Trans-Relativistic Supernovae, Circumstellar Gamma-Ray Bursts, and Supernova 1998bw

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Abstract. Supernova (SN) 1998bw and gamma-ray burst (GRB) 980425 offer the first direct evidence that supernovae are the progenitors of some GRBs. However, this burst was unusually dim, smooth and soft compared to other bursts with known afterglows. Whether it should be considered a prototype for cosmological GRBs depends largely on whether the supernova explosion and burst were asymmetrical or can be modeled as spherical. We address this question by treating the acceleration of the supernova shock in the outermost layers of the stellar envelope, the transition to relativistic flow, and the subsequent expansion (and further acceleration) of the ejecta into the surrounding medium. We find that GRB 980425 could plausibly have been produced by a collision between the relativistic ejecta from SN 1998bw and the star’s pre-supernova wind; the model requires no significant asymmetry. This event therefore belongs to a dim subclass of GRBs and is not a prototype for jet-like cosmological GRBs.

INTRODUCTION

A growing body of indirect evidence links some long duration gamma-ray bursts with regions of recent star formation, and therefore with the core collapse of massive stars (e.g., [1]). The most direct evidence of such a link is provided by the probable association [2, 3] of GRB 980425 with SN 1998bw. However, at the distance of the supernova, this burst was six orders of magnitude dimmer than the brightest of cosmological bursts ($10^{54}$ ergs in $g$-ray isotropic equivalent energy). Should this burst be considered the first of a new class of weak, supernova-related GRBs (as proposed by Bloom et al. 1998 [4]), or should it be counted among those events that could produce strong cosmological GRBs? Central to this question is the degree of asymmetry that must be invoked to understand the supernova and its GRB. Evidence of large-scale asymmetry suggests a jet-like explosion of the core, which is considered a necessary ingredient for cosmological bursts if they involve internal shocks within high Lorentz factor flows [27] from the cores of stars [5]. If instead the event is consistent with spherical symmetry, then it should be inadmissible as evidence of a causal relation between SNe and any model for GRBs requiring a jet.

As a spherical explosion, SN 1998bw possessed about $3 \times 10^{52}$ erg of kinetic energy, thirty times more than what is typical of supernovae. Höflich, Wheeler & Wang [6] argue that SN 1998bw may have been an asymmetric explosion on the basis that this would allow a lower explosion energy, and Nakamura et al. [7] find evidence for asymmetry of the inner ejecta in the late decay of the supernova light curve. Note, however, that the polarization of light from supernova 1993J suggested asymmetry of its inner ejecta [8], whereas radio emission from its outermost ejecta [9] shows no asymmetry; moreover, SN 1998bw exhibited lower polarization than did SN 1993J and most type II supernovae [10]. More compelling would be evidence that the observed GRB originated in a highly asymmetrical event. In this regard, [11], [12], and [13] advocate a scenario in which a beamed, highly relativistic outflow is viewed off-axis to produce GRB 980425.

The competing, more conservative hypothesis construes the burst as the earliest phase of interaction between high-velocity (spherical) stellar ejecta and progenitor star’s wind – the same interaction that gave rise to the later radio emission [14]. This possibility is similar to the suggestion of Colgate [15] that GRBs might be due to shock breakout in supernovae. In this model, the energy that emerged as gamma rays was previously locked up in the kinetic energy of expanding ejecta. Even at the distance of 1998bw, a burst of GRB 980425’s brightness probably required (mildly) relativistic motion in order to avoid excessive self-opacity. Corroborating evidence comes from the very high mean velocity inferred for the
supernova’s synchrotron shell \((c/3 \text{ at 12 days; Kulkarni et al. [16]})\). To assess the viability of a spherical model for GRB 980425, we require:

1. an estimate of the minimum acceptable Lorentz factor that could have produced the GRB;
2. an investigation of whether a model for the supernova explosion that accounts for the optical emission can simultaneously produce sufficient kinetic energy in material above this Lorentz factor; and
3. a determination of whether the pre-supernova stellar wind was dense enough to convert the kinetic energy into gamma rays (without absorbing them) in the duration of the burst.

**MINIMUM LORENTZ FACTOR OF GRB 980425**

The minimum necessary Lorentz factor for GRB 980425 was considered by Lithwick & Sari [17], who found it to be at least 3.8 in order for the burst not to be obscured by electron-positron pairs produced by its radiation field. However, this analysis relied on a power law extrapolation of the observed gamma ray spectrum to energies (in the comoving frame) above \(m_ec^2\). [17] considered spectral slopes no steeper than \(-3 (d \log N_{\gamma}/d \log E_{\gamma})\); further, they adopted \(m_ec^2\) as the maximum observed photon energy. In contrast, the BATSE light curve for this burst exhibited a 37-\(\sigma\) detection in the 50-100 keV channel, 20-\(\sigma\) detection in the 100 – 300 keV channel, and no detection at all (\(< 1-\sigma\)) in the > 300 keV channel. These observations give no evidence for photons with energies above \(m_ec^2\). Interpreted as a power law, the highest two channels give a slope of \(-4\) or steeper; however, they are more suggestive of a spectral cutoff (likely a dilute Wien spectrum; C. Thompson, private communication, 2001) than a power law.

Another estimate of the minimum Lorentz factor, and one that does not depend on the specifics of the emission mechanism, arises from the requirement that \(10^{48}\) ergs of gamma rays be produced in an interaction between stellar ejecta and the pre-supernova stellar wind. For mean ejecta Lorentz factor \(\Gamma\), the wind mass must be about \(1/\Gamma\) of the ejecta mass. This mass of wind must be found in a radius that is roughly \(2\Gamma^2c\) times the observed duration of the burst (\(\sim 15\) seconds). But, the wind cannot be opaque at this radius. Applied to the parameters of GRB 980425, these considerations (including the difference between the velocity of the ejecta and that of the emitting swept-up shell, and the Klein-Nishina opacity correction) give \(\Gamma > 1.9\), roughly; see [18] for a more thorough discussion. Both estimates of the minimum Lorentz factor merit further investigation, preferably careful modeling of both the dynamical interaction and the emission mechanism; we shall adopt the latter as the more robust estimate.

**RELATIVISTIC EJECTA FROM SN 1998BW**

As a supernova explosion engulfs a star’s envelope, the velocity of its leading shock front responds to two competing trends: a general deceleration as increasing mass is swept up, and a tendency to accelerate down any sharply declining density gradient (in a manner analogous to the cracking of a whip). Matzner & McKee [19] have shown that these trends can be combined into a single formula that tracks the behavior seen in numerical simulations remarkably well. After the shock emerges from the stellar surface, the shocked material accelerates further as its residual heat is converted into kinetic energy. The highest velocity attained by the ejecta is set by the fact that the shock front spans a finite optical depth; the star must therefore be relatively compact or have an energetic explosion in order to produce any relativistic ejecta. Matzner & McKee determined that a compact Wolf-Rayet star would most likely satisfy this criterion. Although their formulae did not address relativistic motion, they were able to estimate the kinetic energy in relativistic ejecta by evaluating their formulae at a final velocity of \(c\). This estimate illustrated that an explosion like that of SN 1998bw would indeed produce of order \(10^{48}\) erg in relativistic ejecta, roughly enough to power GRB 980425.

Woosley, Eastman & Schmidt [20] considered the production of relativistic ejecta in the context of specific models developed to fit the light curve of SN 1998bw. The most promising of these is the 6 \(M_\odot\) CO core of a ~25 \(M_\odot\) main-sequence star, exploding with \(2.8 \times 10^{52}\) ergs of final kinetic energy. Woosley et al. used the theory of Gnatyk [21] to extrapolate their nonrelativistic simulations into the relativistic regime. They concluded that the supernova could not have powered GRB 980425; however, this conclusion was flawed on several counts. First, Gnatyk’s formula (an interpolation between non-relativistic [22] and relativistic [23] scaling laws) was of unknown validity. Second, and much more importantly, Woosley et al. made an allowance for the postshock acceleration that was valid in the nonrelativistic regime (in which the four-velocity \(\Gamma\) of a fluid element increases by a factor 2.5), but did not account for the very different character of this acceleration found by Johnson & McKee [23] for relativistic flow (in which \(\log(\Gamma)\) nearly quadruples). This led them to predict a much steeper decline of kinetic energy with increasing Lorentz fac-
tor than actually holds. Lastly, Woosley et al. assumed that the minimum Lorentz factor was at least about 5, whereas we have argued above that this value is not supported by observations and the lower value of $\sim 1.9$ is more appropriate.

To put the theory of this burst on a more solid footing, Tan, Matzner & McKee [18] have considered in detail the evolution of explosions involving a transition from nonrelativistic to relativistic motion. Among the results of this investigation are:

- An extension Matzner & McKee’s analytical theory for the shock velocity into the relativistic regime, more precisely than in Gnatyk’s theory;
- Likewise for the postshock acceleration of fluid elements to their final velocities;
- Formulae for the resulting distribution of kinetic energy among ejecta of different final velocities and Lorentz factors;
- An analysis of what aspects of stellar envelopes enhance the efficiency with which they produce relativistic ejecta;
- Simple formulae for the yield of relativistic ejecta from stars with radiative outer envelopes, in terms of gross parameters like mass, radius, luminosity and composition;
- Formulae to predict the relativistic ejecta in different directions for numerical simulations of asymmetrical explosions (including ejecta produced by shock acceleration in beamed and jet-like events, which could give rise to GRB precursors [5]);
- Generalization to the collapses of compact objects (e.g., accretion-induced collapse of white dwarfs) in which gravity sets the characteristic ejecta velocities; and
- A consideration of the dynamics of putative “hypernova” explosions of very high explosion energy.

These analytical results were verified and calibrated by means of well-resolved, relativistic numerical simulations in spherical and planar symmetry.

Tan et al. verified Matzner & McKee’s prediction of the kinetic energy in relativistic ejecta, demonstrating that this energy is associated with ejecta moving with $\Gamma_{\gamma} > 1.41$. Applying their results to Woosley, Eastman & Schmidt’s model CO6 (kindly provided by Stan Woosley), Tan et al. find that the energy of GRB 980425 emerged in material whose minimum Lorentz factor was 1.7, for which $\Gamma_{\gamma} = 2$. Coupled with the minimum Lorentz factor identified above, this confirms Matzner & McKee’s prediction that SN 1998bw produced enough energy in relativistic ejecta to have powered GRB 980425. For higher Lorentz factors, Tan et al. predict a decline in kinetic energy roughly as $E_k(\Gamma_{\gamma}) \propto 1/\Gamma_{\gamma}$ because of dramatic postshock acceleration in the relativistic regime. This relatively shallow decline indicates that explosions that can produce any relativistic ejecta also channel significant energy into ultrarelativistic motion.

CIRCUMSTELLAR MATERIAL

As discussed above, the interaction that gives rise to the GRB occurs at a radius that is roughly $2\Gamma^2 c t_{\text{obs}}$. Within this radius, a mass $E_r/[c^2 \Gamma (\Gamma - 1)]$ of circumstellar material must be found. The circumstellar material must therefore have a mass per unit radius of $E_r/[2c^4 t_{\text{obs}} \Gamma^3 (\Gamma - 1)]$. The lower limit on $\Gamma$ thus puts an upper limit on the mass per unit length in the circumstellar material; for a stellar wind, this is the ratio of mass loss rate to wind velocity. Evaluated for the parameters of GRB980425, the maximum value of this ratio (attained for the minimum Lorentz factor) is $(3 \times 10^{-4} M_{\odot}/\text{yr})/(1000 \text{ km/s})$, which is dense but within the range of Wolf-Rayet (WC subclass [24]) winds – especially considering that the circumstellar material involved in the GRB was emitted in the last ten hours of the star’s life.

The radio afterglow from SN 1998bw provides a consistency check on any model in which the GRB arises from an early circumstellar interaction. Applying the theory of Chevalier [25] and Nadyozhin [26] to the collision of the nonrelativistic ejecta with the circumstellar wind, Tan et al. find a mean expansion velocity of $0.35 c$ for the first 12 days of this interaction – in excellent agreement with the value $0.3 c$ given by Kulkarni et al. [16]. Similar agreement is found with the detailed modeling of the circumstellar interaction by Li & Chevalier [14].

CONCLUSIONS

We have argued that both the gamma-ray burst and the later radio emission associated with supernova 1998bw can be explained in the context of a spherical model for its explosion – the same spherical model that was proposed by Woosley, Eastman, & Schmidt [20] to explain its light curve. The only additional element that must be included is a relatively dense circumstellar wind, but one within the range observed around Wolf-Rayet stars. The viability of a spherical for GRB 980425 casts significant doubt on the hypothesis that GRB 980425 was intimately related to beamed cosmological bursts. Specifically, there is no evidence for a jet of high Lorentz factor material.

The model we have advocated for GRB 980425 is an external shock model; Sari & Piran [27] have shown (under the assumption of relativistic motion) that such
models can be ruled out for GRBs composed of multiple sub-bursts. However, GRB 980425 exhibited only one smooth pulse, and is consistent with mildly relativistic motion; therefore, the external shock model is tenable.

Tan et al.’s analysis demonstrates that the fraction of a supernova’s energy that winds up in relativistic ejecta is enhanced if the stellar atmosphere is as diffuse as possible compared to its core. Stars whose luminosity is comparable to the Eddington limit are ideal in this regard. A high explosion energy and low envelope mass are even more important, as the energy in relativistic motion scales as $E^{3.6}M_{\text{env}}^{-2.6}$. Pre-explosion mass loss therefore enhances the possibility of a GRB both by increasing the amount of energy in relativistic ejecta, and by giving rise to the circumstellar wind necessary for converting this energy into gamma rays in a brief period.

Even if subsequent investigations indicate that we have underestimated the necessary Lorentz factor or total energy of the interaction that gave rise to GRB 980425, there are two arguments that the hypothesis of a circumstellar origin should not be abandoned. First, the kinetic energy drops only as $1/\Gamma$ to higher Lorentz factors; a higher minimum $\Gamma$ requires only a proportionally higher total energy in relativistic ejecta. Second, this total energy is quite sensitive to the explosion energy and envelope mass, the degree of central concentration of the star’s atmosphere, and any mild asymmetry that may have developed in the explosion. In the model considered, we identified $8 \times 10^{47}$ erg in material expanding with $\Gamma > 1.7$; however, small changes to the model would enhance (or reduce) this yield by factors of several (figure 8 of [18]).

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Final ejecta velocity $\Gamma_f \beta_f$

GRB energy $E_\gamma$

Cumulative energy (erg)

$10^{48}$

$10^{46}$

$10^{44}$

$10^{50}$

$10^0$ $10^1$ $10^2$

$\Gamma_f = 1.7$

FIGURE 2. Kinetic energy contained in ejecta traveling higher than a given final velocity, for the explosion depicted in Figure 1, according to the theory of Tan et al. [18]. The observed energy of GRB 980425 is realized in ejecta with final Lorentz factors above 1.7.

REFERENCES