Signal-Locality in Hidden-Variables Theories

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We prove that all deterministic hidden-variables theories, that reproduce quantum theory for an ’equilibrium’ distribution of hidden variables, give instantaneous signals at the statistical level for hypothetical ’nonequilibrium ensembles’. This generalises another property of de Broglie-Bohm theory. Assuming a certain symmetry, we derive a lower bound on the (equilibrium) fraction of outcomes at one wing of an EPR-experiment that change in response to a shift in the distant angular setting. We argue that the universe is in a special state of statistical equilibrium that hides nonlocality.

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Introduction: Bell’s theorem shows that, with reasonable assumptions, any deterministic hidden-variables theory behind quantum mechanics has to be non-local [1]. Specifically, for pairs of spin-1/2 particles in the singlet state, the outcomes of spin measurements at each wing must depend instantaneously on the axis of measurement at the other, distant wing. In this paper we show that the underlying nonlocality becomes visible at the statistical level for hypothetical ensembles whose distribution differs from that of quantum theory. This generalises yet another property of de Broglie-Bohm theory to all deterministic hidden-variables theories.

Historically, Bell’s theorem was inspired by the pilot-wave theory of de Broglie and Bohm [2–10]. Bell asked [11] if all hidden-variables theories that reproduce the quantum distribution of outcomes have to be nonlocal like pilot-wave theory. He subsequently proved that this is indeed the case [1]. A further property of pilot-wave theory – ‘contextuality’ – was also proved to be universal. In general, quantum measurements are not faithful: they do not reveal the value of an attribute of the system existing prior to the ‘measurement’. This feature of pilot-wave theory was discovered by Bohm [3], and was shown by Kochen and Specker [12] to be a feature of any hidden-variables interpretation of quantum mechanics.

One might ask if there are any other properties of pilot-wave theory that are actually universal, in the sense of necessarily being properties of any viable hidden-variables theory. In this paper we prove that, indeed, yet another feature of pilot-wave theory – the ‘signal-locality theorem’ – is generally true in any deterministic hidden-variables interpretation.

In pilot-wave theory, an individual system with a wavefunction $\psi(x, t)$ has a definite configuration $x(t)$ at all times, with velocity given by the de Broglie guidance equation $\dot{x}(t) = j(x, t)/|\psi(x, t)|^2$ where $j$ is the usual quantum probability current. To recover quantum theory, one must assume that an ensemble of systems with wavefunction $\psi_0(x)$ begins with a distribution of configurations given by $\rho_0(x) = |\psi_0(x)|^2$ at $t = 0$ (which guarantees that $\rho(x, t) = |\psi(x, t)|^2$ at all times). In other words, the Born probability distribution is assumed as an initial condition. In principle, however, the theory – considered as a theory of dynamics – allows one to consider arbitrary initial distributions $\rho_0(x) \neq |\psi_0(x)|^2$. The ‘quantum equilibrium’ distribution $\rho = |\psi|^2$ is analogous to thermal equilibrium in classical mechanics, and may be accounted for by an $H$-theorem [13, 14]. But nonequilibrium $\rho \neq |\psi|^2$ could exist, in violation of the Born rule [5, 10, 14, 15].

Now at the hidden-variable level, pilot-wave theory is nonlocal. For example, for two entangled particles $A$ and $B$ with wavefunction $\psi(x_A, x_B, t)$, operations performed at $B$ (such as switching on an external potential) have an instantaneous effect on the motion of particle $A$ no matter how distant it may be. However, at the quantum level one considers an ensemble with the equilibrium distribution $\rho(x_A, x_B, t) = |\psi(x_A, x_B, t)|^2$ and operations at $B$ have no statistical effect at $A$: as is well known, quantum entanglement cannot be used for signalling at a distance. On the other hand, this masking of nonlocality by statistical noise is peculiar to the equilibrium distribution. If one considers an
ensemble with distribution $\rho_0(x_A, x_B) \neq |\psi_0(x_A, x_B)|^2$ at $t = 0$, changing the Hamiltonian at $B$ generally induces an instantaneous change in the marginal distribution at $A$ [16].

This is the signal-locality theorem of pilot-wave theory: in general, there are instantaneous signals at the statistical level if and only if the ensemble is in quantum nonequilibrium $\rho_0 \neq |\psi_0|^2$ [16]. We wish to show that the same is true in any deterministic hidden-variables theory.

**Bell Nonlocality.** Consider two spin-1/2 particles $A$ and $B$ lying on the $y$-axis and separated by a large distance. If the pair is in the singlet state $|\Psi\rangle = (|z+, z-\rangle - |z-, z+\rangle)/\sqrt{2}$, spin measurements along the $z$-axis at each wing always yield opposite results. But we are of course free to measure spin components along arbitrary axes at each wing. For simplicity we take the measurement axes to lie in the $x-z$ plane, so that their orientations may be specified by the angles $\theta_A$, $\theta_B$ made with the $z$-axis. In units of $\hbar/2$, the possible values of outcomes of spin measurements along $\theta_A$, $\theta_B$ at $A$, $B$ – that is, the possible values of the quantum observables $\hat{\sigma}_A$, $\hat{\sigma}_B$ at $A$, $B$ – are $\pm 1$. Quantum theory predicts that for an ensemble of such pairs, the outcomes at $A$ and $B$ are correlated: $\langle \Psi | \hat{\sigma}_A \hat{\sigma}_B | \Psi \rangle = -\cos(\theta_A - \theta_B)$.

One now assumes the existence of hidden variables $\lambda$ that determine the outcomes $\sigma_A$, $\sigma_B = \pm 1$ along $\theta_A$, $\theta_B$. It is assumed that there exists a ‘quantum equilibrium ensemble’ of $\lambda$ – that is, a distribution $\rho_{eq}(\lambda)$ that reproduces the quantum statistics (where $\int d\lambda \rho_{eq}(\lambda) = 1$). Each value of $\lambda$ determines a definite pair of outcomes $\sigma_A$, $\sigma_B$ (for given angles $\theta_A$, $\theta_B$): for an ensemble of similar experiments – in which the values of $\lambda$ generally differ from one run to the next – one obtains a distribution of $\sigma_A$, $\sigma_B$, which is assumed to agree with quantum theory. In particular, the expectation value $\overline{\sigma_A \sigma_B} = \int d\lambda \rho_{eq} \sigma_A \sigma_B$ must reproduce the quantum result $\langle \hat{\sigma}_A \hat{\sigma}_B \rangle = -\cos(\theta_A - \theta_B)$. Bell was able to show that this is possible only if one has nonlocal equations $\sigma_A = \sigma_A(\theta_A, \theta_B, \lambda)$, $\sigma_B = \sigma_B(\theta_A, \theta_B, \lambda)$, in which the outcomes depend on the distant angular settings [1, 17].

**General Signal-Locality Theorem:** Now given a distribution $\rho_{eq}(\lambda)$, one can always contemplate – purely theoretically – a ‘nonequilibrium’ distribution $\rho(\lambda) \neq \rho_{eq}(\lambda)$, even if one cannot prepare such a distribution in practice. For example, given an ensemble of values of $\lambda$ with distribution $\rho_{eq}(\lambda)$, mathematically one could pick a subensemble such that $\rho(\lambda) \neq \rho_{eq}(\lambda)$.

The theorem to be proved is then the following: in general, there are instantaneous signals at the statistical level if and only if the ensemble is in quantum nonequilibrium $\rho(\lambda) \neq \rho_{eq}(\lambda)$.

**Proof:** Consider an ensemble of experiments, with fixed angular settings $\theta_A$, $\theta_B$, and a ‘quantum equilibrium’ distribution $\rho_{eq}(\lambda)$ of hidden variables $\lambda$ that reproduces quantum statistics. In each experiment, a particular value of $\lambda$ determines an outcome $\sigma_A = \sigma_A(\theta_A, \theta_B, \lambda)$ at $A$. Some values of $\lambda$ yield $\sigma_A = +1$, some yield $\sigma_A = -1$. What happens if the setting $\theta_B$ at $B$ is changed to $\theta'_B$?
In other words: under a shift \( \theta \) contradicts Bell’s theorem. Thus in general otherwise the outcomes \( \sigma \) make the ‘transition’ \( \sigma \) yield (nonequilibrium) ratio of outcomes \( \lambda \) of the equilibrium ensemble of values of \( \lambda \) would have yielded the outcome \( \sigma \) at \( A \) now yield \( \sigma \) = +1; and some \( \lambda \) that would have yielded \( \sigma \) = +1 now yield \( \sigma \) = −1.

Of the equilibrium ensemble with distribution \( \rho_{eq}(\lambda) \), a fraction

\[
\nu_{eq}^{\sigma_{A}(-, +)} = \int_{S_+ \cap S'_+} d\lambda \rho_{eq}(\lambda)
\]

make the nonlocal ‘transition’ \( \sigma_A = -1 \rightarrow \sigma_A = +1 \) under the distant shift \( \theta_B \rightarrow \theta_B' \). Similarly, a fraction

\[
\nu_{eq}^{\sigma_{A}(+, -)} = \int_{S_+ \cap S'_-} d\lambda \rho_{eq}(\lambda)
\]

make the ‘transition’ \( \sigma_A = +1 \rightarrow \sigma_A = -1 \) under \( \theta_B \rightarrow \theta_B' \).

Now with the initial setting \( \theta_A, \theta_B \), quantum theory tells us that one half of the equilibrium ensemble of values of \( \lambda \) yield \( \sigma_A = +1 \) and the other half yield \( \sigma_A = -1 \). (That is, the equilibrium measures of \( S_+ \) and \( S_- \) are both 1/2.) With the new setting \( \theta_A, \theta_B' \), quantum theory again tells us that one half yield \( \sigma_A = +1 \) and the other half yield \( \sigma_A = -1 \) (the equilibrium measures of \( S'_+ \) and \( S'_- \), again being 1/2). The 1:1 ratio of outcomes \( \sigma_A = \pm 1 \) is preserved under the shift \( \theta_B \rightarrow \theta_B' \), from which we deduce the condition of ‘detailed balancing’

\[
\nu_{eq}^{\sigma_{A}(-, +)} = \nu_{eq}^{\sigma_{A}(+, -)}.
\]

The fraction of the equilibrium ensemble that makes the transition \( \sigma_A = -1 \rightarrow \sigma_A = +1 \) must equal the fraction that makes the reverse transition \( \sigma_A = +1 \rightarrow \sigma_A = -1 \).

But for an arbitrary nonequilibrium ensemble with distribution \( \rho(\lambda) \neq \rho_{eq}(\lambda) \), the ‘transition sets’ \( S_- \cap S'_+ \) and \( S_+ \cap S'_- \) will generally have different measures

\[
\int_{S_- \cap S'_+} d\lambda \rho(\lambda) \neq \int_{S_+ \cap S'_-} d\lambda \rho(\lambda)
\]

and the nonequilibrium transition fractions will generally be unequal, \( \nu_{A}^{\sigma_{A}(-, +)} \neq \nu_{A}^{\sigma_{A}(+, -)} \). Thus, if with the initial setting \( \theta_A, \theta_B \) we would have obtained a certain (nonequilibrium) ratio of outcomes \( \sigma_A = \pm 1 \) at \( A \), with the new setting
\(\theta_A, \theta'_B\), we will obtain a different ratio at \(A\). Under a shift \(\theta_B \rightarrow \theta'_B\), the number of systems that change from \(\sigma_A = +1\) to \(\sigma_A = -1\) is unequal to the number that change from \(\sigma_A = -1\) to \(\sigma_A = +1\), causing an imbalance that changes the outcome ratios at \(A\). In other words, the statistical distribution of outcomes at \(A\) is altered by the distant shift \(\theta_B \rightarrow \theta'_B\) [18].

**Quantitative Nonlocality:** The possibility of nonlocal signalling depends on the existence of finite ‘transition sets’ \(S_\pm \cap S'_\pm\) and \(S_+ \cap S'_-\). How large can the signal be? Clearly, the signal vanishes in equilibrium \(\rho(\lambda) = \rho_{eq}(\lambda)\), while if \(\rho(\lambda)\) is concentrated on just one transition set then all the outcomes at \(A\) change. Thus the size of the signal – measured by the fraction of outcomes that change at \(A\) – can range from 0\% to 100\%.

Now Bell’s theorem guarantees that the transition sets have non-zero equilibrium measure: otherwise, as we have seen, we would in effect have a local theory. However, so far we have no idea how large the transition sets are. We know only that, by a detailed-balancing argument, their equilibrium measures must be equal. Their size is important because if the transition sets have a very tiny equilibrium measure, then to obtain an appreciable signal the nonequilibrium distribution \(\rho(\lambda) \neq \rho_{eq}(\lambda)\) would have to be very far from equilibrium – that is, concentrated on a very tiny (with respect to the equilibrium measure) set. We shall therefore try to deduce the equilibrium measure of the transition sets \(S_\pm \cap S'_\pm\) and \(S_+ \cap S'_-\).

In other words, we now ask the following quantitative question: for an equilibrium distribution \(\rho_{eq}(\lambda)\) of hidden variables, what fraction of outcomes at \(A\) are changed by the distant shift \(\theta_B \rightarrow \theta'_B\)?

The quantity \(\alpha \equiv \nu_{eq}^{\alpha}(-,+) + \nu_{eq}^{\alpha}(+,-)\) is the fraction of the equilibrium ensemble for which the outcomes at \(A\) are changed under \(\theta_B \rightarrow \theta'_B\) (irrespective of whether they change from \(-1\) to \(+1\) or vice versa, the fractions doing each being \(\alpha/2\)). There is a ‘degree of nonlocality’ – quantified by \(\alpha\) – and Bell’s theorem tells us that \(\alpha > 0\). A positive lower bound on \(\alpha\) may be obtained, if one assumes a certain symmetry.

First, we derive a general lower bound for the quantity \(\alpha + \beta\), where \(\beta \equiv \nu_{eq}^{\beta}(-,+) + \nu_{eq}^{\beta}(+,-)\) is the equilibrium fraction of outcomes that change at \(B\), under the local shift \(\theta_B \rightarrow \theta'_B\) (with \(\nu_{eq}^{\beta}(-,+)\) and \(\nu_{eq}^{\beta}(+,-)\) defined similarly to \(\nu_{eq}^{\alpha}(-,+)\) and \(\nu_{eq}^{\alpha}(+,-)\) above). In other words, we obtain a lower bound on the sum of the nonlocal and local effects of \(\theta_B \rightarrow \theta'_B\).

**General Lower Bound:** The quantity \(\frac{1}{2} | \sigma_A(\theta_A, \theta'_B, \lambda) - \sigma_A(\theta_A, \theta_B, \lambda) |\) equals 1 if the outcome \(\sigma_A\) changes under \(\theta_B \rightarrow \theta'_B\), and vanishes otherwise. Since \(\rho_{eq}(\lambda) \, d\lambda\) is by definition the fraction of the equilibrium ensemble for which \(\lambda\) lies in the interval \((\lambda, \lambda + d\lambda)\), the fraction for which \(\sigma_A\) changes is

\[
\alpha = \frac{1}{2} \int d\lambda \rho_{eq}(\lambda) | \sigma_A(\theta_A, \theta'_B, \lambda) - \sigma_A(\theta_A, \theta_B, \lambda) |
\]

Similarly, the fraction for which \(\sigma_B\) changes is

\[
\beta = \frac{1}{2} \int d\lambda \rho_{eq}(\lambda) | \sigma_B(\theta_A, \theta'_B, \lambda) - \sigma_B(\theta_A, \theta_B, \lambda) |
\]
Now
\[-\cos(\theta_A - \theta_B) = \int d\lambda \rho_{eq}(\lambda)\sigma_A(\theta_A,\theta_B,\lambda)\sigma_B(\theta_A,\theta_B,\lambda)\]
and so
\[
|\cos(\theta_A - \theta_B') - \cos(\theta_A - \theta_B)| \leq \int d\lambda \rho_{eq}(\lambda)\left|\sigma_A(\theta_A,\theta_B',\lambda)\sigma_B(\theta_A,\theta_B,\lambda) - \sigma_A(\theta_A,\theta_B,\lambda)\sigma_B(\theta_A,\theta_B,\lambda) + \sigma_A(\theta_A,\theta_B,\lambda)\sigma_B(\theta_A,\theta_B',\lambda) - \sigma_A(\theta_A,\theta_B',\lambda)\sigma_B(\theta_A,\theta_B,\lambda)\right|
\]
\[
= \int d\lambda \rho_{eq}(\lambda)\left|\sigma_A(\theta_A,\theta_B',\lambda)\sigma_B(\theta_A,\theta_B',\lambda) - \sigma_A(\theta_A,\theta_B,\lambda)\sigma_B(\theta_A,\theta_B,\lambda)\right|
\]
\[
+ \int d\lambda \rho_{eq}(\lambda)\left|\sigma_B(\theta_A,\theta_B',\lambda) - \sigma_B(\theta_A,\theta_B,\lambda)\right|
\]
\[
= 2\alpha + 2\beta
\]
Thus we have the lower bound
\[
\alpha + \beta \geq \frac{1}{2}|\cos(\theta_A - \theta_B') - \cos(\theta_A - \theta_B)| \tag{1}
\]

The maximum value of the right hand side is 1 (for example with \(\theta_A = 0, \theta_B = 0, \theta_B' = \pi\)). From this inequality alone, then, one could have \(\alpha\) arbitrarily close to zero, with \(\beta \to 1\) – that is, an arbitrarily small fraction could change at \(A\) in response to \(\theta_B \to \theta_B'\), provided virtually all the outcomes change at \(B\). So far, then, we have no lower bound on the nonlocal effect, as quantified by \(\alpha\): it is only the sum \(\alpha + \beta\) of the nonlocal and local effects that is bounded.

**Symmetric Case**: A lower bound on \(\alpha\) may be obtained if we assume (i) an appropriate rotational symmetry at the hidden-variable level, and (ii) that the measurement operations at \(A\) and \(B\) are identical – that is, use identical equipment and coupling – so that there is symmetry between the two wings.

For consider the effect of a shift \(\theta_B \to \theta_B' = \theta_B + \delta\) at \(B\). This changes certain fractions \(\alpha\) and \(\beta\) of the (equilibrium) outcomes at \(A\) and \(B\) respectively. Let us assume that the same changes are effected by the shift \(\theta_A \to \theta_A' = \theta_A - \delta\) at \(A\) [19]. Then if \(\hat{\beta}\) is the fraction of outcomes at \(B\) that change in response to a shift \(\theta_A \to \theta_A' = \theta_A - \delta\) at \(A\), we have \(\hat{\beta} = \beta\). Assuming further an exchange symmetry between \(A\) and \(B\) – specifically, that the effect at \(A\) of a shift \(\theta_B \to \theta_B' = \theta_B + \delta\) at \(B\) equals the effect at \(B\) of a shift \(\theta_A \to \theta_A' = \theta_A - \delta\)
at $A$ – we also have $\alpha = \hat{\beta}$. Thus $\beta = \alpha$ and we obtain a lower bound on $\alpha$ alone

$$\alpha \geq \frac{1}{4} |\cos(\theta_A - \theta'_B) - \cos(\theta_A - \theta_B)| \quad (2)$$

For $\theta_A = \theta_B = 0$, this simplifies to

$$\alpha \geq \frac{1}{4}(1 - \cos \theta'_B) \quad (3)$$

If the measurement angle at $B$ is shifted by $\pi/2$, at least 25% of the outcomes change at $A$; if the angle is shifted by $\pi$, at least 50% change at $A$.

Clearly, in this symmetric case, the transition sets are necessarily very large, and even a mild nonequilibrium $\rho(\lambda) \neq \rho_{eq}(\lambda)$ will entail a significant signal.

**Comparison with Pilot-Wave Theory:** Using the theory of spin measurements due to Bell [20], it may be shown that the above bounds are satisfied – and indeed saturated – by pilot-wave theory [21]. At each wing $A$ and $B$ there is an apparatus coupled to the particle. At $t = 0$ the pointer positions $r_A$ and $r_B$ have localised wavepackets centred at $r_A, r_B = 0$. During the measurement, the packets move ‘up’ or ‘down’ (that is, towards positive or negative values of $r_A, r_B$), indicating an outcome of spin up or down. For an initial wavefunction $\psi_{ij}(r_A, r_B, 0) = \phi(r_A)\phi(r_B)a_{ij}$ – where the indices $i,j = \pm$ denote spin up or down at $A, B$ – the pointers are initially independent but in the singlet state the spins are entangled $a_{ij} \neq b_{c_j}$. One may consider an ideal von Neumann measurement with couplings $g_A(t)$ and $g_B(t)$ at $A$ and $B$ [20].

The quantum equilibrium probability distribution $\rho_{eq}(r_A, r_B, t)$ has associated probability currents $j_A$ and $j_B$. The hidden-variable pointer positions $r_A(t), r_B(t)$ have velocities $v_{A,B} = j_{A,B}/\rho_{eq}$. During the measurement, the packets $\psi_{ij}$ separate in configuration space, the actual configuration $(r_A(t), r_B(t))$ ending up in only one of them – yielding a definite outcome. How do the outcomes $\sigma_A, \sigma_B$ depend on the hidden variables $r_A(0), r_B(0)$, and how many outcomes change under $\theta_B \rightarrow \theta'_B$ [22]?

For the symmetric case, the couplings at $A$ and $B$ are equal: we take $g_A = g_B = a\theta(t)$. Taking square initial pointer packets $\phi$, we find that in the $r_A - r_B$ plane all points above the line $r_B = r_A$ (plotting $r_B$ as ordinate and $r_A$ as abscissa) end up in the packet $\psi_{+}$, yielding outcomes $\sigma_A = -1, \sigma_B = +1$; while those below that line end up in $\psi_{-}$, yielding $\sigma_A = +1, \sigma_B = -1$. If we change the axis of measurement at $B$ to $\theta'_B = \pi/2$, all four packets $\psi'_{ij}$ contribute, each moving into a different quadrant of the $r_A - r_B$ plane. Inspection shows that a point $(r_A, r_B)$ initially in the top-right quadrant ends up in $\psi'_{++}$, yielding $\sigma_A = +1, \sigma_B = +1$. Similarly, points in the bottom-right quadrant yield $\sigma_A = +1, \sigma_B = -1$; those in the top-left yield $\sigma_A = -1, \sigma_B = +1$; and those in the bottom-left yield $\sigma_A = -1, \sigma_B = -1$.

Clearly, some initial points $(r_A, r_B)$ yield the same outcomes with both settings $\theta_A = 0, \theta_B = 0$ and $\theta_A = 0, \theta'_B = \pi/2$, while others do not. A simple count shows that 25% of the outcomes change at $A$ and 25% change at $B$. Thus, under the angular shift at $B$ by $\pi/2$, the fraction $\alpha = 0.25$ that change at $A$ is
indeed equal to the fraction $\beta = 0.25$ that change at $B$. Further, $\alpha$ satisfies the bound (3) – indeed, the bound is exactly saturated. For a general shift $\theta_B \to \theta_B'$, it is similarly found that $\alpha = \beta$ and $\alpha = \frac{1}{4}(1 - \cos \theta_B')$. Thus, our inequality (3) is precisely saturated for all $\theta_B'$. In a precisely defined sense, pilot-wave theory is minimally nonlocal. (For details of the calculations, see ref. [21].)

For an asymmetric case where $g_A = a_A \theta(t)$, $g_B = a_B \theta(t)$ with $a_A = 2a_B$, the packets separate at different speeds along $r_A$ and $r_B$. With the settings $\theta_A = 0$, $\theta_B = 0$, the $r_A - r_B$ plane is divided in a more complicated way into points yielding $\sigma_A = -1$, $\sigma_B = +1$ and $\sigma_A = +1$, $\sigma_B = -1$. When the angle at $B$ is reset to $\theta_B' = \pi/2$, the four quadrants yield distinct outcomes as above. It is found that a fraction $\alpha = 1/8$ of outcomes change at $A$ and a fraction $\beta = 3/8$ change at $B$ [21]. Thus $\alpha \neq \beta$, as expected. The general bound (1) is satisfied, and exactly saturated. In the limit $a_A/a_B \to \infty$ – where the measurement at $A$ takes place much more rapidly than at $B$ (in the sense of rate of packet separation) – we find that $\alpha \to 0$, $\beta \to 1/2$ for the shift $\theta_B = 0 \to \theta_B' = \pi/2$ (again saturating (1)) [21]. Thus it is impossible to deduce a general lower bound on $\alpha$ alone without assuming symmetry between the two wings.

**Conclusion and Hypothesis:** Summarising, we have shown that in any deterministic hidden-variables theory that reproduces quantum statistics for some ‘equilibrium’ distribution $\rho_{eq}(\lambda)$ of hidden variables $\lambda$, a generic ‘nonequilibrium’ distribution $\rho(\lambda) \neq \rho_{eq}(\lambda)$ would give rise to instantaneous signals at the statistical level, as occurs in pilot-wave theory. For an equilibrium ensemble of EPR-experiments, assuming a certain symmetry we have derived a lower bound (2) on the fraction of systems whose outcomes change under a shift in the distant measurement setting. This bound is satisfied by pilot-wave theory.

Bell’s theorem tells us that if hidden variables exist then so do instantaneous influences. But there is no consensus on what to conclude from this. Similarly, one must distinguish between what has been proved above and what this author proposes to conclude from it.

It seems mysterious that nonlocality should be hidden by an all-pervading quantum noise. We have shown that any deviation from that noise would make nonlocality visible. It is as if there is a conspiracy in the laws of physics that prevents us from using nonlocality for signalling. But another way of looking at the matter is to suppose that our universe is in a state of statistical equilibrium at the hidden-variable level, a special state in which nonlocality happens to be hidden. The physics we see is not fundamental; it is merely a phenomenological description of an equilibrium state [16].

This view is arguably supported by quantum field theory in curved spacetime, where there is no clear distinction between quantum and thermal fluctuations [23]. On this basis it has been argued that quantum and thermal fluctuations are really the same thing [24]. This suggests that quantum theory is indeed just the theory of an equilibrium state, analogous to thermal equilibrium.

On this view it is natural to suppose that the universe may have begun in a state of quantum nonequilibrium $\rho(\lambda) \neq \rho_{eq}(\lambda)$, where nonlocal signalling was possible, the relaxation $\rho(\lambda) \to \rho_{eq}(\lambda)$ taking place during the great violence of the big bang [5, 10, 14, 15, 16]. In effect, a hidden-variables analogue of the
classical thermal heat death has actually occurred in our universe. This hypothesis could have observable consequences. In cosmological inflationary theories, early corrections to quantum vacuum fluctuations would change the spectrum of primordial density perturbations imprinted on the cosmic microwave background [10]. And particles that decoupled at sufficiently early times could still be in quantum nonequilibrium today: thus, exotic particles left over from the very early universe might violate quantum mechanics [10, 14, 15].

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[1] J.S. Bell, Physics 1, 195 (1965). It is assumed in particular that there is no ‘conspiracy’ or common cause between the hidden variables and the measurement settings, and that there is no backwards causation (so that the hidden variables are unaffected by the future outcomes). Bell’s original paper addressed only the deterministic case. The later generalisations to stochastic theories are of no concern here.


[17] Here $\lambda$ are the initial values of the hidden variables, for example just after the source has produced the singlet pair. Their later values may be affected by changes in $\theta_A, \theta_B$, and writing $\sigma_A = \sigma_A(\theta_A, \theta_B, \lambda)$, $\sigma_B = \sigma_B(\theta_A, \theta_B, \lambda)$ (where $\lambda$ are initial values) allows for this. See ref. [4], chapter 8.

[18] The signal vanishes for special $\rho(\lambda) \neq \rho_{eq}(\lambda)$ that happen to have equal measures for the transition sets, but not in general.

[19] This assumption refers to the hidden-variable level. (The quantum singlet state is of course rotationally invariant.)


[22] The outcomes $\sigma_A, \sigma_B$ are determined by the initial wavefunction $\psi_{ij}(0)$ and pointer positions $r_A(0), r_B(0)$, so $\lambda = (\psi_{ij}(0), r_A(0), r_B(0))$. Because $\psi_{ij}(0)$ is the same in every run of the experiment, there is no need to consider it.
