Einstein ring system, and thus one of the observable will be the critical radius. Of course, the length of arc is another degeneracy would still persist even by combining with the measurements of the velocity dispersion. We have in mind arc former requiremen twould be an observational challenge (Futamase & Yoshida 2000), while the latter could be accomplished cosmological parameters (Haiman et al. 2000).

Another interesting observable is the location of radial arc (Williams 1999; Molikawa & Hattori 2000; Oguri et al. 2001), which depends on the angular gradient of the projected mass from Eq (6).

3 CONSTRaining Lens Model from Measurement of Velocity Dispersion

We show that there exists degeneracy of lens models to a certain extent on the projected mass density level and that the degeneracy would still persist even by combining with the measurements of the velocity dispersion. We have in mind arc or Einstein ring system, and thus one of the observable will be the critical radius. Of course, the length of arc is another important observable (Kovner 1999) but we will not consider it here.

As an illustration, we pick up the well-known lensing system CL 0024+1654, although our argument is not limited to cluster lenses. Bright multiple arcs were discovered in the cluster CL 0024+1654 at z = 0.39 by Koo (1998) photographically. Five arcs are clearly seen in the HST image (see Fig. 1 in Colley et al. 1999). The redshift of the source galaxy was recently determined spectroscopically, z_s = 1.675 (Broadhurst et al. 2000). The distance of arc from the center of the cluster is \theta_E = 34.6" (Williams 1999), corresponding to 110h^{-1} kpc for the Einstein-de Sitter model. h is the Hubble parameter in units of 100 km/s/Mpc.

Tyson et al. (1998) attempted to construct a high-resolution mass map of the cluster CL 0024+1654 by the Hubble Space Telescope. They found that total mass profile within the arc radius is approximately represented by a power-law model (Schneider et al. 1992):

\[ \Sigma(x) = K(1 + \gamma x^2)^{-1}, \]

where \( x = r/r_{core} \), \( K = 7000 \pm 100hM_\odot pc^{-2} \), \( r_{core} = 35 \pm 3h^{-1} \) kpc, \( \gamma = 0.57 \pm 0.02 \). They also noted that the asymmetry in the mass distribution inside the arcs for CL 0024+1654 is found to be very small (less than 3%).

Recently, however, Broadhurst et al. have suggested that mass profile of CL 0024+1654 is consistent with the NFW profile with \( r_s \approx 400h^{-1} \) kpc and \( \delta_s = \rho_s/\rho_{crit} \approx 8000 \) (Broadhurst et al. 2000). Here \( \rho_{crit} \) is the critical density. Likewise Eq. (1), for the case of the NFW profile, using Eqs. (11), we obtain the following relation around the above set of parameters

\[ \frac{\delta \theta_E}{\theta_E} \approx 2.0 \frac{\delta \rho_s}{\rho_s} + 3.0 \frac{\delta r_s}{r_s} - 0.23 \frac{\delta \Omega_M}{\Omega_M} + 0.19 \frac{\delta w}{w}. \]

On the other hand, Shapiro and Iliev suggested that the projected mass density profile indicated by Tyson et al. (1998) is well fitted by that obtained by the TIS profile with \( \rho_0 \approx 0.06 h^2 M_\odot pc^{-3} \) and \( r_s \approx 20h^{-1} \) kpc (Shapiro & Iliev 2000). They
also pointed out that the mass profile indicated by Broadhurst et al. implies a velocity dispersion ($> 2230\text{km/s}$) that is much higher than the measured value.

In Fig. 1, we show the projected mass density profiles for these three models. We assume the Einstein-de Sitter model and take both critical density ($\rho_{\text{crit}}(z = 0)$ and $\rho_{\text{crit}}(z = z_L)$) for the fit by the NFW profile. We note that the angular resolution of the Hubble Space Telescope is $0.1''$ (Colley et al. 1994), corresponding to $0.32h^{-1}\text{kpc}$. The shaded region is the two-sigma interval of the mass profile determined by Tyson et al. We assume that the parameters $(\kappa, r_e, \gamma)$ are Gaussian-distributed with the dispersions equal to the error bars. Within the uncertainties in fitting parameters, both the power-law profile and the TIS profile look similar, while the NFW profile with $\rho_{\text{crit}}(z = 0)$ appears to deviate slightly from the power-law profile.

As is clear from Fig. 1, the problem of the NFW mass profile fitted by Broadhurst et al. may not be the problem of the NFW model itself but the problem of the definition of $\rho_{\text{crit}}$ as used in the analysis. In the original NFW fit of the cold dark matter halo profile, $\rho_{\text{crit}}$ in $\rho = \delta\rho_{\text{crit}}$ should be evaluated at the redshift of the object. However, as a fitting model, it is not necessarily so, and we can treat $\rho_c$ just as a parameter of the model. If we use $\rho_{\text{crit}}(z = z_L)$ as Shapiro and Iliev (2000) did, then the projected mass density is much higher than the Tyson’s fit of the data from the beginning.

As evident from the lens equation Eq. (14) and Eq. (15), gravitational lensing provides the information regarding the projected two-dimensional mass density. Therefore one may wonder that the degeneracy of the lens model could be broken by combining with three-dimensional data, for example, the velocity dispersion. However, we suggest that it is unlikely. The reason is the following. From Eq. (14), the sensitivity of the velocity dispersion to the parameters of the lens model is evaluated as

$$\frac{\delta v}{v} = \frac{-\pi \delta r_c}{4r} + \frac{\delta \sigma}{\sigma}. \quad (20)$$

Comparing with Eq. (0.7), this indicates that the sensitivity is less dependent on the lens model. The size of the system where the velocity dispersion is measured is larger than the Einstein radius.

In Fig. 2, we show the circular velocity profiles divided by $\sqrt{2}$ for each mass model, although we understand that $v(r) / \sqrt{2}$ exactly coincides with the velocity dispersion only for a singular isothermal lens. The velocity profile of the power-law model is calculated by the use of Abel integral (Binney & Tremaine 1987). As suggested in Fig. 1, the NFW fit with $\rho_{\text{crit}}(z = z_L)$ predicts the velocity profile that is much higher than the measured value, in accordance with the claim by Shapiro and Iliev (2000). We find that each velocity profile look similar. The average velocity dispersion of CL 0024+1654 is measured to be $1150\text{km/s}$ within a radius $r \simeq 600h^{-1}\text{kpc}$, based on 167 galaxy redshifts (Dressler et al. 1999), to an accuracy of roughly $\pm 100\text{km/s}$. For 33 galaxy redshifts, it is also measured to be $1300\text{km/s}$ (Smail et al. 1997).

4 SUMMARY

We have examined the relation between the lensing observables and the lens model and the cosmological parameters. We have found that observables are primarily dependent on the lens model and have assessed the required accuracy to determine cosmological parameters. It is the surface density of the lens which could be determined from the observations of gravitational lensing, and therefore there exists degeneracy of lens models given the observational uncertainties. The degeneracy could not be broken even by combining with the measurements of velocity dispersion. We also have suggested the possible source of the problem of fitting the averaged mass profile of CL 0024+1654 by the NFW profile. The reconstruction of mass profile by detailed shear maps via weak lensing observations (Kaiser & Squires 1993, Seitz & Schneider 1995) may provide more accurate information regarding the mass profile of the lens. In any case, it is not possible to put meaningful constraints on cosmological parameters from gravitational lensing until we have good control of the lens model.

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Using the Davidson-Fletcher-Powell method, we independently fit the Tyson’s profile with the TIS model and find that the Tyson’s profile within $100\text{kpc}$ is also well fitted by the TIS profile with $\rho_0 \simeq 0.0837\text{GeVcm}^{-3}$ and $r_e \simeq 15.4h^{-1}\text{kpc}$.

\* In fact, T. Broadhurst informed us that they adopted $\rho_{\text{crit}}(z = 0)$ to normalize $\rho_c$. 


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Figure 1. The projected mass density profile for CL 0024+1654. The dotted line is the NFW profile with $\rho_s = \delta_c \rho_{\text{crit}}(z_L)$ used by Shapiro and Iliev, and the short-dashed line is the same with $\rho_s = \delta_c \rho_{\text{crit}}(0)$, where $z_L = 0.39$ and we assume the Einstein-de Sitter model. The solid line is the mass profile fitted by Tyson et al. with the 2σ uncertainty by shades. The long-dashed line is the TIS profile fitted by Shapiro and Iliev.
Figure 2. The circular velocity divided by $\sqrt{2}$, which coincides with the velocity dispersion for a singular isothermal lens, for the NFW model with $\rho_s = \delta_c \rho_{\text{crit}}(z_L)$ (the dotted line), the same with $\rho_s = \delta_c \rho_{\text{crit}}(0)$ (the short-dashed line) the power-law model (the solid line), and for the TIS profile (the long-dashed line). Here $z_L = 0.30$ and we assume the Einstein-de Sitter model.