Parton distribution functions in nuclei

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Optimum nuclear parton distributions are determined by an analysis of muon and electron deep inelastic scattering data. Assuming simple $A$ dependence and polynomial functions of $x$ and $1-x$ for nuclear modification of parton distributions, we determine the initial distributions by a $\chi^2$ analysis. Although valence-quark distributions are relatively well determined except for the small-$x$ region, antiquark distributions cannot be fixed at medium and large $x$. It is also difficult to fix gluon distributions.

1 Introduction

Although parton distribution functions in nuclei are often assumed to be equal to those in the nucleon, it is especially important to know the details of nuclear modification in order to find any exotic signature in hadron reactions. For example, we discuss a reaction such as $J/\psi$ production, which is very sensitive to nuclear gluon distributions, as a possible way to find a quark-gluon plasma signature. However, it is unfortunate that the gluon modification is essentially unknown at this stage although there are some implications from lepton deep inelastic data.

There were recent trials to obtain nuclear parton distributions from experimental data, for example, by Eskola, Kolhinen, and Ruuskanen.\textsuperscript{1} Their studies are valuable for providing possible nuclear modification from available data. However, the distributions should be optimized in principle by a $\chi^2$ analysis. Because there are not so many available experimental data in the nuclear case, it is obvious that such an effort is not an easy work at this stage. In Ref. 2, a possible method is developed as a first step trial for the nuclear $\chi^2$ analysis. This talk is based on this work. Our analysis method and results are explained in the following.

2 Analysis method

An important point in the analysis is how to set up a functional form of nuclear parton distributions. Nuclear modification of parton distributions is typically less than 20% for medium size nuclei, so that it is more appropriate
to parametrize the modification instead of the distributions themselves. The nuclear parton distributions are then assumed as

\[ f_i^A(x, Q_0^2) = w_i(x, A, Z) f_i(x, Q_0^2), \]

where \( f_i(x, Q_0^2) \) is the \( i \)-type parton distribution \((i = u, d, \bar{q}, g)\) in the nucleon at \( Q_0^2 \). The distributions in the nucleon are taken from the MRST parametrization.\(^3\) Nuclear antiquark distributions are assumed to be flavor symmetric: \( \bar{u}^A = \bar{d}^A = \bar{s}^A \) at \( Q_0^2 \). We call \( w_i(x, A, Z) \) a weight function, which is determined by a \( \chi^2 \) analysis.

Mass-number dependence of \( w_i \) could be complicated because different physics mechanisms contribute depending on the \( x \) region. On the other hand, it is necessary to simplify the problem as a first effort of nuclear \( \chi^2 \) analysis. Here, we decided to assume the \( A \) dependence as \( 1/A^{1/3} \) as suggested by Sick and Day.\(^4\) Then, the rest is taken as a polynomial function of \( x \) and \( 1-x \):

\[ w_i(x, A, Z) = 1 + \left( 1 - \frac{1}{A^{1/3}} \right) a_i(A, Z) + b_i x + c_i x^2 + d_i x^3 (1-x)^{\beta_i}, \]

where \( a_i, b_i, c_i, d_i, \) and \( \beta_i \) are parameters to be determined by the \( \chi^2 \) analysis. We call this function a “cubic type” and the function without the \( d_i x^3 \) term a “quadratic type”. Analyses are done for both cases.

There are restrictions on the nuclear parton distributions due to the following three conditions:

\begin{align*}
\text{Charge:} & \quad Z = \int dx \frac{A}{3} (2 u_v^A - d_v^A), \\
\text{Baryon Number:} & \quad A = \int dx \frac{A}{3} (u_v^A + d_v^A), \\
\text{Momentum:} & \quad A = \int dx A x (u_v^A + d_v^A + 6 \bar{q}^A + g^A),
\end{align*}

where the distributions are defined by the ones per nucleon. Because of these constraints, three parameters can be expressed by the others. In addition, we fixed some antiquark and gluon parameters which become relevant at large \( x \). Detailed explanations should be found in our first paper.\(^2\)

From the parton distributions in Eq.(1), the structure function \( F_2 \) is calculated by the leading-order expression: \( F_2^A(x, Q^2) = \sum q c_q^2 x [q^A(x, Q^2) + \bar{q}^A(x, Q^2)] \). In calculating \( F_2^A(x, Q^2) \), the initial distributions at \( Q_0^2 \) are evolved to \( Q^2 \) by the ordinary DGLAP evolution equations. Experimental data are taken from the measurements by the EMC, BCDMS, NMC, SLAC-E49, -E87, -E139, -E140, and Fermilab-E665 collaborations. The total number of the data is 309. Various nuclear targets are used experimentally. In our
theoretical analysis, they are assumed as $^4\text{He}$, $^7\text{Li}$, $^{12}\text{C}$, $^{14}\text{N}$, $^{27}\text{Al}$, $^{40}\text{Ca}$, $^{56}\text{Fe}$, $^{63}\text{Cu}$, $^{107}\text{Ag}$, $^{118}\text{Sn}$, $^{131}\text{Xe}$, $^{197}\text{Au}$, and $^{208}\text{Pb}$. In comparison with the experimental data, $\chi^2$ is calculated with the ratio $R_{F_2}^A = F_{2A}^2/F_{2D}^2$ as

$$\chi^2 = \sum_j \frac{(R_{F_2,\text{data}}^A - R_{F_2,\text{theo}}^A)^2}{(\sigma_{\text{data}}^j)^2}. \quad (6)$$

3 Results

Finding the minimum point of $\chi^2$, we determine the parameters. Obtained distributions are compared with the data in Figs.1, 2, and 3, where the dashed and solid curves indicate quadratic and cubic results, respectively. Because the theoretical curves are calculated at $Q^2=5\text{ GeV}^2$ and the data are taken at various $Q^2$ points, they cannot be compared directly. However, the figures seem to indicate reasonable agreement with the data. Obtained $\chi^2$ per degrees of freedom is given by $\chi^2_{\text{min}}/d.o.f.=1.93$ in the quadratic fit and 1.82 in the cubic fit. Because of the additional freedoms, $\chi^2$ is slightly better in the cubic analysis. Obtained $\chi^2_{\text{min}}$ values are not excellent in the sense $\chi^2_{\text{min}}/d.o.f. > 1$, but they are partly due to the scattered experimental data as obvious in Fig.1.

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Figure 1. Comparison with calcium data.

Figure 2. Comparison with iron data.

Figure 3. Comparison with gold data.

Figure 4. Weight functions in calcium.
In order to compare both analysis results, we show the weight functions of the calcium nucleus in Fig. 4. It indicates that the weight functions depend slightly on the assumed functional form. It is noteworthy to mention that the valence-quark distributions do not show any strong shadowing as $F_2$ or the antiquark distributions. It is not clear at this stage whether this is an artifact due to the lack of Drell-Yan data in our analysis. In our studies, we have just set up a method of nuclear $\chi^2$ fit. In future, we need to improve our method and to include other existing data. Nuclear parton distributions can be calculated by obtaining computer codes from our web site. The distributions are provided for nuclei from the deuteron to heavy ones. In addition, analytical expressions of the weight functions are given in Appendix of Ref. 2.

4 Summary

Using electron and muon deep inelastic scattering data, we have investigated optimum parton distributions in nuclei. Valence-quark distributions are well determined except for the small-$x$ region. Antiquark and gluon distributions cannot be determined well at medium and large $x$. The gluon distributions cannot be fixed because of a leading-order analysis and lack of sensitive data.

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References