Generation Structure and Proton Decay Problem

K. Koike
Department of Natural Science
Kagawa University
Takamatsu, 760-8522 Japan

Abstract
Possible generation structure is investigated. We considere alternative possibility of combination of leptons and quarks to compose each generation, where the combination of ($\nu_\tau, \tau$) and ($u, d$) compose a generation. Our model exhibits the hierarchy structure after sea-saw mechanism provied that possible existence of realistic right-handed neutrinos are taken into account. Then, in the GUTs model based on our scheme, possible inter-generation properties are examined together with the classification of gauge bosons. It is shown that if our scheme is realized, the proton decay exhibits different mode from ordinary one.

1 Introduction
The generation structure observed in low energy region seems to exhibit an important suggestion to disclose fundamental existence-form of matter.[1, 2]. It will refrect probably the structure of nature in deeper level. In the standard comprehension, the generation is simply arranged from light to large mass acording to the historical discovery order which is due to the energy frontier. According to this custom, the combination of ($\nu_e, e$) and ($u, d$) compose the first generation. It should be noted that, however, the essential reason to relate leptons and quarks is to satisfy the anomaly free condition, and no restriction is yet known to what lepton must compose a generation together with a certain specific quark. Then, it is worthwhile to investigate another possible generation model as a step to disclose the origin of generation structure. This paper is concerning to this problem.
In this paper, we will propose an alternative model of generation combination of leptons and quarks. In our model, the combination of \((\nu_\tau, \tau)\) and \((u, d)\) compose a generation. It is shown that our model exhibits the new hierarchy structure after sea-saw mechanism provided that the right-handed neutrinos with appropriate mass are taken into account. Then, the generation structure is investigated in the GUTs scheme. In what stage of symmetry breaking of GUTs, is the generation structure appear? We will suppose that a partial GUT structure\([3]\) in each generation in the series of breaking of GUTs symmetry. This picture leads to possible classification of the gauge bosons appearing in GUTs scheme. We will examine possible properties of gauge bosons both within one generation and among inter-generation. Especially, our scheme exhibits the different mode of proton decay in the framework of ordinary GUTs scheme without super symmetry.

2 Possible scheme of generation Structure

Our model of the new generation combination is represented as,

\[
I \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}
\]

\[
II \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix}
\]

\[
III \quad \begin{pmatrix} \nu_{e L} \\ e_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}
\]

(1)

where in Eq.(1), we have written only the left-hand components. We will suppose possible existence of GUTs structure behind of the generation structure, that is, as a result of spontaneous symmetry breaking, our generation structure is realized. The mass spectrum of leptons and quarks will be realized as a result of symmetry breaking with Higgs mechanism.
3 See-saw mechanism and possible new hierarchy structure

In our model, so far as we are restricted in well known particles, a hierarchy structure with simple form seems to not appear. It should be noted that, however, possible new hierarchy structure will appear provided that the existence of right-handed Majorana neutrinos are taken into account together with the sea-saw mechanism. Then, we will sketch this problem.

3.1 See-saw mechanism of neutrino mass splitting

For definiteness, we will make a quick review of see-saw mechanism of neutrino mass generation.\cite{4} As a basis of construction of our scheme, let us consider the D-M (Dirac-Mayorana) mass term \cite{5,7} in the simplest case of one generation labeled by the generation suffix $i$. We have

$$L_{D-M} = -\frac{1}{2} m_{iL} (\nu_{iL})^c \nu_{iL} - m_{iR} \bar{\nu}_{iR} \nu_{iL} - \frac{1}{2} m_{iD} \bar{\nu}_{iR} (\nu_{iR})^c + \text{h.c.}$$

(2)

Here

$$M = \begin{pmatrix} m_{iL} & m_{iD} \\ m_{iD} & m_{iR} \end{pmatrix}, \quad (3)$$

$m_{iL}, m_{iD}, m_{iR}$ are parameters. For a symmetrical matrix $M$ we have

$$M = U m U^\dagger, \quad (4)$$

where $U^\dagger U = 1$, $m_{jk} = m_j \delta_{jk}$. From Eq.(2) and Eq.(4) we have

$$L_{D-M} = -\frac{1}{2} \sum_{\alpha=1}^{2} m_{i\alpha} \bar{\chi}_{i\alpha} \chi_{i\alpha}, \quad (5)$$

where
\[ \nu_{iL} = \cos \theta_i \chi_{i1L} + \sin \theta_i \chi_{i2L} \]
\[ (\nu_{iR})^c = -\sin \theta_i \chi_{i1L} + \cos \theta_i \chi_{i2L}. \] (6)

Here \( \chi_{i1} \) and \( \chi_{i2} \) are fields of Majorana neutrinos with masses \( m_{is} \) (small), \( m_{iB} \) (Big), respectively. The masses \( m_{is}, m_{iB} \) and the mixing angle \( \theta_i \) are connected to the parameters \( m_{iL}, m_{iD} \) and \( m_{iR} \) by the relations

\[ m_{is} = \frac{1}{2} |m_{iR} + m_{iL} - a_i| \]
\[ m_{iB} = \frac{1}{2} |m_{iR} + m_{iL} + a_i| \]
\[ \sin 2\theta_i = \frac{2m_{iD}}{a_i}, \quad \cos 2\theta_i = \frac{m_{iR} - m_{iL}}{a_i} \] (7)

where

\[ a_i = \sqrt{(m_{iR} - m_{iL})^2 + 4m_{iD}^2} \] (8)

It should be noted that the relations Eq.(7) are exact ones. Let us assume now that

\[ m_{iL} = 0, \quad m_{iD} \simeq m_{iF}, \quad m_{iR} \gg m_{iF}, \] (9)

where \( m_{iF} \) is the typical mass of the leptons and quarks of the generation labeled by suffix \( i \). From Eq.(7) we have

\[ m_{is} \simeq \frac{m_{iF}^2}{m_{iR}}, \quad m_{iB} \simeq m_{iR}, \quad \theta_i \simeq \frac{m_{iD}}{m_{iR}} \] (10)

Thus, if the conditions Eq.(9) are satisfied, the particles with definite masses are split to a very light Majorana neutrino with mass \( m_{is} \ll m_{iF} \) and a very heavy Majorana particle with mass \( m_{iB} \simeq m_{iR} \). The current neutrino field \( \nu_{iL} \) practically coincides with \( \chi_{i1L} \) and \( \chi_{i2} \simeq \nu_{iR} + (\nu_{iR})^c \), because \( \theta_i \) is extremely small.

That is, we have assumed such scheme that in D-M mass term Dirac masses are of order of usual fermion masses, the right-handed Majorana masses, responsible for lepton numbers violation, are extremely large and
the left-handed Majorana masses are equal zero. In such a scheme neutrinos
are Majorana particles with masses much smaller than masses of the other
fermions. The predictions of neutrino masses depend on the value of the \( m_{iR} \)
mass. The value of \( m_{iR} \) is often assumed that \( m_{iR} = M_{GUT} \), where \( M_{GUT} \) is
grand unification scale. Though this value depends on the model, a typical
one is \( m_{iR} \approx 10^{19} \) GeV (Planck mass). In the \( M_{GUT} \) region, the ordinary
particle picture may be drastically changed.

It should be emphasized that, however, there is no definite reason why
the mass of \( m_{iR} \) should be \( M_{GUT} \). It is also possible that though the mass
of \( m_{iR} \) is very huge it is below the Planck mass and possessing the picture of
ordinary particle. In such case, possibility of realization of new generation
hierarchy will be implied, which is discussed in the next section.

3.2 Possible hierarchy structure based on the right-
headed Majorana neutrino

The ”standard” structure of generation is composed of each leptons and
quarks, which generation number is labeled according to the sequence of
their historical discovery, depending on the energy frontier. It should be
emphasized that, in the present stage, the only compelling reason to relate
leptons and quarks is the anomaly free condition, and no essential principle to
compose the generation is ever unknown.[8] In our generation scheme given
in Eq.(1), however, a simple hierarchy structure will appear provided that
the right-handed Majorana neutrinos are taken into account.

If the right-handed Majorana neutrinos are realistic particles below the
GUT mass and responsible to the see-saw mechanism, its existence will affect
to the above mentioned classification procedure of the generations. From
Eq.(10), \( m_{iR} \) is expressed as,

\[
m_{iR} \simeq \frac{m^{2}_{iF}}{m_{i\alpha}},
\]

where we should remember to the fact that the current neutrino field \( \nu_{iL} \)
practically coincides with \( \chi_{iL} \) with the mass \( m_{i\alpha} \). Though the values of
neutrino mass has not yet been established, [9] a prominent possibility is
that the mass of neutrinos exhibits an extreme hierarchal structure

\[
m_{\nu_e} \ll m_{\nu_{\mu}} \ll m_{\nu_{\tau}}.
\]
If the degree of the mass difference in Eq.(12) is appropriate magnitude, the new hierarchical structure will be realize. That is, if the degree of mass difference of neutrinos is larger than that of the representative mass of each generation \(m_{iF}\), the magnitude of total mass of \(i\)-th neutrinos, i.e. \(m_\nu_i + m_{iR}\), will become in reverse order to label number \(i\), and this magnitude will dominate the total lepton masses belonging to \(i\)-th label.

Then, we will find the hierarchical structure of "from light to heavy" particles in the following generation classification.

\[
I \quad (\nu_1^R)^c \quad (\nu_{eL} \quad \tau_L \quad (u_L \quad d_L)) \\
II \quad (\nu_2^R)^c \quad (\nu_{\mu L} \quad (\mu_L \quad (c_L \quad s_L))) \\
III \quad (\nu_3^R)^c \quad (\nu_{eL} \quad (e_L \quad (t_L \quad b_L)))
\]

(13)

where we have written only the left-handed components. That is, taking into account of the existence of realistic particles \(\nu_{iR}\), our proposal for new generation structure is different from the ordinary one. The characteristic feature is that the leptons belonging to the first and the third generations are exchanged. It should be noted that this result is caused by the existence of realistic \(\nu_{iR}\) and the see-saw mechanism.

4 "Generational" and "Inter-generational" gauge bosons in GUTs and proton decay problem

The generation structure is the important feature observed in low energy region. This structure will still possess any meaning in high-energy region below GUTs scale. We will suppose that in the symmetry breaking chain of GUT, the single-generation unification structure appear prior to the appearance of each generation structure.[3] This situation leads to the viewpoint to distinguish the gauge bosons appearing in GUTs containing all generations. That is, the gauge bosons common to all generations, and ones which connect particles belonging to different generations. In SU(5) GUT, only the
former type of gauge bosons, $W, Z, A$ and $X, Y$, appear. It should be emphasized that though possible gauge group is not restricted to $SU(5)$, we can see the characteristic feature of GUT in this model. We see that the type of gauge bosons $X$ and $Y$ appearing in $SU(5)$ model will appear in any single-generation unification structure. We will call this type of gauge bosons as "generational gauge bosons"[11]. In the GUTs structure containing all generations, new gauge bosons connecting particles belonging to the different generations appear generally, and we will call them as "inter-generational gauge bosons".[12] Our viewpoint means that the contribution of the "inter-generational gauge boson" should be more suppressed than that of the "generational gauge bosons" in low and intermediate energy region below the GUTs scale.

If we take this viewpoint, our model of new generation structure lead to different feature of proton decay mode in GUTs. Our scheme predict the proton decay mode due to the generational gauge bosons $X$ and $Y$

$$P \rightarrow \pi^+ M_0$$

instead of the well known mode

$$P \rightarrow e^+ M_0$$

where $M_0$ represents $\pi^0, \rho_0, \omega, \eta, \pi^+\pi^- \cdots$. It should be noted that the process in Eq.(14) is forbidden by Q value. The inter-generational gauge bosons may cause the process in Eq.(15), however, this process will be extremely suppressed in low and intermediate energy region below GUTs scale. The other mode is

$$P \rightarrow \pi^+ M^+$$

with $\pi^+, \rho^+, \pi^+\pi^0 \cdots$, and this process is allowed in our scheme.

Thus, the proton decay in our model is different from the "standard" model of generation structure. That is, the well-known difficulty so-called "proton decay problem" is consistent to our scheme. It should be emphasized that when the proton decay is actually observed it will be made clear whether our scheme is realized or not.
5 Discussion

In this paper, we have proposed a possible new structure of generation combination of leptons and quarks. It is discussed that our scheme exhibits a hierarchial structure through the see-saw mechanism provided that the right-handed Majorana neutrinos are actual particles with appropriate mass. It should be noted that this structure appears as a result of the breakdown of a certain GUT structure and successive see-saw mechanism. The profound meaning of the appearance of new hierarchial structure via above two step process may be clarified in possible dynamics of sub-structure or in the process of disclosing further fundamental existence form of matter.[1, 2]

We have classified the gauge bosons appearing in GUTs, that is the "generational gauge bosons" and the "inter-generational gauge bosons". Our model gives some important predictions to so-called "proton decay problem". When the proton decay is actually observed and the neutrino masses are established, it will be made clear whether our scheme is realized or not.

We have taken in this paper so-called constructivity-like approach. That is, supposing the existence of GUTs or partial GUT-like structure behind our discussion, the final form of it doesn’t appear explicitly. In order to give further predictions, some specific assumptions and introduction to many parameters is inevitable in the present stage, then we have restricted ourselves to proposal of the framework of model. In this paper, we have investigated in the framework of ordinary GUT scheme without supersymmetry because the generation structure will be prior to a possible higher symmetry. The investigation in the framework of SUSY GUT is further problem.
References

[3] That is, in a series of breaking of GUTs symmetry, it appears a partial GUT structure such as the SU(5) GUT in each generation.
[8] The mass relation in GUTs means merely indirect condition. If new structure of generation is established, the modified GUTs relation will be constructed on the basis of it.
[9] In order to determine the neutrino masses, the mixing structure should be completely clarified. In the present stage, it will be premature to find the definite masses of neutrinos.
[10] Though possible group is not restricted to SU(5), it should be emphasized that we can see the characteristic feature of GUT in this model. We see that the type of gauge bosons X and Y appearing in SU(5) model will appear in any single-generation unification structure.
[11] In our scheme, researching to possible existence of leptpquarks belonging to the 2nd generation means to investigate one of the "generational gauge boson".See for example, F.Abe et al. Phys. Rev. Letters 81 (1998)4806
[12] "Inter-generational gauge bosons" will generally cause flavor-changinng effects.