Violation of the Equivalence Principle in the light of the SNO and SK solar neutrino results

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Abstract

The SNO result on charged current deuteron disintegration, the SuperKamiokande 1258-day data on electron scattering, and other solar neutrino results are used to revisit the model of neutrino oscillations driven by a violation of the equivalence principle. We use a χ^2 minimization technique to examine oscillation between the ν_e and another active neutrino, both massless, and find that within the Standard Solar Model the fit to the SNO and SuperKamiokande spectra are moderately good while a very good fit is obtained when the absolute normalizations of the 8B and hep neutrino fluxes are allowed to vary. The best fit prefers large, but not maximal, mixing, essentially no hep neutrinos, and a 40% reduction in the 8B neutrino flux. The fit to the total rates from the different experiments is not encouraging but when the rates and spectra are considered together the situation is much improved. We remark on the expectations of the VEP model for the neutral current measurements at SNO.

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1 Introduction

The Sudbury Neutrino Observatory (SNO) collaboration has presented results from 241 days running of their solar neutrino experiment [1], a reminder that it is still early days yet in this fruitful area of astro-particle physics. From the data on charged current (CC) deuteron disintegration (ν_e + d → p + p + e^-) they observe a ν_e flux which is 3.3σ less than the neutrino flux measured by the Super-Kamiokande collaboration (SK) via ν_e scattering (ES) [2]. This further strengthens the oscillation interpretation of the solar neutrino problem.

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1SNO also has results from ES events [1] which agree with the SK results [2] but is of poorer statistics currently.
neutrino problem and, at least in a two neutrino scenario, disfavours the mixing of the \( \nu_e \) with a sterile species.

Violation of the Equivalence Principle (VEP) has been examined in the literature as a possible solution to the solar neutrino problem [3, 4, 5, 6, 7], one of the alternatives to the mass-mixing vacuum oscillation and the matter induced Mikheyev-Smirnov-Wolfenstein (MSW) approaches [8]. In this note we revisit the VEP solution in a two-flavor picture with the \( \nu_e \) oscillating to another active species both of which are assumed to be massless.

Neutrino oscillation in VEP can occur if (a) the weak equivalence principle is not satisfied, i.e., the coupling of neutrinos to the gravitational field is nonuniversal and (b) the flavor eigenstates are not identical to the states that couple to gravity [9]. It does not require neutrinos to carry a non-zero mass. The weak equivalence principle demands the coupling of particles to the ambient gravitational potential \( \phi \) to be of the form \( f\phi E \), where \( E \) is the particle energy, and \( f \) a universal coupling constant. If the latter varies from one neutrino species to another then that would constitute a violation of the equivalence principle. If \( f_1 \neq f_2 \) in a two-neutrino framework, then these states define a basis in the two-dimensional space which, in general, could be different from the flavor basis.

\[
\nu_e = \nu_1 \cos \theta + \nu_2 \sin \theta; \quad \nu_x = -\nu_1 \sin \theta + \nu_2 \cos \theta, \quad (1)
\]

where \( x = \mu, \tau \). The effect due to a small splitting \( \Delta f \) will manifest itself in the form of flavor oscillations, the wavelength going to infinity as \( \Delta f \) tends to zero. The essential difference between this approach and the mass-mixing solution appears in the energy dependence of the survival probability. For a two-neutrino picture, the general expression for the survival probability for an initial \( \nu_e \) after propagation through a distance \( L \) is given by:

\[
P_{ee}(E_\nu, L) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\pi L}{\lambda} \right), \quad (2)
\]

where, for the VEP induced case, the oscillation length \( \lambda \) is:

\[
\lambda = \frac{2\pi}{E_\nu \phi \Delta f}. \quad (3)
\]

\( E_\nu \) is the neutrino energy. Thus, unlike the mass-mixing case where \( \lambda \propto E_\nu \), for VEP \( \lambda \propto 1/E_\nu \). Due to the different energy dependences of the survival probability in the mass-mixing and the VEP alternatives, their predictions can be quite different for solar neutrinos of different energies. We follow the prevalent practice of choosing the gravitational potential, \( \phi \), to be a constant over the neutrino path. This is the case if the potential due to the Great Attractor [10] dominates over that due to the sun and other heavenly bodies in our neighborhood. In such an event, writing \( \Delta F = \phi \Delta f/2 \), the expression for the oscillation wavelength, Eq. (3), becomes

\[
\lambda = \frac{6.20 \times 10^{-13} \text{m}}{\Delta F} \left( \frac{1\text{MeV}}{E_\nu} \right). \quad (4)
\]
An alternate physics scenario which also leads to neutrino oscillations with \( \lambda \propto 1/E_\nu \) is a picture of violation of special relativity (VSR) [11]. If special relativity is violated, the maximum attainable speed of a particle *in vacuo* need not universally be the speed of light \( c \). In particular, if the maximum possible velocities of the two types of neutrinos be \( v_1 \) and \( v_2 \) and these *velocity eigenstate* neutrinos be related to the \( \nu_e \) and \( \nu_x \) through a mixing angle \( \theta \) (see Eq.(1)) then in this case the expression for \( \lambda \) is:

\[
\lambda = \frac{2\pi}{E_\nu \Delta v},
\]

where \( \Delta v \) is the velocity difference for the neutrinos \( \nu_1 \) and \( \nu_2 \). Comparing Eqs. (5) and (3) one finds that the energy dependence of the oscillation length is identical in the two cases\(^2\) and the role of \( \Delta v \) in the VSR case is the same as that of \( \phi \Delta f \) in the VEP formalism. Here, we use the terminology of the VEP mechanism but the results can be taken over *mutatis mutandis* to the VSR situation.

In this work we use the BBP2000 calculation [13] of the solar neutrino flux as the SSM reference. In addition to this, we explore the possibility of the absolute normalizations of the solar \(^8\)B- and hep-neutrino spectra\(^3\) being different from their BBP2000 SSM predictions. If \( X_B \) and \( X_{hep} \) denote the factors by which the absolute normalizations are multiplied, we use the data to find the best-fit values for these. We find that all the fits are improved in a noteworthy manner when \( X_B \) is permitted to be less than unity.

### 2 Update of VEP for the SK ES results

In contrast to other recent work on VEP [5, 6] based on the 825-day SK spectral data, in a previous publication [3], the SK ES results from 1117 days of running [14] were considered. It was found that the predictions were by-and-large robust. One trend, which was remarked upon, was that the softening of the highest energy SK ES data had resulted in a reduction of \( X_{hep} \). In fact, it was noted that the best-fit to the day-night spectrum of the 1117-day SK data preferred a negligibly small value of \( X_{hep} \).

For a further update, we have performed a similar analysis with the SK 1258-day data. The SK results are presented in the form of number of events (normalized to the SSM expectation) in 19 electron recoil energy bins of width 0.5 MeV in the range 5 MeV to 14 MeV and a 19th bin which covers the events in the range 14 to 20 MeV [2].

The definition of \( \chi^2 \), the error correlations, *etc.* are chosen in the same manner as in [3]. Suffice it to say that we have included the statistical error, the uncorrelated systematic errors, and the energy-bin-correlated experimental errors [15] as well as those from the calculation of the shape of the expected spectrum [16]. When we allow the

\(^2\)It has been shown that inclusion of CPT-violating interactions in addition to Violation of Special Relativity can lead to more general energy dependences involving \( 1/E_\nu, E_\nu \), and constant terms [12].

\(^3\)The SNO and SK experiments are sensitive only to the \(^8\)B and hep neutrinos.
normalizations of the $^8$B and hep fluxes to vary, we do not include their astrophysical uncertainties separately. The results are presented in Table 1.

<table>
<thead>
<tr>
<th>Fitted Data</th>
<th>Case</th>
<th>$\sin^2 2\theta$</th>
<th>$\Delta F$ ($10^{-24}$)</th>
<th>$X_B$</th>
<th>$X_{hep}$</th>
<th>$\chi^2_{min}$/d.o.f.</th>
<th>g.o.f. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK1258 ES Day-Night averaged Spectrum</td>
<td>1a</td>
<td>0.72</td>
<td>0.22</td>
<td>1.0</td>
<td>1.0</td>
<td>29.10/17</td>
<td>3.37</td>
</tr>
<tr>
<td></td>
<td>1b</td>
<td>1.0</td>
<td>0.99</td>
<td>0.77</td>
<td>2.4 $\times 10^{-5}$</td>
<td>11.38/15</td>
<td>72.52</td>
</tr>
<tr>
<td>SK1258 ES separate Day &amp; Night Spectra</td>
<td>1c</td>
<td>0.72</td>
<td>0.21</td>
<td>1.0</td>
<td>1.0</td>
<td>43.71/36</td>
<td>17.67</td>
</tr>
<tr>
<td></td>
<td>1d</td>
<td>1.0</td>
<td>0.98</td>
<td>0.77</td>
<td>9.3 $\times 10^{-8}$</td>
<td>24.68/34</td>
<td>87.92</td>
</tr>
</tbody>
</table>

Table 1: The best-fit values of the parameters, $\sin^2 2\theta$, $\Delta F$, $X_B$, $X_{hep}$, the $\chi^2_{min}$, and the goodness of fit (g.o.f.) for fits to the SK1258 ES spectra.

We find that the fits are by-and-large robust; using the separate day and night data or the day-night averaged data affects them only marginally. Further, the best-fit parameters have not changed much and they are well within the 90% C.L. allowed regions of the previous analysis using the 1117-day data. For example, for the fit to the the day-night averaged data sample, in the SSM case ($X_B = X_{hep} = 1$), the best-fit values of $\sin^2 2\theta$ and $\Delta F$ have changed < (5 - 10)% but the quality of fit has significantly dropped (g.o.f. = 76.7% for the fit to the 1117-day data). When $X_B$ and $X_{hep}$ are allowed to vary, the best-fit value of (a) $X_{hep}$ is tiny, and (b) $\Delta F$ is larger than earlier and closer to the value for the SSM fit, a welcome feature. The goodness of fit is also a bit lower. For the fit to the separate day and night data, the differences between the fits to the 1258-day and 1117-day data broadly follow the same pattern as above.

### 3 VEP Fits to the SNO CC and SK ES spectra

The SNO collaboration has published the first measurement of the spectrum of electron kinetic energy produced from CC deuteron disintegration by solar neutrinos. The results are presented in eleven electron kinetic energy bins ranging from 6.5 MeV to 13.0 MeV. This provides a new check on solutions to the solar neutrino puzzle. The best-fit values of the VEP parameters obtained from fitting this data sample are shown in Table 2.

When the SNO CC and SK 1258-day ES data (separate day and night or day-night averaged) are simultaneously addressed within the VEP picture, the best-fit values are remarkably close to each other in the SSM fits (2a), (2c), and (2e) though the goodness of fit is low. When $X_B$ and $X_{hep}$ are allowed to vary, then the fits to the combined SK ES and SNO CC spectra improve. These results are also presented in Table 2.
As seen in the fit to the SK ES spectra alone (Table 1), here again the fits do not change whether the separate day and night data or the day-night averaged data are used. In view of this, in the next section, we use only the day-night averaged data.

We show in Fig. 1, the allowed regions for the parameters $\sin^2 2\theta$ and $\Delta F$ at 90\% C.L. obtained by fitting the SK 1258-day ES spectrum (dot-dashed line), the SNO CC spectrum (small-dashed line), and for a combined fit to the SNO CC and the SK ES spectra (solid – for the SSM case – and dotted – when $X_B$ and $X_{hep}$ are allowed to vary). It is noteworthy that for the SSM, the allowed regions from the different fits overlap very nicely.

## 4 VEP Fits to total rates and the SNO CC and SK ES spectra

The SK ES and SNO CC spectrum measurements are excellent filters for choosing between alternative solutions of the solar neutrino problem. But it has to be borne in mind that they are both sensitive only to the highest energy $^8$B neutrinos. Further constraints on any putative solution come from the total solar neutrino rates measured by the Chlorine and Gallium experiments. In order to be acceptable, the VEP solution must also be confronted with these results, to which we now turn. The data used in the $\chi^2$ analysis of total rates are given in Table 3.
Table 3: The ratio of the observed solar neutrino rates to the corresponding BBP2000 SSM predictions used in this analysis. The results are from Refs. [17, 2, 1]. The Gallium rate is the weighted average of the Gallex, SAGE, and GNO results.

The best-fit parameters obtained by fitting the total rates are presented in Table 4 (sets 4a and 4b). Here the goodness of fits are not high. In order to ascertain how the total rate measurements mesh with the SK ES and SNO CC measurements, we have performed combined fits to the rates and spectral data. These results are also shown in Table 4. Note that the SSM is disfavoured by this analysis.

<table>
<thead>
<tr>
<th>Fitted Data</th>
<th>Case</th>
<th>$\sin^2 2\theta$</th>
<th>$\Delta F$ (10^{-24})</th>
<th>$X_B$</th>
<th>$X_{hep}$</th>
<th>$\chi^2$/d.o.f.</th>
<th>g.o.f. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Rates</td>
<td>4a</td>
<td>1.0</td>
<td>1.66</td>
<td>1.0</td>
<td>1.0</td>
<td>5.50/2</td>
<td>6.39</td>
</tr>
<tr>
<td></td>
<td>4b</td>
<td>1.0</td>
<td>1.66</td>
<td>0.74</td>
<td>1.0</td>
<td>2.40/1</td>
<td>12.13</td>
</tr>
<tr>
<td>SNO CC, SK</td>
<td>4c</td>
<td>0.72</td>
<td>0.21</td>
<td>1.0</td>
<td>1.0</td>
<td>117.62/32</td>
<td>1.0 \times 10^{-9}</td>
</tr>
<tr>
<td>ES spectra &amp; Total Rates</td>
<td>4d</td>
<td>1.0</td>
<td>1.60</td>
<td>0.77</td>
<td>6.8 \times 10^{-8}</td>
<td>33.54/30</td>
<td>29.96</td>
</tr>
<tr>
<td>SNO CC, SK</td>
<td>4e</td>
<td>0.72</td>
<td>0.21</td>
<td>1.0</td>
<td>1.0</td>
<td>92.43/30</td>
<td>2.8 \times 10^{-6}</td>
</tr>
<tr>
<td>ES spectra &amp; Total Rates (incl. SNO &amp; SK)</td>
<td>4f</td>
<td>1.0</td>
<td>1.62</td>
<td>0.77</td>
<td>1.8 \times 10^{-6}</td>
<td>31.04/28</td>
<td>31.53</td>
</tr>
</tbody>
</table>

Table 4: The best-fit values of the parameters, $\sin^2 2\theta$, $\Delta F$, $X_B$, $X_{hep}$, the $\chi^2_{min}$, and the g.o.f. for fits to the total rates along with the SNO CC and SK ES spectra.

When we fit the total rates and the SNO CC and SK ES spectra together, the SNO and SK total rates and the corresponding spectra cannot be considered to be statistically independent pieces of data. However, their exclusion from the fits would also not be entirely correct as the fitted spectral ratios (observed/SSM) do not completely determine the total rates. We have therefore chosen to examine two extreme alternatives; i.e., in the fits to the spectra and the total rates together, (a) in one case we include the SK and SNO total rates in the fit, and (b) in the other case we have excluded them. It is gratifying that the best-fit values of the parameters of the two cases (4c, 4e) or (4d, 4f) are not very different, a reflection of the comparative energy independence of the (observed/SSM) spectra.

The best-fit values of the combined total rates + spectra fits for the SSM case (4c, 4e) are within the 90% C.L. allowed regions obtained from fits to the spectra alone (see Fig. 1).
5 SNO NC expectation, Comparison of different fits

SNO will soon start taking data for a calorimetric measurement of neutral current (NC) deuteron disintegration ($\nu + d \rightarrow \nu + p + n$) to which all active neutrinos contribute equally. A measure of neutrino oscillations is provided by the ratio of the NC and CC rates, $R_{NC}$ and $R_{CC}$, which is somewhat less sensitive to theoretical uncertainties than the rates themselves. We define

$$R_{SNO} = \frac{R_{NC}}{R_{CC}}$$  \hspace{2cm} (6)

For the best fit values of parameters obtained in the earlier sections, we present the predicted values of $R_{SNO}$ in Table 5. For an assessment of the various fits, for every set of best-fit parameters we also show in Table 5 the following quantities: the calculated rates for the Chlorine, Gallium, SK ES and SNO CC observations, and also the $\chi^2$ obtained for the calculated SK ES and SNO CC electron spectra.

<table>
<thead>
<tr>
<th>Case</th>
<th>Cl</th>
<th>Ga</th>
<th>SK Rate</th>
<th>$\chi^2$ (spect)</th>
<th>SNO Rate</th>
<th>$\chi^2$ (spect)</th>
<th>$R_{SNO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>0.516</td>
<td>0.940</td>
<td>0.479</td>
<td>29.10</td>
<td>0.353</td>
<td>11.65</td>
<td>1.00</td>
</tr>
<tr>
<td>1b</td>
<td>0.437</td>
<td>0.796</td>
<td>0.462</td>
<td>11.38</td>
<td>0.380</td>
<td>20.96</td>
<td>0.71</td>
</tr>
<tr>
<td>1c</td>
<td>0.508</td>
<td>0.940</td>
<td>0.476</td>
<td>29.40</td>
<td>0.341</td>
<td>10.73</td>
<td>1.04</td>
</tr>
<tr>
<td>1d</td>
<td>0.436</td>
<td>0.798</td>
<td>0.458</td>
<td>11.50</td>
<td>0.380</td>
<td>21.62</td>
<td>0.71</td>
</tr>
<tr>
<td>2a</td>
<td>0.522</td>
<td>0.941</td>
<td>0.502</td>
<td>45.50</td>
<td>0.356</td>
<td>9.74</td>
<td>0.99</td>
</tr>
<tr>
<td>2b</td>
<td>0.523</td>
<td>0.942</td>
<td>0.522</td>
<td>78.60</td>
<td>0.356</td>
<td>9.70</td>
<td>1.07</td>
</tr>
<tr>
<td>2c</td>
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<td>0.940</td>
<td>0.481</td>
<td>29.96</td>
<td>0.343</td>
<td>10.35</td>
<td>1.03</td>
</tr>
<tr>
<td>2d</td>
<td>0.496</td>
<td>0.858</td>
<td>0.461</td>
<td>13.14</td>
<td>0.418</td>
<td>12.53</td>
<td>0.51</td>
</tr>
<tr>
<td>2e</td>
<td>0.506</td>
<td>0.940</td>
<td>0.479</td>
<td>30.25</td>
<td>0.336</td>
<td>10.15</td>
<td>1.05</td>
</tr>
<tr>
<td>2f</td>
<td>0.495</td>
<td>0.859</td>
<td>0.459</td>
<td>13.20</td>
<td>0.416</td>
<td>12.56</td>
<td>0.51</td>
</tr>
<tr>
<td>4a</td>
<td>0.443</td>
<td>0.609</td>
<td>0.605</td>
<td>355.6</td>
<td>0.520</td>
<td>23.18</td>
<td>0.67</td>
</tr>
<tr>
<td>4b</td>
<td>0.339</td>
<td>0.596</td>
<td>0.448</td>
<td>23.60</td>
<td>0.385</td>
<td>12.37</td>
<td>0.67</td>
</tr>
<tr>
<td>4c</td>
<td>0.511</td>
<td>0.940</td>
<td>0.482</td>
<td>30.26</td>
<td>0.344</td>
<td>10.29</td>
<td>1.03</td>
</tr>
<tr>
<td>4d</td>
<td>0.352</td>
<td>0.613</td>
<td>0.456</td>
<td>18.40</td>
<td>0.400</td>
<td>12.94</td>
<td>0.67</td>
</tr>
<tr>
<td>4e</td>
<td>0.511</td>
<td>0.940</td>
<td>0.480</td>
<td>29.75</td>
<td>0.344</td>
<td>10.44</td>
<td>1.03</td>
</tr>
<tr>
<td>4f</td>
<td>0.350</td>
<td>0.608</td>
<td>0.456</td>
<td>18.40</td>
<td>0.400</td>
<td>12.97</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 5: The total rates for the different experiments obtained by using the best-fit values of the VEP parameters presented in Tables 1, 2, and 4. Also presented are the $\chi^2$ for fits to the observed SK ES and SNO CC spectra using these values of the parameters. The predictions for $R_{SNO}$ are also shown.

From Table 5 it can be seen that fits to the spectra by themselves give best-fit parameters grossly disfavoured by the total rates measured in the Chlorine and Gallium
experiments. The most acceptable results are obtained in cases (4d, 4f) – combined fits to the SK ES spectra, SNO CC spectra, and the Chlorine and Gallium total rates measurement with $X_B = 0.77$ (see Table 4). These values of parameters will be further strengthened if $R_{SNO}$ is measured around 0.67.

6 Conclusions and Discussions

In this work, we have considered the VEP oscillation explanation of the solar neutrino problem within a two-flavor (active) scenario in the light of the SNO CC measurement, the SK 1258-day ES data, and the total rates from the Chlorine and Gallium experiments.

We find that the VEP fits to the SK ES data are robust. The best-fit parameters have changed only marginally from the earlier fit to the 1117-day data.

In the literature there are many variants of the data-fitting procedure for solar neutrinos. The solution preferred by Nature should fit all the solar neutrino results collectively and also individually. A good fit to one (or some) of the results may do rather poorly when confronted with another piece of datum. A good fit to all the data taken together may mask bad fits to some of the results taken in isolation. This caveat is overlooked in much of the current literature. We have attempted to fit the SK ES and SNO CC spectra individually and jointly, and also the total rates by themselves and jointly with the spectral data in an attempt to find a region of parameter space that is acceptable in every respect. These results are given in Table 5 and in Tables 1, 2, and 4. We find that the best-fit parameters of the cases (4d) and (4f) (see Tables 4 & 5) best meet the requirements alluded to above. Note that these fits correspond to $X_B \sim 0.77$ and predict $R_{SNO} \sim 0.67$.

During the passage of the neutrinos from their point of production to the solar surface, interactions with the ambient matter, responsible for the MSW effect, become important. In the presence of VEP and this MSW contribution, the effective neutrino mass matrix in flavor space takes the form

$$M = \frac{1}{2} \begin{pmatrix} E_\nu \Delta F \cos 2\theta & E_\nu \Delta F \sin 2\theta & \frac{E_\nu}{2} \sqrt{2} G_F n_e(r) \end{pmatrix},$$

(7)

where we have dropped an irrelevant part proportional to the identity matrix. The MSW contribution in (7) inside the sun turns out to be several orders of magnitude larger than the terms due to VEP that we have discussed in this work. Recall that we have assumed the neutrinos to be massless. For maximal mixing (e.g., the preferred 4d and 4f cases), there is no resonance effect and, in fact, till such time that the neutrino emerges from the sun, the MSW contribution controls the masses in (7). Inside the sun, the $\nu_e$ is, therefore, a mass eigenstate to a very good approximation. The effect of VEP oscillations begins to manifest itself only from then onwards.
The VEP oscillation wavelengths favored by the data are comparable to the distance of the sun to the earth. Therefore, seasonal effects are expected in this picture. When more seasonal data from SK accumulates, it will provide a good check on this scenario.

Since the publication of the first results by the SNO collaboration, a number of analyses of the data in the mass-mixing vacuum oscillation and MSW approaches have appeared [8]. The quality of these fits are comparable to those found in the VEP picture. Here too, the parameters which best fit the rates alone are somewhat different from those that best fit the rates together with the spectra [8]. Further sharpening of the data from new and ongoing experiments, it is hoped, will help to distinguish between these alternatives.

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References


Figure 1: The 90\% C.L. allowed regions in the $\sin^2 2\theta - \Delta F$ plane. The area enclosed by the solid (dotted) line is allowed by the SNO CC + SK1258 ES spectral data for the SSM (the model where $X_B$ and $X_{hep}$ are allowed to vary). The area allowed by the SNO CC (SK1258 ES) spectrum alone for the SSM is enclosed by the small dashed (dot-dashed) line. The best fit points have been indicated.
\[ \Delta F(10^{-24}) \]

Spectrum
90\% C.L. Allowed
VEP

\( \sin^2 2\theta \)