Limits on the size of extra dimensions from LEP experiments

V. Zhuravlov

Joint Institute for Nuclear Research, Dubna

Abstract. We give the summary of the bounds on extra dimensions obtained from LEP running at energy above Z-pole. We present limits on “effective” Planck mass M_D and radius of extra dimensions from direct graviton production, limits on cut-off parameter M_s from virtual graviton exchange contribution and limits on compactification scale M_c from electroweak observables and Bhabha scattering.

I INTRODUCTION

The idea that we live in more than 3 spatial dimensions has a long history. It was born in the 20’s when Kaluza and Klein introduced one additional compact dimension. After that the possibility of extra dimensions was being considered by physicist for a long time, and in the 80’s the string theory appeared. The theory required ten or eleven dimensions compactified with the radius $\sim 1/M_{Planck}$. At the end of the 90’s it was recognized that the radius of compactification could be much higher than Planck scale, which gives an opportunity to experimentalists to test the consequences of extra dimensions at present colliders.

In this note we review the limits on extra dimensions for the following two theoretical models. The first proposed by Arkani-Hamed, Dimopoulos and Dvali assumes that only gravitons live in extra dimensions. In the second model gauge bosons are also allowed to propagate in extra dimensions. In the first model collider signatures can be divided into direct graviton production (γ and missing energy or Z and missing energy) and virtual graviton exchange in the fermion and boson pairs production. For the second model the constraints can be obtained from electroweak precision measurements and from Bhabha scattering mediated by Kaluza-Klein modes of gauge bosons.

II THEORETICAL MODELS

In the model proposed by Arkani-Hamed, Dimopoulos and Dvali [1] all Standard Model particles live in 3-brane of $(3+N)$ -space, while gravitons can propagate in

N extra dimensions. Extra dimensions should be hidden for us – and the good possibility to provide this is to compactify them into a torus. To obey the Newton law the radius of compactification should be less than 1 mm and the number of extra dimension should be greater than one. One of the main advantage of this model is a natural solution of “hierarchy problem”. One can introduce “effective” $(3+N)$ -dimensional Planck mass which obeys the relation

$$M_{Pl}^2 = (M_D)^{2+N} \cdot r^N$$

where M_{Pl} is the ordinary Planck mass and r is the radius of compact dimensions. The “effective” Planck mass can be chosen as small as $\sim 1\text{TeV}$ if number of extra dimensions greater than one.

Due to compact dimensions a tower of massive Kaluza-Klein (KK) gravitons arise with the mass spectrum

$$m_k = 2\pi \frac{k}{r}, k = 1, 2, \dots$$

and the usual small coupling to SM particles $\sim \frac{1}{M_{Pl}}$. Summation over all KK states gives the coupling $\sim \frac{1}{M_D}$ which is on the order of electroweak scale. Using this fact one can make an optimistic conclusion: present colliders are sensitive to extra dimensions and quantum gravity.

Massive KK states of gravitons can be directly produced by colliders. As gravitons escape detectors without an interaction the experimental signature is the missing energy. Another possibility to look for extra dimensions is measurements of cross section of fermion and boson pairs production, which is subject to the influence of virtual graviton exchange.

In the second theoretical model SM fermions live in 3-brane and SM bosons reside on a p -brane, where $3 \leq p \leq D$, D - full number of dimensions. As in the first model, gravitons are allowed to propagate in all dimensions. In this case the main influence of extra dimension arise from KK modes of gauge bosons with a common mass spectrum

$$M_k = kM_c, k = 1, 2, \dots$$

where $M_c = 1/r$ is compactification scale. The effect of gravity is negligible in comparison with the effect of gauge bosons. It was shown in [2] that gauge couplings could be unified at the TeV scale.

Experimental manifestation of this model at the present colliders can be detected as (i) deflection of electroweak precision measurements from their SM values due to mixing of “ordinary” gauge bosons with their KK excitations and (ii) modification of the cross section of fermion pair production due to exchange of a KK mode of γ and Z^0 .

Both models can be considered as realizations of the string theory and, in contrast to “conservative” string theory, the prediction of both models can be tested at present colliders and in the nearest future.

III EXPERIMENTAL SIGNATURES FOR ADD MODEL

A $e^+e^- \rightarrow \gamma G$

Experimental signature for this channel is one (or more) γ and missing energy. The same signature has Standard Model process $e^+e^- \rightarrow \nu\bar{\nu}\gamma(\gamma)$. The main background is produced by processes $e^+e^- \rightarrow e^+e^-\gamma$, where both electrons escape detection and $e^+e^- \rightarrow \gamma\gamma$ with one detected photon.

The differential cross section is given by [3]

$$\frac{d^2\sigma}{dx d\cos\theta} = \frac{\alpha}{32s} \frac{\pi^{N/2}}{\Gamma(N/2)} \left(\frac{\sqrt{s}}{M_D}\right)^{N+2} f(x, \cos\theta) \quad (1)$$

where

$$f(x, \cos\theta) = \frac{2(1-x)^{\frac{N}{2}-1}}{x(1-\cos^2\theta)} \times \\ [(2-x)^2(1-x+x^2) - 3x^2(1-x)\cos^2\theta - x^4\cos^4\theta] \\ x = \frac{E_\gamma}{E_{beam}}$$

The production of graviton and photon has been searched for at LEP II. Experiments used slightly different techniques to derive $e^+e^- \rightarrow \gamma G$ cross section. ALEPH collaboration made binned maximum likelihood fit of Standard Model prediction added to graviton production cross section to the 2-dimensional $(E_{miss}, \cos\theta)$ distribution for all γ candidates. Value of $(\frac{1}{M_D})^{N+2}$ was used as a fit parameter. DELPHI found the kinematic region which is more sensitive to the signal: $6 < E_\gamma < 50$ GeV for the barrel calorimeter and $18 < E_\gamma < 50$ GeV for the forward calorimeter. Then a comparison of the full number of observed events in the described kinematic region with the integrated predicted cross section has been done. L3 collaboration performed the fit of the SM prediction plus graviton production to the E_γ spectrum of γ candidates. OPAL used “cut based” method: cut $E_\gamma < 34$ GeV was applied to minimize expected upper limit on signal cross section. The results of LEP experiments in the form of 95% C.L. limits on the “effective” Planck mass M_D are presented in table 1

An illustration of ALEPH analysis is presented in fig. III A, where on the plot (a) the invariant mass distribution of the system recoiling against the γ candidate is shown and on the plot (b) the polar angle distribution of photon candidate is shown.

Fig. III A presents the 95% C.L. cross section limit for $e^+e^- \rightarrow \gamma G$ production and expected cross sections with 2 and 4 extra dimensions.

Value of $(\frac{1}{M_D})^{N+2}$ was used to find the average of the LEP experiments. Results of averaging and results of the LEP experiments are shown in fig. 3.

TABLE 1. 95% C.L. lower limits on mass scale M_D , TeV

		$N = 2$	$N = 4$	$N = 6$
ALEPH	189 - 202 GeV	1.10	0.70	0.52
DELPHI	192 - 202 GeV	1.250	0.792	0.57
L3	189 GeV	1.018	0.674	0.506
OPAL	189 GeV	1.086	0.710	0.528

95% C.L. lower limits on the “effective” Planck mass and upper limits on radius of compact dimensions derived from averaging of ALEPH, DELPHI and OPAL data are presented in table 2

TABLE 2. Average 95% C.L. limits from ALEPH 183 - 202 GeV, DELPHI 189 - 202 GeV and OPAL 189 GeV

	$N = 2$	$N = 4$	$N = 6$
M_D , TeV	1.29	0.80	0.58
r , cm	$2.9 \cdot 10^{-2}$	$1.4 \cdot 10^{-9}$	$4.5 \cdot 10^{-12}$

B $e^+e^- \rightarrow Z^0 G$

Gravitons in ADD model can also be produced in association with Z bosons, but phase space available for graviton is less then in the case of $e^+e^- \rightarrow \gamma G$ because of the Z boson mass. Therefore experimental sensitivity to extra dimensions in this channel is suppressed. Experimental signature of this processes is a single Z^0 and missing energy. A search for this processes done only by L3 collaboration with hadronic decayed Z’s. Obtained upper limits on the cross section and corresponding common radius of extra dimensions and lower limit on the “effective” Planck mass are shown in the table III B. Visible mass distribution of the candidates together with the Standard Model expectation and its modification by graviton production is shown in fig. 4.

C $e^+e^- \rightarrow Z^0 \rightarrow Z^* G_0$

Projection of the spin of a graviton propagating in extra dimensions onto world brane have three possible values: 1, 2 and 3. From the 3-dimensional point of view it means that we have three types of gravitons. Coupling of spin-0 gravitons to the Standard Model particles is proportional to their masses, spin-1 gravitons

FIGURE 1. a - the invariant mass distribution of the system recoiling against the γ candidate, b - the polar angle distribution of photon candidate. Points stand for real data, dashed histograms are Standard Model MC and open histograms corresponds to graviton production with two extra dimensions and “effective” Planck mass $M_D = 0.99 \text{ TeV}$.

are decoupled from matter and coupling of spin-2 gravitons is proportional to the energy-momentum tensor. The most prominent channel for a search for spin-0 gravitons at LEP is the graviton emission by Z boson: $e^+e^- \rightarrow Z \rightarrow Z^*G_0$ and $Z^* \rightarrow f\bar{f}$. Z branching ratio is proportional to $(M_Z/M_D)^{N+2}$, where N is the number of extra dimension. Topology of this channel is identical to invisible Higgs decay. Search for spin-0 gravitons has been done by ALEPH collaboration with LEP1 data. Hadronic decayed Z's were used, no candidate events were found and 1.4 events were expected from background. The branching ratios and limits on M_D from this analysis are shown in table 4

FIGURE 2. 95% C.L. cross section limit for $e^+e^- \rightarrow \gamma G$ production and expected cross sections for 2 and 4 extra dimensions

$$D \quad e^+e^- \rightarrow \gamma\gamma(\gamma)$$

Beside the direct production of gravitons extra dimensions can manifest themselves by modification of the cross sections of Standard Model processes by the graviton exchange. We start with two gamma process.

Modification of differential cross section by s-channel exchange of a spectrum of virtual gravitons is given by [4]

$$\frac{d\sigma(e^+e^- \rightarrow \gamma\gamma)}{d\cos\theta} = \frac{\pi}{s} \left[\alpha F_1\left(-\sin^2\frac{\theta}{2}\right) - \lambda \frac{4s^2}{\pi M_s^4} F_2\left(-\sin^2\frac{\theta}{2}\right) \right]^2$$

$$F_1(x) = \sqrt{\frac{1+x+2x^2}{-x(1+x)}} \quad (2)$$

$$F_2(x) = \sqrt{\frac{-x(1+x)(1+2x+2x^2)}{16}}$$

FIGURE 3. Value of $(\frac{1}{M_D})^{N+2}$ obtained from LEP experiments data and average over ALEPH, DELPHI and OPAL.

Cut off parameter M_s was introduced to regularise ultraviolet divergence due to infinite KK tower of massive gravitons, parameter λ describes the topology of extra dimensions and can not be calculated without full knowledge of the model. As

$$\lambda = \mathcal{O}(\infty)$$

we will consider two cases: $\lambda = 1$ with positive interference and $\lambda = -1$ with negative interference.

In table 5 we present the lower limits on cutoff parameter M_s obtained by LEP collaborations. Angular distribution of two gamma candidates obtained from data collected by L3 experiment is shown on fig. 5. Also shown are the Standard Model expectation and its distortion by the graviton exchange with $M_s = 0.65$ TeV.

$$\text{E} \quad e^+e^- \rightarrow \mu\mu, \tau\tau$$

Differential cross section of fermion pair production is given by [5]

TABLE 3. Upper limit on the cross section σ_{ZG} and r and lower limit on M_D , 95% C.L., 189 GeV L3 collaboration, hadronic Z^0 decays, $N = 2$

σ_{ZG} , pb	M_D , TeV	r , cm
0.29	0.42	0.27

TABLE 4. Branching ratios and 95% C.L. lower limits on M_D ALEPH collaboration (LEP1 data): Hadronic Z^0 decays

	$N = 2$	$N = 4$	$N = 6$
$BR(Z \rightarrow Z^* G_0)$,	$\frac{4.0 \cdot 10^{-8}}{M_D^4}$	$\frac{4.5 \cdot 10^{-11}}{M_D^6}$	$\frac{7.5 \cdot 10^{-14}}{M_D^8}$
M_D in TeV			
M_D (TeV)	0.35	0.17	0.12

$$\frac{d\sigma}{dz} = \frac{\pi\alpha^2}{2s} P_{ij} [A_{ij}^2(1+z^2) + 2B_{ij}^2 z] - \frac{\lambda}{M_s^4} \frac{s\alpha P_i}{4} [2z^3 v_i^2 - (1-3z^2)a_i^2] + \frac{\lambda^2}{M_s^8} \frac{s^3}{32\pi} [1-3z^2+4z^4] \quad (3)$$

where i, j are summed over γ and Z exchange

$$z = \cos\theta$$

$$A_{ij} = v_i v_j + a_i a_j, \quad B_{ij} = v_i a_j + v_j a_i$$

P_{ij}, P_i – propagator factors

The change of the total cross section given by the graviton exchange is negligible since integration of the interference term in eq. 3 over full range of the polar angle is vanishing. Consequently one should consider angular distribution of the fermion pairs. Therefore in this analysis one can use fermions with well reconstructed charge, i.e. muon and tau pairs. Bhabha scattering is a special case and will be considered in the following sections.

In contrast to $e^+e^- \rightarrow \gamma G$ the dependence on the number of extra dimensions is weak and included into λ parameter. “Non-radiative” events are studied.

The lower limits on the cutoff parameter M_s obtained by LEP experiments are shown in table 6

Differential cross section of muon pairs production and data – Monte Carlo ratio for 183 GeV and 189 GeV data collected by OPAL experiment are shown in fig. III E.

FIGURE 4. Visible mass distribution of Zg candidates. Points are real data, solid line is Standard Model expectation and dashed line is modification due to graviton production. L3 collaboration, 189 GeV data, hadronic Z decays.

F Bhabha scattering

Differential cross section of $e^+e^- \rightarrow e^+e^-$ can be presented in the a form

$$\frac{d\sigma}{d\cos\theta} = SM(s, t) + \frac{\lambda}{M_s^4} \cdot A_{int}(s, t) + \left(\frac{\lambda}{M_s^4}\right)^2 \cdot B_{graviton}(s, t)$$

In contrast to muon and tau pairs production integrated interference is not vanishing because of t-channel contribution. This fact and much higher statistics of Bhabha scattering lead to the best sensitivity of this process to gravitons in extra dimension.

Theoretical error in the Standard Model cross section of Bhabha forward scattering is a limiting factor for the constrains on extra dimensions, while in the case of $e^+e^- \rightarrow \gamma\gamma$, $\mu\mu$ and $\tau\tau$ statistical error is dominated.

FIGURE 5. Angular distribution of $e^+e^- \rightarrow \gamma\gamma(\gamma)$ candidates. Points stands for 189 GeV data, solid line - Standard Model expectation, dashed (dotted) line - deflection by graviton exchange with positive (negative) interference, $M_s = 0.65$ TeV, L3 collaboration.

FIGURE 6. Differential cross section of muon pair production for 183 GeV (a) and 189 GeV (b) and ratio data – Monte Carlo ratio for 183 GeV (c) and 189 GeV (d). Points are real data, solid line – Standard Model prediction, dashed line - best fit of the cutoff parameter M_s to data, dotted (dashed-dotted) line – graviton exchange with positive (negative) interference

TABLE 5. Lower limits on M_s for $e^+e^- \rightarrow \gamma\gamma(\gamma)$ process

		M_s , GeV $\lambda = 1$	M_s , GeV $\lambda = -1$
ALEPH	189-202 GeV	810	813
DELPHI	130-202 GeV	713	691
L3	183-189 GeV	770	760
OPAL	183-189 GeV	660	634

TABLE 6. Lower limits on M_s for $e^+e^- \rightarrow \mu\mu$ and $e^+e^- \rightarrow \tau\tau$ processes

		$\mu\mu$		$\tau\tau$	
		$\lambda = 1$	$\lambda = -1$	$\lambda = 1$	$\lambda = -1$
ALEPH	130-189 GeV	607	563	527	509
DELPHI	183-202 GeV	725	598	645	557
L3	183-189 GeV	690	560	540	580
OPAL	183-189 GeV	600	630	630	500

In order to set the limit on M_s Global fit to 4 LEP experiments data was done in [6]. In this paper the following data set was taken: differential cross sections measured by ALEPH and L3 experiments for 183 and 189 GeV data and total cross sections and forward-backward asymmetries measured by DELPHI and OPAL experiments for 183 and 189 GeV. Obtained 95 % C.L. lower limits on the cutoff parameter M_s are: $M_s > 1.261$ TeV for $\lambda = 1$ and $M_s > 0.962$ TeV for $\lambda = -1$.

Ratio of observed and predicted cross section measured by ALEPH experiment together with an expectation for graviton exchange is shown on fig. 7.

Figure 8 presents values $\frac{\lambda}{M_s^4}$ obtained by LEP experiments (95% C.L. limits only available for L3 collaboration) from γ , muon and tau pairs. Averaged value for γ , muon and tau pairs and averaged value for electron pairs production are also shown.

Average 95% C.L. limits from $e^+e^- \rightarrow \mu\mu$, $e^+e^- \rightarrow \tau\tau$, $e^+e^- \rightarrow \mu\mu$, $e^+e^- \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow e^+e^-$ are $M_s > 1.27$ TeV for $\lambda = 1$ and $M_s > 0.98$ TeV for $\lambda = -1$.

G $e^+e^- \rightarrow ZZ$ and $e^+e^- \rightarrow WW$

Massive gauge boson pairs production is also sensitive channel to extra dimensions. In contrast to fermion pair production, in the case of boson pair production additional graviton exchange diagram changes not only differential cross section but also full cross section. Searches for extra dimensions in W pairs and Z pairs

FIGURE 7. Observed / Predicted cross section, ALEPH 189 GeV Data, inner bars – data errors, outer bars – data+theory errors. Dashed (dotted) line stands for positive (negative) interference with graviton exchange. Parameter Λ is connected to M_s as $\Lambda = (2\pi)^{1/4} M_s$.

FIGURE 8. $\frac{\lambda}{M_s^4}$ obtained from γ , muon and tau production, average of $\frac{\lambda}{M_s^4}$ over ALEPH, DELPHI and OPAL data for γ , muon and tau production, average of $\frac{\lambda}{M_s^4}$ over ALEPH, DELPHI, L3 and OPAL data for Bhabha scattering

was done by L3 collaboration. Fig. 9 shows the reconstructed Z boson mass for $e^+e^- \rightarrow ZZ \rightarrow q\bar{q}l^+l^-$ channel and fig. 10 presents the polar angle distribution for $e^+e^- \rightarrow WW$ candidates with hadronic and semileptonic decays of W's.

Inspite of small number of events observed for these channels, they give quite strong 95% limits on M_s : $M_s > 0.77$ TeV ($\lambda = 1$) and $M_s > 0.76$ TeV ($\lambda = -1$) for Z boson pairs production and $M_s > 0.79$ TeV ($\lambda = 1$) $M_s > 0.68$ TeV ($\lambda = -1$) for W boson pairs production.

IV GAUGE BOSONS IN EXTRA DIMENSIONS

The second theoretical model with gauge bosons propagating in extra dimensions is hardly constrained by electroweak observables measurements. It was shown in [7] that the global fit of the electroweak observables with the compactification scale as a free parameter and one extra dimension give the limits $M_c > 3.8$ TeV with Higgs propagating in extra dimension and $M_c > 3.3$ TeV with Higgs confined in "3-brane". Transforming these numbers to the limits on radius of extra dimension one can obtain $r < 5.2 \cdot 10^{-5}$ fm and $r < 6.0 \cdot 10^{-5}$ fm.

Constraints on compactification scale also can be found from Bhabha scattering. Cross section modification is given by

$$\frac{d\sigma}{d\cos\theta} = \left(\frac{d\sigma}{d\cos\theta}\right)_{SM} \cdot |S(s, t)|^2$$

$$S(s, t) = \frac{\Gamma(1 - \frac{s}{M_c^2})\Gamma(1 - \frac{t}{M_c^2})}{\Gamma(1 - \frac{s}{M_c^2} - \frac{t}{M_c^2})} \approx (1 - \frac{\pi^2}{6} \frac{st}{M_c^4})$$

$s, t \ll M_c^2$

In [8] the limit of $M_c = 0.631$ TeV, $r < 3 \cdot 10^{-4}$ fm was found from ALEPH, L3 and OPAL 183 and 189 GeV e^+e^- differential cross section.

V CONCLUSION

In this talk we present the following limits on extra dimensions.

Limits on M_D and r in from direct graviton production

	$N = 2$	$N = 4$	$N = 6$
M_D , TeV	1.28	0.80	0.58
r , cm	$2.9 \cdot 10^{-2}$	$1.4 \cdot 10^{-9}$	$4.5 \cdot 10^{-12}$

Limits on cut-off parameter M_s from virtual graviton exchange contribution

$$M_s > 1.27 \text{ TeV} \quad \lambda = 1$$

$$M_s > 0.98 \text{ TeV} \quad \lambda = -1$$

Limits on compactification scale M_c from electroweak observables

FIGURE 9. Reconstructed Z mass for $e^+e^- \rightarrow ZZ \rightarrow q\bar{q}l^+l^-$, L3 data at 189 GeV. Points stand for real data, solid line – Standard Model expectation, dashed (dotted) line – graviton exchange with $\lambda = 1$ ($\lambda = -1$).

FIGURE 10. Polar angle distribution for $e^+e^- \rightarrow WW$, L3 data at 189 GeV. Points stand for real data, solid line – Standard Model expectation, dashed (dotted) line – graviton exchange with $\lambda = 1$ ($\lambda = -1$)

$$M_c > 3.8 \text{ TeV} \quad \text{Higgs in extra dimensions}$$

$$M_c > 3.3 \text{ TeV} \quad \text{Higgs confined in "3-brane"}$$

Limit on compactification scale M_c from Bhabha scattering

$$M_c = 0.631 \text{ TeV}$$

REFERENCES

1. N.Arkani-Hamed, S.Dimopoulos, G. Dvali, Phys. Lett. **B429**, 263 (1998);
I.Antoniadis, N.Arkani-Hamed, S.Dimopoulos, G. Dvali, Phys. Lett. **B436**, 257 (1998); N. Arkani-Hamed, S.Dimopoulos, G. Dvali, Phys. Rev. D**59**, 086004 (1999);
2. K.Dienes, E.Dudas, T.Gherghetta, Nucl. Phys **B537**, 47 (1999)
3. G.Giudice, R. Rattazzi and D.Wells hep-ph/9811291, E.Mirabelli, M.Perelstein and M.Peskin hep-ph/9811337, K.Cheung and W.-Y.Keung hep-ph/9903294
4. G.Giudice, R.Rattazzi, J.Wells hep-ph/9811291
K.Agashe and N.Deshpande OITS-669
5. J.Hewett hep-ph/9811356, T.Rizzo hep-ph/9901209, K.Cheung and W.-Y.Keung hep-ph 9903294, K.Agashe and N.Deshpande OITS-669
6. D.Bourilkov hep-ph/9907380
7. T.Rizzo and J.Wells hep-ph/9906234
8. D.Bourilkov hep-ph/0002172