Massive vector trapping as a gauge boson on a brane

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Abstract

We propose a mechanism to trap massive vector fields as a photon on the Randall-Sundrum brane embedded in the five dimensional AdS space. This localization-mechanism of the photon is realized by considering a brane action, to which a quadratic potential of the bulk-vector fields is added. We also point out that this potential gives very severe constraints on the fluctuations of the vector fields in the bulk space.
1 Introduction

People expect that our four dimensional world would be embedded in the higher dimensional space-time where the geometry would be determined by the superstring theory. The Randall-Sundrum brane-model (RS brane) [1, 2] would be a probable candidate of such a simplified theory with more extra-dimensions. It is possible to consider such that the RS brane is embedded at some point of the transverse coordinate in the five-dimensional anti-de Sitter (AdS$_5$) space. And AdS$_5$ is realized near the horizon of the background geometry constructed by the stack of many D3-branes.

In order to consider this RS brane as our four-dimensional world, it would be necessary to confine all observed fields in this brane. Up to now, it is known that the gravity and scalars can be trapped on the brane of positive tension [2, 3], and fermions are localizable on the one of negative tension [4, 5, 3] due to the gravitational force coming from the background configuration, AdS$_5$. However no one knows how the gauge bosons can be trapped on the RS brane by the same situation.

Some ideas [6, 7, 8, 9] for the localization of the gauge fields have been proposed without relating the mechanism to the gravitational force only. Another interesting mechanism has been proposed in [10], where a special mass term has been introduced in the bulk action through three-form field. As a result, the zero-mode of the photon becomes localizable on the brane. However, a resultant four-dimensional action contains a mass term of the gauge boson characterized by the introduced topological mass. In this sense, this mechanism should be modified to a more reasonable one.

The purpose of this paper is to propose a new localization-mechanism of the gauge field by considering a massive vector in the bulk and a slightly modified brane-action. The modification of the brane-action is performed by adding a localized potential of the vector fields which are living in the bulk. The idea to add such a potential of the bulk field is also seen in [11] for the case of scalar fields, and its possible origin might be found in the quantum corrections in the bulk [12]. Here the potential on the brane is given by a quadratic form of this vector field, which can be regarded as a localized mass-term of the vector. Then we can show that the zero-mode, which means zero four-dimensional mass of this mode, of the vector is localized on the brane as a gauge field. In other words, the broken gauge-symmetry in the bulk is restored on the brane of reduced dimensions. Then our mechanism would be inverse of the one given in [10], where the bulk gauge-symmetry seems to be broken on the brane due to the induced mass-term of the localized vector-field.

In the next section, our localization-mechanism is shown by solving the field equations of the vector fields according to the parallel method used in [13, 14]. In the section three, this result is further assured through the study of the Green function of the vector in the bulk space. Final section is devoted to the summary.
2 Localized state of the vector

Here we start from the following effective action,

\[ S = S_{\text{gr}} + S_A. \]  \hfill (1)

The first term denotes the gravitational part,

\[ S_{\text{gr}} = \frac{1}{2\kappa^2} \left\{ \int d^5X \sqrt{-G} (R + \Lambda) + 2 \int d^4x \sqrt{-g} K \right\} - \frac{\tau}{2} \int d^4x \sqrt{-g}. \]  \hfill (2)

where \( K \) is the extrinsic curvature on the boundary, and \( \tau \) represents the tension of the brane. The background configuration with RS brane,

\[ ds^2 = e^{-2|y|/L} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \]  \hfill (3)

is determined by this action, where \( \eta_{\mu\nu} = (-, +, \cdots, +) \). Here \( \tau = 6/(L\kappa^2) \) and \( L = \sqrt{6/\kappa^2 \Lambda} \) which denotes the radius of five-dimensional AdS space. The coordinates parallel to the brane are denoted by \( x^\mu \) and \( y \) is the coordinate transverse to the brane.

The second part \( S_A \) denotes the action for the massive vector, which is denoted by \( A_M(x, y) \),

\[ S_A = \int d^5X \sqrt{-G} \left( -\frac{1}{4} G^{MN} G^{PQ} F_{MP} F_{NQ} - \frac{1}{2} M^2 G^{MN} A_M A_N \right) - c \int d^4x \frac{1}{2} \sqrt{-g} g^{MN} A_M A_N, \]  \hfill (4)

where the second integral is defined on \( X^5 = y = 0 \). The parameter \( M \) and \( c \) denote the bulk mass of the vector and the coupling of the vector potential and the brane respectively. Here no field other than the vector and gravity is considered, so we ignore to consider the origin of the mass \( (M) \) of \( A_M \) and the gauge symmetry expected before the generation of \( M \) in the bulk. And this action is the starting point.

Here we concentrate our attention on the behaviours of the vector-fields fluctuation around the background (3) according to the analysis of the massive scalar given in [13]. Then the field equation of \( A_M \) is given as

\[ \frac{1}{\sqrt{-G}} \partial_A [\sqrt{-G} (G^{AB} G^{CD} - G^{AC} G^{BD}) \partial_B A_C] - [M^2 + c \delta(y)] G^{DB} A_B = 0. \]  \hfill (5)

This equation can be solved by separating to the one of \( A_y \) and \( A_\mu \), the transverse and parallel parts to the brane. The parallel part \( A_\mu \) is further separated to the transverse and longitudinal parts with respect to the four-dimensional momentum on the brane,

\[ A_\mu = A_\mu^T + A_\mu^L, \]  \hfill (6)

where \( A_\mu^T = (\eta_{\mu\nu} - \partial_\mu \partial_\nu / \square) \eta^{\rho\sigma} A_\rho \) and \( A_\mu^L = (\partial_\mu \partial_\nu / \square) \eta^{\rho\sigma} A_\rho \) with \( \square = \eta^{\mu\nu} \partial_\mu \partial_\nu \).
The separation of the equation is performed by using the following equation

\[ [M^2 e^{-2|y|/L} + c\delta(y)]\eta^{\mu\nu}\partial_{\mu}A_{\nu} + M^2 \partial_y \tilde{A}_y + c\partial_y \delta(y)A_y = 0, \]  

(7)

where \( \tilde{A}_y = \sqrt{-G}A_y \). This equation is obtained by operating \( \partial_D \) on (5) times \( \sqrt{-G} \).

And we use this equation to relate \( A_y \) and \( A^L_\mu \) and to give the boundary conditions of these fields at \( y = 0 \). Considering the singular \( \delta \)-function, the boundary conditions at \( y = 0 \) are obtained as,

\[ \eta^{\mu\nu}\partial_{\mu}A_{\nu}|_{y=0} = A_y|_{y=0} = 0. \]

(8)

And from the regular part, we obtain the relation between \( A_y \) and \( A^L_\mu \),

\[ \eta^{\mu\nu}\partial_{\mu}A_{\nu} + e^{2|y|/L}\partial_y \tilde{A}_y = 0. \]

(9)

Further, considering the limit \( y \to 0 \) of this equation (9), we get another boundary condition at \( y = 0 \),

\[ \partial_y \tilde{A}_y|_{y=0} = 0. \]

(10)

Then using (5) and (9), we can obtain the equations for \( A^T_\mu \) and \( A_y \). In order to write these equations in the form of one-dimensional “Schrödinger equation”, \( A^T_\mu \) and \( A_y \) are replaced in the form,

\[ A^T_\mu = (|z|/L + 1)^{1/2}\hat{\psi}(x,z), \quad A_y = (|z|/L + 1)^{5/2}\hat{\psi}(x,z), \]

(11)

where \( z = \text{sgn}(y)L(e^{y/L} - 1) \). Further imposing \( \Box \hat{\psi}(x,z) = m^2 \hat{\psi}(x,z) \) for \( \hat{\psi}(x,z) \), where \( m \) denotes the four-dimensional mass of the bulk fields. Then we obtain

\[ [-\partial^2_z + V(z)]\hat{\psi}(x,z) = m^2 \hat{\psi}(x,z), \]

(12)

where

\[ V(z) = \frac{a}{(|z|/L + 1)^2} - b\delta(z), \quad a = \frac{3}{4L^2} + M^2. \]

(13)

The parameter \( b \) differs for \( A^T_\mu \) and \( A_y \), which are given as \( b = 1/L - c \) and \( b = -3/L - c \) for \( A^T_\mu \) and \( A_y \) respectively. And \( A^L_\mu \) is given by (9) after solving \( A_y \), as

\[ A^L_\mu = -e^{2|y|/L}\partial_y \left( \frac{\partial_\mu \tilde{A}_y}{\Box} \right). \]

(14)

In (12), the \( x \)-dependent part can be factored out, so we concentrate on its \( z \)-dependence hereafter except for a special case.

For \( A^T_\mu \), the equation (12) can be solved with \( b = 1/L - c \) and the following boundary condition,

\[ \partial_z \hat{\psi}(z)|_{z=0} = -\frac{1}{2}(1/L - c)\hat{\psi}(z)|_{z=0}, \]

(15)

which is required from the equation (12) due to the existence of the \( \delta \)-function in the potential. Then the solution for \( m > 0 \) is obtained as

\[ \hat{\psi}(z) = N(|z|/L + 1)^{1/2}[J\nu(m||z| + L)] + \alpha Y\nu(m||z| + L)] \]

(16)

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where \( N \) denotes the normalization factor and
\[
\alpha = \frac{-mJ'_\nu(mL) + (1/L - c/2)J_\nu(mL)}{mY'_\nu(mL) + (1/L - c/2)Y_\nu(mL)}.
\] (17)

But we know that this mode does not localize on the brane. It is seen from the equation (12). For \( 1/L > c \) and \( a > 0 \), we obtain the so-called volcano-type potential which is necessary for the localization of the mode given by the solution of this equation. However any mode with \( m > 0 \) would decay into the bulk with a finite life-time as shown in [13, 15]. The modes with \( m > 0 \) are identified with the continuous Kaluza-Klein (KK) modes. Then the localizable and stable state is restricted to the “zero-mode” of \( m = 0 \) [13].

The normalizable solution of zero-mode is given by
\[
\hat{\psi}(z) = N(|z|/L + 1)^{1/2 - \nu},
\] (18)

In this case, the parameter \( c \) is determined from the boundary condition (15) as,
\[
c = -\frac{2(\nu - 1)}{L}.
\] (19)

Then, this mode is localized on the brane as a massless vector boson, namely as the gauge boson, when the parameter \( c \) is chosen as above.

Next, we examine other components of the original bulk vector. As for \( A_y \), its solution can be obtained similarly to the case of \( A_T^\mu \) by solving (12) with \( b = -3/L - c \). In this case, we could expect zero-mode bound state for \( b > 0 \) or \( c < -3/L \). Before considering the zero mode, we consider the KK modes of \( m > 0 \). The solution of (12) contains two integral-constants, while there are three independent boundary conditions at \( z = 0 \) or \( y = 0 \), i.e. (8), (10) and the one required from the equation (12). The third condition is given as
\[
\frac{\partial}{\partial z}\hat{\psi}(z)|_{z=0} = \frac{1}{2}(3/L + c)\hat{\psi}(z)|_{z=0}.
\] (20)

Then we obtain the trivial solution, \( A_y = 0 \). The situation is the same for the zero-mode. \( A_\mu^L \) is obtained from (14) in terms of \( A_y \), so this is also trivial, i.e. \( A_\mu^L = 0 \). This fact means that the potential of the vector on the brane introduced here provides severe boundary conditions for the fluctuation modes of the bulk vector fields, and all mass-modes of \( A_\mu^L \) and \( A_y \) can not be allowed to be excited.

The effective action for the localized zero-mode of \( A_\mu^L \) can be written by denoting it as \( A_\mu(x, y) = a_\mu(x)u(y) \) and substituting the solution obtained above for \( u(y) \), which is given by
\[
u(y) = e^{-(\nu - 1)y/L}
\] (21)

where we set \( u(0) = 1 \) since the normalization can be absorbed into \( a_\mu(x) \). Then the effective action is obtained as
\[
\int d^4x \left( -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} \right),
\] (22)

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where the suffices are contracted by $\eta_{\mu\nu}$ and $f_{\mu\nu} = \partial_\mu \tilde{a}_\nu - \partial_\nu \tilde{a}_\mu$. Here
\[
\tilde{a}_\mu(x) = \sqrt{\int_0^\infty dy u(y)^2 a_\mu(x)} = \sqrt{\frac{L}{2(\nu - 1)}} a_\mu(x).
\] (23)
The above integral with respect to $y$ is finite for $\nu > 1$ or $M^2 > 0$, which is the case considered here.

If we choose a more soft form of the potential
\[
-\frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} A_\mu A_\nu,
\] (24)
where the potential for $A_y$ is dropped. Then the condition (8) for $A_y$ disappears since the last term of (7) is absent in this case. Further we can see that (10) and (20) are equivalent since the parameter $c$ in (20) is absent, so we can find a non-trivial solution for $A_y$ in this case. Then we get $A^L_\mu$ from (14) in terms of this non-trivial solution of $A_y$. However we must require that this solution should satisfy the original equation for $A^L_\mu$ derived from (5). In this equation, the potential coming from the mass term added on the brane is included, then we must impose the boundary condition $\partial_y A^L_\mu = 0$. And it produces new boundary condition for $A_y$,
\[
\partial_y^2 \tilde{A}_y|_{y=0} = 0.
\] (25)
As a result, we find only one non-trivial solution for the mode of $m = M$, and all other modes ($m \neq M$) are trivial. Then we should consider that this potential (24) is also severe.

In any case, we can see that the zero mode of the bulk vector is localized on the brane as a gauge boson when a quadratic form of potential of this vector is added on the brane. In the next section, we see this point from the Green function for the vector fields.

### 3 Vector propagator

In this section we analyze the Green function of massive vector in the $d+1$ dimensional AdS space to see its effective propagator observed in $d$ dimensional flat space of the brane. After obtaining the Green function, we go back to the case of $d = 4$. We work in the brane background of the following AdS metric,
\[
ds^2 = G_{MN}dX^M dX^N = \frac{L^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2).
\] (26)
We can consider that the brane is located at $z = L$, where $z = L \exp(y/L)$, which is different from $z$ used in the previous section. The action integral for free massive vector $A_M(X)$ with external source $J_M(X)$ is given by
\[
S_{d+1} = S_{Ad+1} - \int d^{d+1}X \sqrt{-G} G^{MN} A_M J_N,
\] (27)
where $S_{A d+1}$ is the $d + 1$ dimensional version of $S_A$ defined by (4). Then the Green function $\Delta_{MN}(X, X')$ is defined by

$$D^{BN}(X)\Delta_{NP}(X, X') = \frac{1}{\sqrt{-G}} \delta^B_P \delta^{d+1}(X - X'),$$

(28)

$$D^{BN} \Delta_{NP} = \frac{1}{\sqrt{-G}} \partial_A \left( \sqrt{-G} (G^{AM} G^{BN} - G^{AN} G^{BM}) \partial_M \Delta_{NP} \right) - [M^2 + c\delta(z - L)] G^{BN} \Delta_{NP}.$$ 

(29)

From this definition it follows the relation

$$\frac{1}{\sqrt{-G}} \partial_B \left( \sqrt{-G} [M^2 + c\delta(z - L)] G^{BN} \Delta_{NP} \right) = - \frac{1}{\sqrt{-G}} \partial_P \delta^{d+1}(X - X'),$$

(30)

as a consequence of the identity

$$\partial_B \partial_A \left( \sqrt{-G} (G^{AM} G^{BN} - G^{AN} G^{BM}) \partial_M \Delta_{NP} \right) = 0.$$ 

(31)

The components $D^{BN}$ in the definition (28) are given by the followings;

$$D^{\beta\nu} = \left( \frac{z}{L} \right)^4 \left[ \eta^{\beta\nu} D_1 - \eta^{\alpha\nu} \eta^{\beta\mu} \partial_\alpha \partial_\mu - \eta^{\beta\nu} c \delta(z - L) \right],$$

(32)

$$D^{\beta z} = - \left( \frac{z}{L} \right)^4 \eta^{\beta\mu} \partial_\mu \left( \partial_z - \frac{d - 3}{z} \right),$$

(33)

$$D^{z\nu} = - \left( \frac{z}{L} \right)^4 \eta^{\alpha\nu} \partial_\alpha,$$

(34)

$$D^{zz} = \left( \frac{z}{L} \right)^4 \left[ - \left( \frac{ML}{z^2} \right)^2 - c \delta(z - L) \right],$$

(35)

$$D_1 \equiv \partial_z^2 - \frac{d - 3}{z} \partial_z - \frac{(ML)^2}{z^2} + \Box$$

(36)

while the relation (30) reads in components as

$$[M^2 + c\delta(z - L)] \left[ \eta^{\beta\nu} \partial_\beta \Delta_{\nu P} + \left( \partial_z - \frac{d - 1}{z} \right) \Delta_{z P} \right] + c \partial_z \delta(z - L) \Delta_{z P} = - \left( \frac{z}{L} \right)^{d-1} \partial_P \delta^{d+1}(X - X').$$ 

(37)

The defining equations (28) for the Green functions are solved off brane ($z \neq L$) as follows. (i) With the aid of relations (37), $\eta^{\beta\nu} \partial_\beta \Delta_{\nu P}$ are written in terms of $\Delta_{z P}$. (ii) Then closed equations are obtained for $\Delta_{zz}$ and $\Delta_{zp}$ by themselves and they are easily solved. (iii) As for $\Delta_{az}$ and $\Delta_{ap}$, the equations become inhomogeneous equation again with the aid of relations (37). Since the inhomogeneous terms for $\Delta_{az}$ ($\Delta_{ap}$) is already solved $\Delta_{zz}$ ($\Delta_{zp}$) and $\delta(z - z')$, they are solved with as yet undetermined homogeneous
solutions. (iv) The undetermined homogeneous solutions are determined in such a way that \( \Delta_{MN} \) satisfy the relation (30). The resulting solutions are given by the followings;

\[
\Delta_{\alpha \rho} = \Delta_{\alpha \rho}^{T} + \Delta_{\alpha \rho}^{L},
\]

(38)

\[
\Delta_{\alpha \rho}^{T} = \left( \frac{z z'}{L^2} \right)^{d/2-1} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \left( \eta_{\alpha \rho} - \frac{p_\alpha p_\rho}{p^2} \right) \tilde{\Delta}_1(p, z, z'),
\]

(39)

\[
\Delta_{\alpha \rho}^{L} = -\frac{1}{M^2} \left( \frac{z z'}{L^2} \right)^{d/2} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \frac{p_\alpha p_\rho}{p^2} \left[ \frac{L}{z'} \delta(z - z') \right.
\]

\[
+ \left( \partial_z - \frac{d/2 - 1}{z} \right) \left( \partial''_z - \frac{d/2 - 1}{z'} \right) \tilde{\Delta}_2(p, z, z'),
\]

(40)

\[
\Delta_{\alpha z} = -\frac{1}{M^2} \partial_\alpha \left( \partial_z - \frac{d - 1}{z} \right) \Delta_2(X, X'),
\]

(41)

\[
\Delta_{zp} = -\frac{1}{M^2} \left( \partial'_z - \frac{d - 1}{z'} \right) \Delta'_2(X, X'),
\]

(42)

\[
\Delta_{zz} = -\frac{1}{M^2} \left( \frac{z z'}{L^2} \right)^{d+1} \delta^{d+1}(X - X') + \frac{M^2}{M^2} \Delta_2(X, X')
\]

\[
= -\frac{1}{M^2} \left[ \partial'_z \left( \partial'_z - \frac{d - 1}{z'} \right) - \frac{(ML)^2}{z'^2} \right] \Delta_2(X, X'),
\]

(43)

(44)

where \( \partial'_z \) (\( \partial'_\alpha \)) denotes differentiation with respect to \( z' (x'^\alpha) \). The expression (43) is convenient to verify that the above \( \Delta_{MN} \) is the solution, while the expression (44) is convenient to recast the boundary condition for \( \Delta_{zz} \) into the one for \( \Delta_\alpha \) defined below. The scalar-type propagators \( \Delta_1(X, X') \) and \( \Delta_2(X, X') \) are defined by the followings;

\[
D_1(X)\Delta_1(X, X') = \left( \frac{z'}{L} \right)^{d-3} \delta^{d+1}(X - X'),
\]

(45)

\[
D_2(X)\Delta_2(X, X') = \left( \frac{z'}{L} \right)^{d-1} \delta^{d+1}(X - X'),
\]

(46)

\[
D_2 \equiv \partial'^2_z - \frac{d - 1}{z} \partial_z - \frac{-(d - 1) + (ML)^2}{z^2} + \Box,
\]

(47)

\[
\Delta_1(X, X') = \left( \frac{z z'}{L^2} \right)^{d/2-1} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \tilde{\Delta}_1(p, z, z'),
\]

(48)

\[
\Delta_2(X, X') = \left( \frac{z z'}{L^2} \right)^{d/2} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \tilde{\Delta}_2(p, z, z'),
\]

(49)

\[
\left[ \partial'^2_z + \frac{1}{z} \partial_z + \left( q^2 - \frac{\nu^2}{z^2} \right) \right] \tilde{\Delta}_i(p, z, z') = \frac{L}{z'} \delta(z - z'), \quad (i = 1, 2)
\]

(50)

\[
q^2 = -p^2, \quad \nu \equiv [(d/2 - 1)^2 + (ML)^2]^{1/2}
\]

(51)

It should be noted that the \( \delta \)-function singularity is canceled by the second term in the square bracket of (40) and that \( \tilde{\Delta}_1 \) only contributes to transverse parts while \( \tilde{\Delta}_2 \)
contributes to longitudinal parts. It is not hard to directly verify that (38)∼(43) satisfy the relation (30) and the defining equation (28) by using the following relations;
\[
\Delta_i(X, X') = \Delta_i(X', X), \quad (i = 1, 2)
\]
\[
\tilde{\Delta}_i(p, z, z') = \tilde{\Delta}_i(p, z', z), \quad (i = 1, 2)
\]
\[
D_1 \left( \partial_z - \frac{d-1}{z} \right) = \left( \partial_z - \frac{d-3}{z} \right) D_2 - \frac{2}{z} \Box.
\]

Having obtained the Green functions in the bulk, we now proceed to impose boundary conditions on them on the brane. They are obtained by matching \( \delta(z - L) \) in the defining equation (28) and the relation (30). Since \( \partial_z^2 \Delta_{\alpha P} \) contain \( \delta(z - L) \) by the \( Z_2 \) symmetry, \( \delta(z - L) \) matching conditions obtained from the defining equation (28) become as follows;
\[
\partial_z \Delta_{\alpha \rho} \big|_{z=L} = \frac{c}{2} \Delta_{\alpha \rho} \big|_{z=L},
\]
\[
\partial_z \Delta_{\alpha z} \big|_{z=L} = \frac{c}{2} \Delta_{\alpha z} \big|_{z=L},
\]
\[
\Delta_{z \rho} \big|_{z=L} = 0,
\]
\[
\Delta_{zz} \big|_{z=L} = 0.
\]

On the other hand, since the relations (37) do not contain \( \partial_z^2 \Delta_{MN} \), \( \delta(z - L) \) matching conditions from them simply read as
\[
\eta^{\beta \nu} \partial_\beta \Delta_{\nu \rho} \big|_{z=L} = 0,
\]
\[
\eta^{\beta \nu} \partial_\beta \Delta_{\nu z} \big|_{z=L} = 0,
\]
and the conditions identical to (57) and (58) are obtained. In terms of \( \hat{\Delta}_1 \) and \( \hat{\Delta}_2 \), (55)∼(60) lead to the following by using (38)∼(42) and (44);
\[
\partial_z \hat{\Delta}_1(p, z, z') \big|_{z=L} = \left( \frac{c}{2} - \frac{d/2 - 1}{L} \right) \hat{\Delta}_1(p, z, z') \big|_{z=L},
\]
\[
-(q^2 - M^2) \hat{\Delta}_2(p, z, z') \big|_{z=L} = c \left( \partial_z - \frac{d/2 - 1}{L} \right) \hat{\Delta}_2(p, z, z') \big|_{z=L},
\]
\[
\left( \partial_z - \frac{d/2 - 1}{z} \right) \hat{\Delta}_2(p, z, z') \big|_{z=L} = 0,
\]

where \( z' > L \) is assumed. It turns out that it is convenient for our purpose to set as
\[
c = -\frac{2(\nu - d/2 + 1)}{L},
\]
which coincide with (19) when \( d = 4 \). This value is chosen in such a way that the leading pole of \( \Delta_{\mu \nu}^T \) is massless (see (71)∼(73)).
First we discuss a consequence of the boundary conditions (62)~(64) for $\hat{\Delta}_2$. From these boundary conditions, it follows immediately

$$\partial_z \hat{\Delta}_2(p, z, z')|_{z=L} = \hat{\Delta}_2(p, z, z')|_{z=L} = 0.$$  \hspace{1cm} (66)

The solution of $\hat{\Delta}_2$ for the equation (50) are given by a linear combination of two types of Bessel function

$$\hat{\Delta}_2(p, z, z') = B(p, z') J_\nu(qz) + C(p, z') Y_\nu(qz).$$  \hspace{1cm} (67)

Then boundary conditions (66) force to be

$$\hat{\Delta}_2(p, z, z') = 0.$$  \hspace{1cm} (68)

But $\hat{\Delta}_2$ should satisfy the inhomogeneous equation (50) so that there is no solution for $\hat{\Delta}_2$. This result is consistent with the result in the previous section that $A_\mu = 0 = A_\mu^L$ since the longitudinal sources as well as transverse sources are contained in the definition (28) of the Green functions in the present case.

Next we discuss about a solution for $\Delta_1$. The boundary condition (61) is obtained as a consequence of the potential on the brane and it differs from the Neumann condition. It is a mixed boundary condition of Dirichlet type and of Neumann type, which is briefly commented in the previous paper [13] and is employed in [16]. As we have imposed the boundary condition (61) on the brane, the procedure to obtain the propagator is parallel to [14]. The result is given by

$$\hat{\Delta}_1(p, z, z') = \frac{i\pi L}{2} \left[ \frac{J_{\nu-1}(qL) H_{\nu}^{(1)}(qz) H_{\nu}^{(1)}(qz') - J_\nu(qz\zeta) H_\nu^{(1)}(qz)}{H_{\nu-1}^{(1)}(qL)} \right],$$  \hspace{1cm} (69)

where $z_{\zeta}$ ($z_{\zeta}$) denotes the greater (lesser) of $z$ and $z'$. A case that is of particular interest here is that where the arguments of $\Delta_{T\mu}^T(x, z; x, z')$ is on the brane, $z = z' = L$. In this case, the propagator is expressed as

$$\Delta_{T\alpha\rho}^T(x, L; x', L) = \int \frac{d^dp}{(2\pi)^d} e^{ip(x-x')} \left( \eta_{\alpha\rho} - \frac{p_\alpha p_\rho}{p^2} \right) \frac{1}{q} \frac{H_{\nu}^{(1)}(qL)}{H_{\nu-1}^{(1)}(qL)}.$$  \hspace{1cm} (70)

From this result, some interesting features are observed. Hereafter we consider the case of $d = 4$.

As in Ref. [14], $\Delta_{T\mu}^T(x, L; x', L)$ is separated to the 4-dimensional massless propagator and the exchange of the Kaluza-Klein states;

$$\Delta_{T\alpha\rho}^T(x, L; x, L) = \frac{2(\nu - 1)}{L} \Delta_{T\alpha\rho}^{(0)}(x, x') + \Delta_{T\alpha\rho}^{T(KK)}(x, x'),$$  \hspace{1cm} (71)

$$\Delta_{T\alpha\rho}^{T(0)}(x, x') = \int \frac{d^dp}{(2\pi)^d} e^{ip(x-x')} \left( \eta_{\alpha\rho} - \frac{p_\alpha p_\rho}{p^2} \right) \frac{1}{q^2},$$  \hspace{1cm} (72)

$$\Delta_{T\alpha\rho}^{T(KK)}(x, x') = - \int \frac{d^dp}{(2\pi)^d} e^{ip(x-x')} \left( \eta_{\alpha\rho} - \frac{p_\alpha p_\rho}{p^2} \right) \frac{1}{q} \frac{H_{\nu}^{(1)}(qL)}{H_{\nu-1}^{(1)}(qL)}.$$  \hspace{1cm} (73)
The propagator $\Delta T^{(0)}(x,x')$ represents the localized massless state in the 4-dimensional brane. This localized mode is precisely the gauge boson (photon) trapped on the brane. This is a consequence of the choice (65) as previously noted. The leading part of the summation of the KK exchanges gives $1/r^3$ potential for $\nu > 2$ as in the case of the gravity while it gives $1/r^{\nu+1}$ potential for $1 < \nu < 2$. Thus we have succeeded to trap a photon on the brane as a leading massless mode of vector.

In order to obtain this result the potential on the brane has played an important role. If another potential is considered the results may be changed. As the another potential, we consider (24), and discuss its consequences. In this case, boundary conditions (57) and (58) are not imposed so that (64) is not subjected for $\hat{\Delta}$. In spite of the lack of the boundary condition (64), we can still conclude that $\hat{\Delta}$ has no solution when $q \neq M$ as is seen from the boundary conditions (62) and (63). When $q = M$, we have a nontrivial solution for $\hat{\Delta}_2$. As for $\hat{\Delta}_1$, the boundary condition is unchanged so that the same results are obtained. Thus almost same results are obtained when (24) is adopted instead of the second term in $S_A$.

4 Summary

We have examined the massive vector field in the AdS background, in which the Randall-Sundrum three-brane is embedded. We find the localization of the leading massless mode of the transverse part of the massive vector on the brane. This result is assured by both wave-function analysis and propagator analysis. The leading massless mode is nothing but a gauge boson (photon) as is seen from the effective action (22). In contrast to [10], the mass of the trapped gauge boson is strictly vanishing in our analysis. For the purpose of obtaining the above conclusion, the potential on the brane, which take forms of tachyonic mass terms, play an essential role. Although the origin of the potentials on the brane is obscure at present, we would like to point out some resemblance of them to the potential on the brane for scalar considered in [11], where the potential contains tachyonic mass term.

On the other hand, the potentials on the brane have imposed severe boundary conditions for the longitudinal modes ($A_{\mu}^L$ mode) and transverse to the brane mode ($A_y$ mode). Since they can be considered as over-determining conditions, the corresponding wave-functions are not allowed to have solutions other than trivial solutions and the corresponding propagator $\hat{\Delta}_2$ is not allowed to have even trivial solution. As a consequence, the massive vector field has a configuration parallel to the brane. This result might be related to the “layered structure” of the gauge fields pointed out in terms of the lattice calculation [8].

In addition to the potential on the brane, the mass term of the bulk vector is necessary to obtain the normalizable wave-function. The mechanism proposed here is curious in the sense that the broken gauge symmetry in the bulk is restored on the brane. It could be considered as a kind of inverse Higgs mechanism into the reduced dimensions.
It will be interesting to study the case including the charged particles [17, 18], since our model could provide concrete form of $z$-dependent wave-function and propagator of the gauge fields. We will give the discussion related to these in the future.

References
