Charged and Uncharged D-branes in various String Theories

E. Dudas\textsuperscript{a}, J. Mourad\textsuperscript{a,b} and A. Sagnotti\textsuperscript{a,c}

\textsuperscript{a} LPT\textsuperscript{⋆}, Bâtiment 210, Université de Paris-Sud
F-91405 Orsay FRANCE

\textsuperscript{b} LPTM, Université de Cergy-Pontoise, Site de Neuville III
F-95031 Neuville sur Oise FRANCE

\textsuperscript{c} Dipartimento di Fisica, Università di Roma “Tor Vergata”
INFN, Sezione di Roma II
Via della Ricerca Scientifica 1
I-00133 Roma ITALY

Abstract

We describe how the D-brane spectra of the various ten-dimensional string theories can be related to general properties of the open-closed duality, encoded in the $S$ and $P$ matrices of the conformal field theory. We also complete the classification and the description of non-BPS branes in these string theories, elucidating their non-Abelian structures and the nature of the corresponding super-Higgs mechanisms. We find that the type 0 theories and their orientifolds have two types of uncharged branes, distinguished by their couplings to the closed string tachyon. We also find that the 0A orientifold has the unusual feature of having charged and uncharged branes with identical world-volume dimensions. We conclude with some comments on fractional branes, elucidating their role in connection with the boundary states of $D_{odd}$ SU(2) WZW models.

\textsuperscript{⋆}Unité mixte du CNRS, UMR 8627.

July 19, 2001
1. Introduction

The last few years have witnessed a widespread interest in the role of D-branes [1] in String Theory, after Polchinski [2] elucidated their role in relation to R-R charges. To some extent, this interest was also spurred by the relative simplicity with which the low-energy spectra of these complicated systems can be described, as compared to those of other types of branes responsible for non-perturbative aspects of String Theory. D-branes and orientifold planes are also key ingredients in the construction of open descendants, or orientifolds [3], vacuum configurations for type I strings [4] or for their non-supersymmetric counterparts [5, 6] related to the type 0 strings [7].

The study of orientifolds has so far relied on two apparently distinct approaches. The first, more rooted in the world-sheet boundary Conformal field Theory (CFT), has led to early constructions of orientifold models in various dimensions with a number of exotic features [3], but has not been widely applied to the study of D-brane configurations as such. The second, essentially based on the systematic use of boundary states [8, 9] and more rooted in the space-time picture of branes, has formed the basis of most D-brane studies [10, 11].

Most of the early work dealt with BPS brane configurations, or with corresponding orientifold models with a number of residual supersymmetries, but more recently Sen [12] initiated a systematic study of additional types of branes, that do not saturate the BPS bound but can at times be stable nonetheless. Supersymmetry is in general fully broken by the presence of these defects, as is also generally the case when BPS branes and anti-branes are simultaneously present, and indeed these non-BPS branes can be related to suitable orbifolds of combinations of this type. One of the purposes of this paper is to show how boundary CFT methods neatly encode the known properties of D-branes, allowing a systematic discussion of additional, less known or new, properties of charged and uncharged branes in the ten-dimensional non-supersymmetric orientifolds.

In orientifolds, supersymmetry breaking can be dealt with to a level of generality comparable to what previously attained for oriented closed strings [13], and can be realized in essentially four different contexts. In the first, supersymmetry is broken from the start, so that no gravitinos are present in the spectrum. All models of this type are descendants of
the ten-dimensional type 0 strings of [7], as in [5], or of their compactifications, and are in general fraught with tachyons. A special projection, however, leads to the so-called 0′B model, that is free of tachyons both in the open and in the closed spectrum [6], an interesting property shared by corresponding orbifolds [14]. In the second, the Scherk-Schwarz deformations [15] of momenta or windings in “parent” oriented closed models induce supersymmetry breaking at the *compactification scale* in the descendants [16, 17], in a way that reflects the geometry of the corresponding brane configurations [17]. In the third, one resorts to a new option provided by open strings that, consistently with conformal invariance, can be exactly deformed by constant magnetic fields in internal tori [18]. As a result, supersymmetry, unbroken to lowest order in the closed sector, is broken in these models by the magnetic moment couplings of the brane excitations [19]. The resulting scale of supersymmetry breaking is again tied to the *compactification scale*, and more precisely to the areas of the tori affected by the magnetic flux, but again one has generally to face the presence of tachyons [20]. T-dual descriptions relate this setting to configurations with branes at angles [21], and special choices, corresponding to instantons in the internal space, can actually lead to additional supersymmetric vacua where D5 branes, blown up uniformly on the internal tori, are exactly described in terms of magnetized D9 branes [22]. Finally, in the fourth [23, 24] suitable configurations of BPS (anti)branes and orientifold planes induce the breaking of supersymmetry at the *string scale* in the open sector. This “brane supersymmetry breaking”, induced by non-BPS collections of BPS objects, differs from constructions based on stacks of genuine non-BPS branes [12], and leads to tachyon-free vacua, while its low-energy description [25] can be related to a non-linear realization of local supersymmetry *à la* Volkov-Akulov, along the lines of [26].

The spectrum of charged branes for the 0′B model, recently studied in [27], gives a rationale for the results of [6, 14], since it displays the potential ingredients of orbifolds of the ten-dimensional model, showing in particular which combinations do not introduce tachyon instabilities. Fairly enough, a full catalogue of the available branes substantially exhibits the whole variety of phenomena bound to be met in their presence, and with this in mind we shall study in detail the uncharged branes of type 0 models, the analogues of the non-BPS branes present in supersymmetric strings. Our general conclusion will be that, both in supersymmetric and in non-supersymmetric strings, tachyon instabilities are
generally present in stacks of coincident \textit{uncharged} branes, that indeed do not experience a R-R repulsion, while they are systematically absent in stacks of coincident \textit{charged} branes of a given type. We also find that the 0A orientifold of [5] has \textit{both} charged and uncharged branes with identical dimensions. In this paper brane configurations without open-string tachyons will be loosely called “stable” although, strictly speaking, our analysis does not suffice to establish their stability at the quantum level. In a similar fashion, for the sake of brevity we shall refer to the Chan-Paton groups of brane stacks as gauge groups, even for D0 and D(−1) branes.

The description of branes in flat space in the formalism of [5] allows a simple and general analysis of the non-Abelian structure of their excitations. In addition, and more importantly, the resulting constructions do not differ substantially from what is needed to discuss branes in (rational) curved geometries. These settings, although conceptually richer, differ only in the choice of boundary CFT [28], and no essential novelties are met in the construction of string partition functions, although a number of important issues related to their space-time interpretation still await a proper clarification.

O-planes can be discussed along similar lines. Aside from the well-known four types present in type II and type I models, others appear in 0A and 0B orientifolds. Thus, the tachyon-free 0′ B orientifold [6] contains tensionless O9 planes with negative R-R charge, but since its R-R spectrum comprises all even-dimensional forms, additional ones, with $p = 1, 3, 5, 7$, will appear in suitable compactifications. In a similar fashion, the other 0B orientifold in [5] contains O9 planes coupling only to the dilaton, while the third 0B orientifold has O9 planes coupling only to the closed tachyon. The 0A orientifold [5] contains uncharged O9-planes of four-types, depending on all possible signs of their dilaton and tachyon couplings, and corresponding lower-dimensional O-planes will appear in its orbifold compactifications.

The plan of this paper is as follows. In Section 2 we describe the general rules underlying the brane spectra of the ten-dimensional string models, and illustrate them referring to a few simple examples. These suffice, in particular, to exhibit the key role played by the $S$ and $P$ matrices in determining the type I D-branes. In Section 3 we study the properties of stacks of uncharged D-branes in type 0 theories, generalizing the analysis in [5], and show that they are of two types, here called $Dp_{\pm}$, distinguished by the sign of their couplings to
the closed tachyon. In Section 4 we study the properties of stacks of non-BPS Dp-branes in type I strings. We show that the resulting gauge groups are orthogonal or symplectic for even \( p \) and unitary for odd \( p \), and study the cancellation of gauge and gravitational anomalies for non-BPS D3 and D7 branes, whose massless spectra are chiral. Section 5 is devoted to some comments on the super-Higgs mechanism, and in particular to the issue of the gravitino mass, both for non-BPS branes and for non-supersymmetric configurations of BPS (anti)branes with “brane supersymmetry breaking”. Our conclusion will be that, while the former can host a standard super-Higgs mechanism, the latter are bound to lead to non-standard realizations, along the lines of [25]. In Sections 6 and 7 we describe the charged and uncharged branes of the 0B and 0A orientifolds. In all cases we determine gauge groups and matter spectra for brane stacks and describe the cancellation of all potential anomalies and the resulting Wess-Zumino terms. In one of the 0B orientifolds, we find two types of branes of identical dimensionalities, with orthogonal and symplectic gauge groups, respectively. In addition, for the 0A orientifold we find the novel feature that two types of branes with identical dimensions, one charged and one uncharged, are simultaneously present. We conclude in Section 8 with some comments on the fractional branes of these models. These are typically characterized by new types of R-R charges related to orbifold fixed points, that play an important role in orientifold models, being directly responsible for their generalized Green-Schwarz mechanisms [29]. In particular, we present an interesting four-dimensional example with fractional D3 branes at a \( Z_2 \) orbifold singularity in the 0’B model, and relate the peculiar features of boundaries in \( D_{odd} \) WZW models, first noticed in [30], to the appearance of corresponding fractional branes. Finally, the Appendix describes in some detail the compact notation of [5] used for the amplitudes.

2. General rules and some examples

A rational boundary CFT is characterized by a central charge \( c \) and by a finite number of characters \( \{ \chi_i \} \), of conformal weights \( h_i \), acted upon by two matrices, \( S \) and \( P \). The \( S \) matrix, that we shall assume symmetric and unitary, implements on the \( \{ \chi_i \} \) the transformation \( \tau \rightarrow -1/\tau \), and is quite familiar from the bulk CFT of oriented closed strings. On the other hand, the somewhat less familiar \( P \) matrix plays a ubiquitous role in the deter-
mination of non-orientable spectra. It is fair to say that, in these constructions, $P$ replaces the more familiar $T$ matrix, that implements on the $\{\chi_i\}$ the transformation $\tau \rightarrow \tau + 1$ and acts diagonally on them as $T_{ij} = \exp[2i\pi(h_i - c/24)]\delta_{ij}$. $S$ and $T$ actually determine $P$ as

$$P = T^{1/2}ST^2ST^{1/2},$$

(2.1)

where the $T^{1/2}$ factors, with $T^{1/2}_{ij} = \exp[i\pi(h_i - c/24)]\delta_{ij}$, are introduced by phase redefinitions to a convenient real basis of “hatted” characters for the Möbius amplitude. In addition, $S^2 = P^2 = C$, where $C$ is the conjugation matrix of the CFT. $S$ relates the direct and transverse channels of the Klein-bottle and annulus amplitudes, that in the following will be denoted by $\mathcal{K}$, $\bar{\mathcal{K}}$ and $\mathcal{A}$, $\bar{\mathcal{A}}$, while $P$ plays a similar role for the Möbius amplitudes $\mathcal{M}$ and $\bar{\mathcal{M}}$. Annulus and Möbius strip bring about an additional subtlety: their transverse channels describe the tree-level propagation of the closed spectrum, that lives in the embedding space-time, between lower-dimensional branes, while their direct channels are one-loop amplitudes for the open spectra of brane modes [8]. As a result, in describing lower-dimensional branes one is to resort to character bases adapted to their reduced symmetry. This subtlety is particularly relevant for even-$p$ D$p$ branes, that have odd dimensional world volumes and therefore non-chiral spinors.

In this paper, our focus will be on flat-space branes, and therefore the CFT’s we shall need are related to level-one orthogonal affine algebras. The corresponding $S$ and $P$ matrices are collected in the Appendix, together with some other useful properties. As we shall see, $P$ encodes the more peculiar properties of the brane spectra of orientifold models. In order to illustrate its role, let us begin by recovering, in this language, a few simple and well-known results on D-branes in type II and type I models.

The BPS branes of the type IIB model have even-dimensional world volumes and chiral massless spectra. In the notation of [5], briefly reviewed in the Appendix, the annulus amplitude for a stack of these D$p$-branes is

$$\mathcal{A}_{pp} = d\bar{d} \left(V_{p-1}O_{9-p} + O_{p-1}V_{9-p} - S_{p-1}S_{9-p} - C_{p-1}C_{9-p}\right),$$

(2.2)

where we have decomposed the $O(8)$ characters with respect to the $(p - 1)$ light-cone directions longitudinal to the branes. In space-time language, $V_{p-1}O_{9-p}$ describes gauge bosons, $O_{p-1}V_{9-p}$ describes scalars (internal components of ten-dimensional vector fields)
and $S_{p-1}S_{q-p}$ and $C_{p-1}C_{q-p}$ describe space-time fermions. As in [5], the “complex multiplicities” $d(\bar{d})$ label the fundamental (conjugate fundamental) representations of the corresponding $U(d)$ gauge groups, and the coefficients in the corresponding transverse-channel amplitudes,

$$\tilde{A}_{pp} = 2^{-(p+1)/2} d\bar{d} \left( V_{p-1}O_{q-p} + O_{p-1}V_{q-p} - S_{p-1}S_{q-p} - C_{p-1}C_{q-p} \right),$$  \hspace{1cm} (2.3)$$

that determine both the brane tensions and their R-R charges, depend in this case on the \textit{squared absolute values} of the complex multiplicities $d$. In the closed (transverse-channel) amplitude (2.3), the characters have a different interpretation: the NS-NS terms, $V_{p-1}O_{q-p} + O_{p-1}V_{q-p}$, describe the tree-level exchange of dilaton and internal graviton modes, while the R-R terms, $S_{p-1}S_{q-p} + C_{p-1}C_{q-p}$, reflect the R-R charges of the various branes. The decomposition of closed-channel contributions with respect to $SO(p-1) \times SO(9-p)$ characters plays an important role in the D9-Dp amplitudes, that in the transverse channel read

$$\tilde{A}_{qp} = 2^{-5} \left[ (n\bar{d} + \bar{n}d)(V_{p-1}O_{q-p} - O_{p-1}V_{q-p}) \\
+ (e^{-i(p-1)\pi/4} n\bar{d} + e^{i(p-1)\pi/4} \bar{n}d)(S_{p-1}S_{q-p} - C_{p-1}C_{q-p}) \right].$$  \hspace{1cm} (2.4)$$

The $S$ matrix of eq. (A8) determines the corresponding direct-channel amplitudes,

$$A_{qp} = \left. \frac{1}{2} (n\bar{d} + \bar{n}d) \left[ (O_{p-1} + V_{p-1})(S_{q-p} + C_{q-p}) - (S_{p-1} + C_{p-1})(O_{q-p} + V_{q-p}) \right] \right|$$

$$+ \frac{1}{2} (n\bar{d} + e^{i(p-5)\pi/2} \bar{n}d)(O_{p-1} - V_{p-1})(S_{q-p} - C_{q-p})$$

$$+ \frac{1}{2} (e^{-i(p-1)\pi/2} n\bar{d} + \bar{n}d)(O_{p-1} - V_{p-1})(S_{q-p} - C_{q-p}),$$  \hspace{1cm} (2.5)$$

only consistent for odd values for $p$, that in these cases give the chiral spectra

$$A_{qp} = (n\bar{d} + \bar{n}d)(O_{p-1}S_{q-p} + V_{p-1}C_{q-p} - C_{p-1}O_{q-p} - S_{p-1}V_{q-p})$$  \hspace{1cm} (2.6)$$

for $p = 1, 5$ and

$$A_{qp} = n\bar{d} (O_{p-1}S_{q-p} + V_{p-1}C_{q-p} - S_{p-1}O_{q-p} - C_{p-1}V_{q-p})$$

$$+ \bar{n}d (O_{p-1}C_{q-p} + V_{p-1}S_{q-p} - C_{p-1}O_{q-p} - S_{p-1}V_{q-p})$$  \hspace{1cm} (2.7)$$
for \( p = -1, 3, 7 \). The BPS branes for the type IIA string are very similar, being related to these by T-dualities along odd numbers of coordinates \([1, 2]\), but have odd-dimensional world volumes, and thus non-chiral spectra.

In moving to a stack of BPS D-branes in the SO(32) type I model, one has to face the presence of background D9-branes and O9-planes. These are encoded in the familiar amplitudes

\[
\begin{align*}
\mathcal{K} &= \frac{1}{2}(V_8 - S_8), \\
A_{99} &= \frac{n^2}{2}(V_8 - S_8), \\
M_9 &= -\frac{n}{2}(\hat{V}_8 - \hat{S}_8),
\end{align*}
\]

where the “hatted” characters are defined in the Appendix and \( n \) equals 32 on account of tadpole cancellation, and their presence has two important consequences. First, the D\( p \)-D\( p \) amplitude is to be supplemented with additional ones accounting for the propagation of the bulk spectrum between the probe D\( p \) and the background D9 and O9. Moreover, the (overall real) Chan-Paton multiplicities of the probe branes now lead to transverse-channel coefficients that are perfect squares, so that the resulting closed-channel amplitudes are

\[
\begin{align*}
\tilde{A}_{pp} &= \frac{2^{-(p+1)/2}d^2}{2} (V_{p-1}O_{9-p} + O_{p-1}V_{9-p} - S_{p-1}S_{9-p} - C_{p-1}C_{9-p}), \\
\tilde{A}_{p9} &= 2^{-5} n \times d (V_{p-1}O_{9-p} - O_{p-1}V_{9-p} + S_{p-1}S_{9-p} - C_{p-1}C_{9-p}), \\
\tilde{M}_p &= -d (\hat{V}_{p-1}\hat{O}_{9-p} - \hat{O}_{p-1}\hat{V}_{9-p} + \hat{S}_{p-1}\hat{S}_{9-p} - \hat{C}_{p-1}\hat{C}_{9-p}).
\end{align*}
\]

The closed-channel Möbius amplitude \( \tilde{M}_p \) and the D9-D\( p \) amplitude \( \tilde{A}_{p9} \) thus involve, again, a relative sign between the different contributions that breaks the SO(8) space-time symmetry. In all cases, this sign reflects the presence of \( p \)-dimensional extended objects in the embedding ten-dimensional space-time, and can be neatly ascribed, in the open-string channel of the Möbius amplitude, to the additional parity operation carried along by the \( \Omega \) projection when acting in the Dirichlet-Dirichlet sector. In the D\( p \)-D9 sector, as we have seen, a similar relative sign reflects the presence of \( (9 - p) \) Neumann-Dirichlet coordinates, and in more general boundary CFT’s all this is precisely in the spirit of [31], where boundaries preserving only part of the bulk symmetry were studied in detail as the proper general setting for D-brane configurations.

The breaking of the SO(8) symmetry has a clearcut role in the low-energy effective field theory, where the background D9 and probe D\( p \) branes would interact with the ten-
dimensional dilaton $\phi_{10}$ according to
\[ \mathcal{L} = -T_9 \int d^{10}x \sqrt{-g} \ e^{-\phi_{10}} - T_p \int d^{p+1}x \sqrt{-g} \ e^{-\phi_{10}}. \tag{2.10} \]

After a reduction to $p + 1$ dimensions in a compact internal volume $V$, in terms of the $(p + 1)$-dimensional dilaton, defined by
\[ e^{-\phi_{p+1}} = \sqrt{V} \ e^{-\phi_{10}}, \tag{2.11} \]
the resulting couplings would be proportional to
\[ \text{D9} : \sqrt{V} \ e^{-\phi_{p+1}}, \quad \text{Dp} : \frac{1}{\sqrt{V}} \ e^{-\phi_{p+1}}. \tag{2.12} \]

In the closed channel, the coefficient of $V_{p-1}O_{9-p}$ thus determines the identical couplings of D9 and Dp branes to the fluctuation $\delta\phi_{p+1}$ of the $(p + 1)$-dimensional dilaton, while the coefficient of $O_{p-1}V_{9-p}$ determines their opposite couplings to the “breathing mode”, the fluctuation $\delta V$ of the $(9 - p)$-dimensional volume field $V$.

The direct annulus amplitudes derived from (2.9) are
\[ \mathcal{A}_{pp} = \frac{d^2}{2} \ (V_{p-1}O_{9-p} + O_{p-1}V_{9-p} - S_{p-1}S_{9-p} - C_{p-1}C_{9-p}), \tag{2.13} \]
\[ \mathcal{A}_{p0} = \frac{n \times d}{2} \left[ (O_{p-1} + V_{p-1})(S_{9-p} + C_{9-p}) - (S_{p-1} + C_{p-1})(O_{9-p} + V_{9-p}) \right. \]
\[ + e^{-\frac{(9-p)\pi i}{4}}(O_{p-1} - V_{p-1})(S_{9-p} - C_{9-p}) + e^{-\frac{(p-1)\pi i}{4}}(S_{p-1} - C_{p-1})(O_{9-p} - V_{9-p}) \left. \right], \]
and the D9-Dp amplitudes are thus inconsistent unless $p = 1, 5, 9$, that identify the allowed BPS branes in the type I string. The sign in (2.9) has also a crucial effect on the structure of the direct-channel amplitude $\mathcal{M}_p$, determined by the $P$ transformation in eq. (A8) to be
\[ \mathcal{M}_p = -\frac{d}{2} \left[ \sin \frac{(p-5)\pi}{4} (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) + \cos \frac{(p-5)\pi}{4} (\hat{O}_{p-1}\hat{V}_{9-p} - \hat{V}_{p-1}\hat{O}_{9-p}) \right. \]
\[ - i \sin \frac{(p-5)\pi}{4} (\hat{C}_{p-1}\hat{S}_{9-p} - \hat{S}_{p-1}\hat{C}_{9-p}) - \cos \frac{(p-5)\pi}{4} (\hat{S}_{p-1}\hat{S}_{9-p} - \hat{C}_{p-1}\hat{C}_{9-p}) \right]. \tag{2.14} \]

This Möbius projection of $\mathcal{A}$ is thus clearly inconsistent, unless $\sin(p-5)\pi/4$ vanishes, a condition that simply recovers the other allowed BPS D-branes in the SO(32) type I string, that are indeed only D5 and D1. Moreover, since in these two cases the left-over
cosines are equal to ±1, stacks of these D-branes have, as is well known, USp and SO gauge groups respectively. Antibranes can be similarly discussed, reversing the signs of the R-R contributions to the $\mathcal{M}_p$ and $\tilde{A}_{pq}$ amplitudes. In a similar fashion, one can see that the non-supersymmetric USp(32) model of [23] also allows only D5 and D1 branes, albeit with non-supersymmetric spectra, and with SO and USp gauge groups, respectively.

Similar considerations determine the spectra of all charged and uncharged branes of the ten-dimensional string models, and the $P$ matrix always encodes interesting properties of the resulting orientifolds.

The following sections are devoted to a systematic discussion of the ten-dimensional models, and all the results rest on the following, by now standard, criteria:

a. The open-string spectrum, described by the one-loop amplitudes in the open channel, should be compatible with a correct space-time particle interpretation, and in particular with the appropriate spin-statistics relation for bosons and fermions.

b. After suitable modular transformations, the same amplitudes should describe the tree-level propagation of closed strings, in a way consistent with the closed-string spectrum. This also fixes the relative tensions of the various branes.

c. In the ten-dimensional orientifolds, one is also to account for the background O9-planes and D9-branes. The $\Omega$ projection is encoded in the $P$ matrix of the conformal field theory, while in the closed-channel Möbius amplitude the dimensions of the branes determine the decomposition of the corresponding $O(8)$ characters with respect to the $O(p - 1)$ subgroups. This reflects the presence of $(9 - p)$ Dirichlet coordinates in the $D_p$ boundary states, that are folded into the conventional O9-planes.

d. The charged (BPS-like) branes also couple to the R-R fields of the closed sector, while the uncharged (non-BPS-like) ones couple only to NS-NS fields.

Aside from the BPS $D_p$ branes for odd (even) $p$, the type IIB (IIA) models contain non-BPS branes for even (odd) $p$. These additional branes do not carry R-R charges, and are thus potentially unstable [12]. They can be generated subjecting brane-antibrane pairs, that in type IIB would be described by

$$\tilde{A}_{pp} = 2^{-(p+1)/2} \left[ |m + n|^2 (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) - |m - n|^2 (S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) \right]$$
\[ \mathcal{A}_{pp} = (m\bar{m} + n\bar{n})(V_{p-1}O_{9-p} + O_{p-1}V_{9-p} - S_{p-1}S_{9-p} - C_{p-1}C_{9-p}) \]
\[ + (m\bar{m} + n\bar{n})(O_{p-1}O_{9-p} + V_{p-1}V_{9-p} - S_{p-1}C_{9-p} - C_{p-1}S_{9-p}) , \] (2.15)

to an orbifold operation that interchanges them, to be combined with a corresponding \( Z_2 \) operation in the closed spectrum. This involves the left space-time fermion number, and as a result turns the original type IIB into type IIA. Hence, one is finally relating non-BPS branes in type IIA to brane-antibrane pairs in type IIB. In the open sector all this corresponds to identifying \( n \) and \( m \) with a single charge multiplicity \( N \), while rescaling the amplitudes by an overall factor \( \frac{1}{2} \), so that
\[ \tilde{\mathcal{A}}_{pp} = 2 \times 2^{-(p+1)/2} N \bar{N} (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) \]
\[ \mathcal{A}_{pp} = N \bar{N} [(O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) - (S_{p-1} + C_{p-1})(S_{9-p} + C_{9-p})] . \] (2.16)

The low-lying open spectrum in (2.16) contains a vector boson, \((9-p)\) scalars, a tachyon and a non-chiral fermion, all in the adjoint representation of the \( U(N) \) gauge group. It is worth stressing that, from the CFT viewpoint, these are simply branes associated to non-diagonal bulk modular invariants, \( i.e. \) defined in settings that are more general than the Cardy case [32]. These non-BPS branes interact with the dilaton, with a tension \( \sqrt{2} \) times larger than that of the BPS branes, as needed for a correct particle interpretation of their open-string states, and consistently with their instability. It was conjectured by Sen that, after tachyon condensation, they should decay into the vacuum. We shall return to this issue later, since it poses interesting questions related to the super-Higgs effect triggered by non-supersymmetric branes.

One can discuss with no further difficulties systems of different branes, although for the sake of brevity we shall mostly refrain from doing it in the following sections. For instance, the strings stretching between \( n \) \( Dp \) and \( d \) \( Dq \) non-BPS branes, where \( p - q = 0 \) mod 2 and, for definiteness, \( p > q \), have \( q + 1 \) Neumann-Neumann (NN) coordinates, \( p - q \) Neumann-Dirichlet (ND) coordinates and \( 9 - p \) Dirichlet-Dirichlet (DD) coordinates. The corresponding annulus amplitudes read
\[ \tilde{\mathcal{A}}_{pq} = 2 \times 2^{-(p+1)/2} (n\bar{d} + \bar{n}d) (V_{8-p+q}O_{p-q} - O_{8-p+q}V_{p-q}) , \] (2.17)
\[ \mathcal{A}_{pq} = (n\bar{d} + \bar{n}d) [(O_{8-p+q} + V_{8-p+q})(S_{p-q} + C_{p-q}) - (S_{8-p+q} + C_{8-p+q})(O_{p-q} + V_{p-q})] . \]
In order to exhibit the resulting spectrum, the characters are to be decomposed with respect to the SO\((q - 1)\) little group, making use of eq. (A2) of the Appendix, but in all cases there are non-chiral space-time fermions in bi-fundamental representations of \(U(n) \times U(d)\). In addition, this \(D_p-D_q\) spectrum contains tachyons for \(|p - q| < 4\), massless scalars for \(|p - q| = 4\) and only massive bosons for \(|p - q| > 4\). One can similarly write the \(D_p-D_q\) amplitude between a BPS and a non-BPS brane \((p - q = 1 \mod 2)\), and the corresponding annulus amplitudes, similar to the previous ones, read

\[
\tilde{A}_{pq} = \sqrt{2} \times 2^{-(p+1)/2} \left( n\bar{d} + \bar{n}d \right) \left( V_{8-p+q}O_{p-q} - O_{8-p+q}V_{p-q} \right),
\]
\[
A_{pq} = (n\bar{d} + \bar{n}d) \left[ (O_{8-p+q} + V_{8-p+q})S'_{p-q} - S'_{8-p+q}(O_{p-q} + V_{p-q}) \right].
\]

The novelties in (2.18) are the \(\sqrt{2}\) factor in the closed channel, that results from the geometric average of BPS and non-BPS brane tensions, and the appearance of the non-chiral fermion characters (A3) in the open channel, due to the odd number of ND coordinates.

Sen [12] actually introduced an additional selection rule for these non-BPS branes. For instance, in the D9 case, if one starts with brane-antibrane stacks in type IIB, as we have seen one is led to introduce a pair of Chan-Paton multiplicities \(m\) and \(n\). These integers are dimensions of sub-blocks of large Chan-Paton matrices, of size \(m + n\), whose charges are split among the different states. In order to arrive at the non-BPS D9 brane in type IIA, one begins by noting that type IIA can be obtained from type IIB by the orbifold operation \((-1)^{G_L}\), where \(G_L\) denotes the left space-time fermion number, that in the open sector induces precisely the interchange of branes and antibranes. This is reflected in the analytic dependence on the charge multiplicities, proportional to \((n - m)\), of the R-R boundary one-point functions in eq. (2.15). The induced operation is a symmetry of the open spectrum only if \(m = n = N\), and in this case the resulting projection breaks the gauge group to the diagonal combination of the two original ones. This is precisely the gauge group for a stack of non-BPS D9 branes in type IIA, but this construction somehow leaves behind a reducible representation of the resulting Chan-Paton gauge group, with matrices \(\lambda_{V,S} = M \otimes I_2\) and \(\lambda_{O,C} = M \otimes \sigma_1\), with \(M\) a more familiar \(N \times N\) hermitian matrix associated to the adjoint of \(U(N)\). The reducible matrices enforce a selection rule: in all non-vanishing amplitudes any boundary must contain an even number of \(\sigma_1\) factors. A related observation is that some of the closed-string vertex operators, when inserted in
amplitudes, are to decorate boundaries with additional powers of $1 \otimes \sigma_1$. These may be regarded as end-points of cuts originating from these vertices, and one is to sum over the possible decorations of this type, in analogy with corresponding sums over cuts familiar from GSO projections of fermionic systems or from orbifold constructions. The meaning of these additional insertions can be understood noting that, when a closed-string vertex is moved toward a boundary, eventually it is to turn into open-string vertices with proper Chan-Paton assignments, that in this framework have acquired additional labels. The R-R bulk fields of this type belong to the $S_8 \bar{C}_8$ sector and, when they come close to boundaries, turn into open-string ones of the $O_8$ sector.

We can now move on to complete our description of charged and uncharged branes in the various ten-dimensional string theories. This, as anticipated by this discussion, can be done in rather general and efficient terms using the formalism of [5]. One of the results of this work is that, in order to obtain non-Abelian gauge groups from stacks of coincident branes with no open string tachyons, the branes are to be charged under some R-R fields of the theory under consideration. In addition, all the ten-dimensional orientifold models allow charged tachyon-free brane stacks of this type.

3. The D-branes of type 0 string theories

The two ten-dimensional type 0 theories [7]

$$T_{0A} = |O_8|^2 + |V_8|^2 + S_8 \bar{C}_8 + C_8 \bar{S}_8,$$

$$T_{0B} = |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2,$$

(3.1)

contain a tachyon in their NS-NS spectra. The 0B theory has four types of odd-$p$ D$p$ branes, characterized by a pair of R-R charges relative to its two R-R sectors, whose annulus amplitudes

$$\tilde{A}_{pp} = \frac{2^{-(p+1)/2}}{2} \left[ |n_1 + n_2 + n_3 + n_4|^2(V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) + |n_1 + n_2 - n_3 - n_4|^2(O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) - |n_1 - n_2 + n_3 - n_4|^2(S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) \right]$$
- \|n_1 - n_2 - n_3 + n_4\|^2 (S_{p-1}C_{9-p} + C_{p-1}S_{9-p}) \right], \\
A_{pp} &= (n_1\tilde{n}_1 + n_2\tilde{n}_2 + n_3\tilde{n}_3 + n_4\tilde{n}_4)(O_{p-1}V_{9-p} + V_{p-1}O_{9-p}) \\
&\quad + (n_1\tilde{n}_2 + n_2\tilde{n}_1 + n_3\tilde{n}_4 + n_4\tilde{n}_3)(O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) \\
&\quad - (n_1\tilde{n}_3 + n_3\tilde{n}_1 + n_2\tilde{n}_4 + n_4\tilde{n}_2)(S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) \\
&\quad - (n_1\tilde{n}_4 + n_4\tilde{n}_1 + n_2\tilde{n}_3 + n_3\tilde{n}_2)(S_{p-1}C_{9-p} + C_{p-1}S_{9-p}) \right), \quad (3.2)

are essentially as in [5], although of course they involve four types of complex charges, so that the resulting gauge groups are \(U(n_1) \times U(n_2) \times U(n_3) \times U(n_4)\). Notice that the signs of the couplings of these branes to the dilaton, the tachyon and the two R-R sectors, determined by the \(S_8\) and \(C_8\) contributions to \(\tilde{A}_{pp}\), are \((+, +, +, +)\), \((+, +, -, -)\), \((+,-,+,+)\), \((+,-,-,+)\), so that the branes of the second and fourth types can be regarded as antibranes of those of the first \((D_{p1})\) and third \((D_{p2})\) types. The uncharged (non-BPS-like) D9 branes of the 0A model are now obtained as \(D9_1-\overline{D9_1}\) and \(D9_2-\overline{D9_2}\) combinations, along the lines of what was reviewed in the previous section. As in that simpler case, the orbifold of the closed spectrum by \((-1)^{G_L}\) turns type 0B into type 0A, and in the open sector interchanges branes and antibranes. This operation is a symmetry when their numbers are equal, and the end result is

\[ \tilde{A}_{pp} = 2^{-(p+1)/2} \left| n + m \right|^2 (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) + \left| n - m \right|^2 (O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) \right], \\
A_{pp} &= (n\tilde{m} + m\tilde{n}) (O_{p-1} + V_{p-1}) (O_{9-p} + V_{9-p}) \\
&\quad - (n\tilde{m} + \tilde{n}m) (S_{p-1} + C_{p-1}) (S_{9-p} + C_{9-p}) \right), \quad (3.3)

where the suffix \(p\) anticipates the fact that \(T\)-duality connects the D9 branes of type 0A to corresponding lower-dimensional uncharged \(Dp\) branes, present for even values of \(p\) in type 0B and for odd values of \(p\) in type 0A. The four original gauge groups of charged type 0B D9 branes are broken to a pair of diagonal combinations, so that the general gauge group for a stack of these uncharged branes is \(U(m) \times U(n)\). With a reducible representation of the Chan-Paton group, the vector \((V_8)\) and tachyon \((O_8)\) sectors, although both valued in the adjoint representation, would actually be distinguished by additional tensor factors 1 or \(\sigma_1\), and the same would be true for the two spinorial sectors, valued in corresponding bi-fundamental representations. Notice that these uncharged branes are actually of two
types, distinguished by the relative sign of their couplings to the closed-string tachyon. The branes of the first type, called in the following D$p_+$ branes, have a positive coupling to the dilaton, \textit{i.e.} a positive tension, and a positive coupling to the tachyon. On the other hand, the branes of the second type, called in the following D$p_-$ branes, have a positive tension and a negative coupling to the tachyon\textsuperscript{1}. These neutral branes of type 0 theories have a tension equal to that of corresponding BPS branes in type II theories, but larger than that of the charged type 0 branes by a factor $\sqrt{2}$. Notice that, in analogy with other examples in the literature [5, 6, 34, 27], the open spectrum contains space-time fermions, although the closed sector contains only bosons.

The strings stretching between $n$ D$p_+$ and $d$ D$q_\pm$ branes, for definiteness with $p > q$ ($p-q = 0 \text{ mod } 2$), have $q+1$ Neumann-Neumann (NN) coordinates, $p-q$ Neumann-Dirichlet (ND) coordinates and $9-p$ DD coordinates. The corresponding annulus amplitudes read

$$\tilde{A}_{pq} = 2^{-(p+1)/2}(n\bar{d} + \bar{n}d) \left( O_{8-p+q}O_{p-q} - V_{8-p+q}V_{p-q} + V_{8-p+q}O_{p-q} - O_{8-p+q}V_{p-q} \right) ,$$

$$A_{pq} = (n\bar{d} + \bar{n}d) \left( O_{8-p+q} + V_{8-p+q}\right) (S_{p-q} + C_{p-q}) , \quad (3.4)$$

and as a result the spectra contain tachyons for $|p-q| < 4$, massless scalars for $|p-q| = 4$ and only massive bosons for $|p-q| > 4$. On the other hand, for a system of $n$ D$p_\pm$ and $d$ D$q_\mp$ branes, the amplitudes read

$$\tilde{A}_{pq} = 2^{-(p+1)/2}(n\bar{d} + \bar{n}d) \left( -O_{8-p+q}O_{p-q} + V_{8-p+q}V_{p-q} + V_{8-p+q}O_{p-q} - O_{8-p+q}V_{p-q} \right) ,$$

$$A_{pq} = -(n\bar{d} + \bar{n}d) \left( S_{8-p+q} + C_{8-p+q}\right) (O_{p-q} + V_{p-q}) , \quad (3.5)$$

and the corresponding open spectra contain non-chiral massless fermions.

All type II and type 0 uncharged branes are unstable, as signalled by the presence of tachyons in their spectra. Some of these tachyons, however, can be eliminated compactifying on suitable orbifolds [12], if the branes are placed at fixed points.

\textsuperscript{1}This distinction, also noticed in the recent preprint [46], that appeared while this paper was being typed, is also manifest in the D9 spectra of the 0A orientifolds in [5].
4. The non-BPS branes of type I strings

There are two types of ten-dimensional type I strings: aside from the usual supersymmetric model with an SO(32) gauge group [4], there is indeed a second, non-supersymmetric model, with a USp(32) gauge group [23]. Whereas in the first model there are 32 D9 branes and 32 conventional O9+ planes, with negative tension and negative R-R charge, in the second there are 32 O9− planes, with positive tension and positive R-R charge, together with 32 anti D9-branes [23]. In the latter case, local supersymmetry is non-linearly realized à la Volkov-Akulov in the brane sector [25]. Both variants of type I strings have BPS D9, D5 and D1 branes, as well as non-BPS branes for the remaining values of \( p \). In this section we briefly present their construction in the formalism of [5], that allows a straightforward generalization of previous results in [36, 11] to arbitrary stacks. In type I strings, these D-branes are immersed in the proper D9 and O9 background, and therefore in this section the \( p-p \) annulus amplitude will always be accompanied by a Möbius amplitude, originating from the O9-D\( p \) exchange, and by a D9-D\( p \) amplitude, describing the spectrum of strings stretched between the probe D\( p \) branes and the background D9 branes.

Stacks of \( d \) non-BPS D\( p \) branes for even \( p \) (\( p = 0, 2, 4, 6, 8 \)) can be obtained applying the orientifold projection to the corresponding non-BPS branes of the parent type IIB. As they are uncharged with respect to the R-R fields, the \( \Omega \) projection acts diagonally on their Chan-Paton factors, and therefore one expects orthogonal or symplectic gauge groups. The corresponding D\( p \)-D\( p \) annulus amplitudes are thus

\[
\bar{A}_{pp} = 2^{-(p+1)/2} d^2 (V_{p-1} O_{9-p} + O_{p-1} V_{9-p}) ,
\]

\[
A_{pp} = \frac{d^2}{2} (O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) - 2 \times \frac{d^2}{2} S'_{p-1} S'_{9-p} ,
\]

where the non-chiral fermion characters \( S' \) for odd space-time dimensions are defined in eq. (A3). In this case the fermions cannot contribute to \( \mathcal{M}_p \), since they do not flow in \( \bar{A}_{pp} \), and thus, \( a \) fortiori, in \( \bar{M}_p \). However, the presence of two R-R contributions allows a clearcut interpretation of the spectrum, in terms of two sectors, one symmetrized and one anti-symmetrized, consistently with the absence of a net contribution to \( \mathcal{M}_p \). This is actually one more instance of a general phenomenon in boundary CFT, first met in WZW models in [33, 30]: the contributions of states with identical charges that enter \( \mathcal{A} \) with even
multiplicities need not be matched by corresponding terms in $\mathcal{M}_p$. Equivalently, in general $\mathcal{A}$ and $\mathcal{M}$ need only match modulo 2 for such diagonal terms, and whenever they do not match one is describing one or more (symmetric+antisymmetric) pairs of representations of the gauge group.

The O9-Dp contribution is encoded in the Möbius amplitude, whose precise normalization is unambiguously determined by the non-BPS tension in (4.1) and by the O9 tension. The result reads

$$\tilde{\mathcal{M}}_p = -\epsilon \sqrt{2} d (\hat{V}_{p-1} \hat{O}_{9-p} - \hat{O}_{p-1} \hat{V}_{9-p}),$$

$$\mathcal{M}_p = -\frac{\epsilon d}{\sqrt{2}} \left[ \sin \left( \frac{(p-5)\pi}{4} \right) (\hat{O}_{p-1} \hat{O}_{9-p} + \hat{V}_{p-1} \hat{V}_{9-p}) + \cos \left( \frac{(p-5)\pi}{4} \right) (\hat{O}_{p-1} \hat{V}_{9-p} - \hat{V}_{p-1} \hat{O}_{9-p}) \right],$$

where the sign $\epsilon$ is +1 for the SO(32) string and -1 for the USp(32) string. In relating the open and closed channels, we have used again the $P$ transformation in eq. (A8), that introduces crucial additional factors of $\sqrt{2}$ in $\mathcal{M}_p$ for all even $p$. In a similar fashion, the D9-Dp spectrum can be easily extracted from the annulus amplitudes

$$\tilde{\mathcal{A}}_{9\theta} = 2^{-5} \sqrt{2} \times 32 \times d (V_{p-1} O_{9-p} - O_{p-1} V_{9-p}),$$

$$\mathcal{A}_{9\theta} = 32 d \left[ (O_{p-1} + V_{p-1}) S'_{9-p} - S'_{p-1} (O_{9-p} + V_{9-p}) \right],$$

where, again, the non-chiral fermion characters $S'$ for odd space-time dimensions are defined in eq. (A3) of the Appendix. For all these branes, the tension is $\sqrt{2}$ times larger than it would be for BPS branes of the same dimension.

These expressions summarize the complete open spectra for the various non-BPS Dp branes ($p$ even) in the two type I strings. For the SO(32) model they are as follows:

- **D0-brane**: SO($d$) Chan-Paton group, tachyons in the adjoint, scalars (including the position of the branes) in the symmetric representation and fermions in the symmetric and antisymmetric representations. The massless D0-D9 spectrum contains only fermions in the $(32, d)$ of SO(32) × SO($d$). The tachyon is projected out if $d=1$, and therefore a single D particle is stable, as correctly pointed out in [12].

- **D2-brane**: SO($d$) gauge group, tachyons and scalars (including the position of the branes) in the symmetric representation and fermions in the symmetric and antisymmetric representations. The massless D2-D9 spectrum contains only fermions in the
(32, d) of SO(32) × SO(d). The tachyon cannot be eliminated, and therefore the D2 brane is unstable.

- **D4-brane**: USp(d) gauge group, tachyons in the adjoint representation, scalars (including the position of the branes) in the antisymmetric representation and fermions in the symmetric and antisymmetric representations. The massless D4-D9 spectrum contains only fermions in the (32, d) of SO(32) × USp(d). The tachyon cannot be eliminated, and therefore the D4 brane is unstable.

- **D6-brane**: USp(d) gauge group, tachyon and scalars (including the position of the branes) in the antisymmetric representation and fermions in the symmetric and antisymmetric representations. The D6-D9 spectrum contains tachyons and massless fermions in the (32, d) of SO(32) × USp(d), and therefore the D6 brane is unstable.

- **D8-brane**: The D8-D8 spectrum is similar to the D0-D0 spectrum above, and reduces to it upon dimensional reduction of all spatial coordinates. The D8-D9 spectrum contains tachyons and massless fermions in the (32, d) of SO(32) × SO(d), and therefore the D8 brane is unstable.

The corresponding spectra for the USp(32) string can be obtained from these interchanging orthogonal and symplectic gauge groups, as well as the related symmetric and antisymmetric representations for the matter modes. In particular, in this case a single D4 brane, rather than a single D0 brane, is stable.

Type I strings have also non-BPS $D(-1)$, $D3$ and $D7$ branes, but these have a more peculiar structure, since for these dimensions $\Omega$ can be defined only for type IIB brane-antibrane pairs, and interchanges them. As a result, stacks of these additional branes have unitary gauge groups, and the corresponding annulus amplitudes are

$$\tilde{A}_{pp} = \frac{2^{-p+1/2}}{2} \left[ (d + \bar{d})^2 (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) + (d - \bar{d})^2 (S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) \right]$$

$$A_{pp} = d\bar{d} (O_{p-1}V_{9-p} + V_{p-1}O_{9-p} - S_{p-1}S_{9-p} - C_{p-1}C_{9-p})$$

$$+ \frac{d^2 + \bar{d}^2}{2} (O_{p-1}O_{9-p} + V_{p-1}V_{9-p} - S_{p-1}C_{9-p} - C_{p-1}S_{9-p}) . \quad (4.4)$$

Notice that the R-R coupling in the closed channel is actually unphysical, a familiar state of affairs whenever “complex” charges are present, in agreement with the fact that these non-
BPS branes are uncharged. As usual, the corresponding closed-channel Möbius amplitude

\[
\tilde{\mathcal{M}}_p = (d + \bar{d})(\hat{O}_{p-1}\hat{V}_{9-p} - \hat{V}_{p-1}\hat{O}_{9-p}) - (d - \bar{d})(\hat{S}_{p-1}\hat{S}_{9-p} - \hat{C}_{p-1}\hat{C}_{9-p}) , \quad (4.5)
\]
can be obtained as a “geometric mean” of the probe Dp-Dp (cylinder) and background O9-O9 (Klein) amplitudes, while

\[
\mathcal{M}_p = \frac{d + \bar{d}}{2} \sin \left( \frac{(p-5)\pi}{4} \right) (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) \\
- \frac{d - \bar{d}}{2} e^{i\frac{(p-5)\pi}{4}} (-i) \sin \left( \frac{(p-5)\pi}{4} \right) (\hat{S}_{p-1}\hat{C}_{9-p} - \hat{C}_{p-1}\hat{S}_{9-p}) , \quad (4.6)
\]
follows from it after a \( P \) transformation. We thus found, as anticipated, a \( U(d) \) gauge group, with \( 9 - p \) scalars and fermions in the adjoint representation, the latter obtained dimensionally reducing a ten-dimensional Majorana-Weyl fermion to the Dp-brane world-volume.

For the D3 (D7 and D(−1)) brane there are also complex tachyons in (anti)symmetric representations, Weyl fermions of positive chirality in the symmetric representation and Weyl fermions of negative chirality in the antisymmetric representation of the gauge group. Finally, the low-lying Dp-D9 spectrum, encoded in the amplitudes

\[
\tilde{A}_{p9} = 2^{-5} n \left[ (d + \bar{d})(V_{p-1}O_{9-p} - O_{p-1}V_{9-p}) - i(d - \bar{d})(S_{p-1}S_{9-p} - C_{p-1}C_{9-p}) \right] , \\
A_{p9} = d n \left( O_{p-1}S_{9-p} + V_{p-1}C_{9-p} - C_{p-1}O_{9-p} - S_{p-1}V_{9-p} \right) \\
+ \bar{d} n \left( O_{p-1}C_{9-p} + V_{p-1}S_{9-p} - S_{p-1}O_{9-p} - C_{p-1}V_{9-p} \right) , \quad (4.7)
\]
where \( n \), equal to 32, is the D9 Chan-Paton multiplicity, comprises in both cases massless Weyl fermions in the \((32, d)\) of \( SO(32) \times U(d) \), and for the D7 and D(−1) branes also complex tachyons in the \((32, d)\). These chiral spectra embody a non-trivial cancellation of irreducible gauge anomalies between the Dp-Dp and Dp-D9 sectors. The corresponding results for the USp(32) type I string can again be obtained interchanging symmetric and antisymmetric representations, while also flipping the (space-time and internal) chiralities in the Dp-D9 sector. Notice that the \( D(-1) \) brane (D-instanton) in the \( SO(32) \) string and the D3 brane in the USp(32) string are stable, being free of tachyons. The tensions of these branes are twice the values one would expect for BPS type I branes of the same dimension, if they existed.

Since the non-BPS D3 and D7 branes have chiral spectra, it is instructive to verify how the resulting anomalies cancel. The potentially anomalous groups are the \( SO(1, p) \) Lorentz
group relative to the world-volume of the branes, the transverse $\text{SO}(9-p)$ rotation group, the ten-dimensional $\text{SO}(32)$ gauge group and, finally, the $U(d)$ gauge group for the brane stack [37]. The anomaly polynomials are the 6 and 10-form contributions to

$$
\hat{A}(R) [\hat{A}(N)]^{-1} \left[ \frac{1}{2} (c_+ (N) - c_- (N)) \, tr_{ade} e^G 
+ c_- (N) \, tr_{se} e^G - c_+ (N) \, tr_{ae} e^G - tr_{e} e^F \right],
$$

(4.8)

where $R$, $N$, $G$ and $F$ are curvature forms for the $\text{SO}(1,9)$, $\text{SO}(9-p)$, $U(d)$ and $\text{SO}(32)$ gauge groups. An explicit calculation reveals that all irreducible anomalies indeed cancel in (4.8), while the residual anomaly polynomials are

$$
I_6 = Y_2 \, X_4 , \quad (4.9)
$$

$$
I_{10} = Y_2 \, X_8 + Y_6 \, X_4 - 2 \, N \, Y_2 \, Y_6 . \quad (4.10)
$$

Here

$$
Y_2 = -i \, tr G , \quad Y_6 = \frac{i}{6} \, tr G^3 + i \, tr G \frac{p_1 (R)}{48} - \frac{i}{24} \, tr G \, N^2 , \quad (4.11)
$$

$$
X_4 = -p_1 (R) - \frac{1}{2} \, tr F^2 , \quad X_8 = \frac{tr F^4}{24} + \frac{p_1 (R) \, tr F^2}{96} + \frac{3p_1 (R)^2 - 4p_2 (R)}{192} . \quad (4.12)
$$

factorize the ten dimensional anomaly polynomial as $I_{12} = X_4 \, X_8$, and

$$
p_1 (R) = -\frac{1}{2} \, tr R^2 , \quad p_2 (R) = \frac{1}{8} \, [(tr R^2)^2 - 2 \, tr R^4] . \quad (4.13)
$$

It should be appreciated that the adjoint fermions play a crucial role in canceling the irreducible part of the anomaly due to the curvature of the normal bundle. The residual anomaly polynomials in (4.10) can then be canceled by the Wess-Zumino terms

$$
S_{WZ}(D_3) = T_1 \int_{D_3} Y_2 \, B_2 ,
$$

$$
S_{WZ}(D_7) = T_5 \int_{D_7} Y_2 \, B_6 + T_1 \int_{D_7} Y_6 \, B_2 . \quad (4.14)
$$

in the effective actions for the D-branes. In verifying the cancellation one needs to use the relations

$$
\delta(D3)|_{D3} = \chi(N) , \quad \delta(D7)|_{D7} = N \quad (4.15)
$$
between the $\delta$ functions on the brane world-volumes and the Euler characters of the normal bundles [39]. The mixing of U(1) gauge fields with R-R forms in the Wess-Zumino couplings implies that they become massive, leaving only SU($d$) unbroken gauge groups. It is interesting to note that wrapping the D7 brane on a magnetized torus would give rise to a non-BPS, but nevertheless charged, D5 brane.

5. Comments on the super-Higgs mechanism on non-BPS branes

The D-branes and O-planes of supersymmetric (type II or type I) strings can trigger the complete breaking of supersymmetry, and this asks for an understanding of the corresponding super-Higgs mechanisms. This issue was recently analyzed in [25] for the non-BPS combinations present in the USp(32) type I string [23], where supersymmetry is broken at the tree level in the open sector due to the simultaneous presence of $\overline{D}9$ branes and $O9_-$ planes. More precisely, the minimal ten-dimensional supersymmetry is realized linearly in the closed sector and non-linearly in the open sector, and the goldstino, present in the massless brane spectrum, has consistent interactions, although the Majorana-Weyl ten-dimensional gravitino does not allow a mass term in the Lagrangian. The super-Higgs mechanism is thus taking an unconventional form in this ten-dimensional model. This peculiar fact is also revealed by a simple counting: the 64 combined degrees of freedom of the massless gravitino and of the brane goldstino are far fewer than the 128 proper of a massive ten-dimensional gravitino. Still, they are compatible with a massive nine-dimensional gravitino, that together with the corresponding internal component $\psi_9$ would have precisely 64 components, and indeed the theory has a vacuum with an SO(1,8) symmetry group, smaller than the maximal symmetry groups compatible with the ten-dimensional brane world-volume [40]. Further arguments to this effect are provided in [41].

A similar phenomenon appears at work also in lower-dimensional models with “brane supersymmetry breaking” [24]. In all these cases one has non-BPS combinations of BPS (anti)branes, that in type I are D9, D5 and D1, and corresponding O-planes. The simplest manifestation of D5 branes in this context is provided by the six-dimensional $T^4/Z_2$ type I model with 32 D9 branes and 32 $\overline{D}5$ branes [24], with a supersymmetric closed spectrum including 16 (1,0) tensor multiplets from the twisted sector, one per fixed point. In world-
sheet language, the Ω projection has here a flipped sign in the whole twisted sector, a feature reminiscent of the WZW models discussed in [33, 30]. In space-time language, the vacuum includes 32 O9$^+$ planes and 32 O5$^-$ planes, and as a result supersymmetry is broken at the string scale on all D5 branes sitting at orbifold fixed points. The fermion counting relative to a D5-brane world volume now goes as follows: a massive six-dimensional gravitino would have 32 degrees of freedom, while the original massless one has 12, and the brane goldstino only 4, but again a similar five-dimensional counting does not contradict a standard lower-dimensional realization of the super-Higgs mechanism, since a massive five-dimensional gravitino together with the corresponding internal component $\psi_5$ would have 16 degrees of freedom. These two examples thus capture all basic features, in this respect, of four-dimensional models with “brane supersymmetry breaking”.

A similar question is clearly raised by the other non-BPS branes of type II and type I theories discussed in the previous sections. Again, the D$p$-D$p$ sector always contains a candidate goldstino, with the correct chirality, and a quick case-by-case analysis shows that massless gravitinos and goldstinos always provide the proper numbers of degrees of freedom for massive gravitinos on brane world-volumes. For instance, the non-BPS D9 branes of type IIA have in their world volumes a Majorana goldstino, with 16 degrees of freedom, that can be eaten by a massless ten-dimensional Majorana gravitino, with 112 degrees of freedom, to give a massive ten-dimensional Majorana gravitino, with 128 degrees of freedom. As another example, the D8 branes of type I have in their world volumes a nine-dimensional Majorana goldstino, that can turn a massless nine-dimensional Majorana gravitino, with 48 degrees of freedom, into a massive one. As a last example, the D3 branes of type I have in their world volumes four Majorana goldstinos, which can mix with the four Majorana gravitinos to give four massive Majorana gravitinos. All these examples thus point toward standard realizations of the super-Higgs mechanism on non-BPS branes. In principle, this could be verified explicitly from the effective Lagrangian for the ten-dimensional supergravities coupled to non-BPS D$p$ branes, following lines similar to those in [25]. Further support to this conjecture is provided by the classical supergravity solution corresponding to D$p$-D$p$ systems [45], that has the full SO($p+1$) symmetry along directions tangential to the world-volume of the non-BPS branes, as expected for a conventional super-Higgs mechanism on them. However, the classical backgrounds of non-BPS configurations
typically have naked metric singularities, whose resolution is clearly important in order to
gain a better control of these super-Higgs mechanisms and of the corresponding low-energy
physics.

6. The D-branes of the 0B orientifolds

The 0B orientifolds were constructed in [5, 6]. There are three possible choices, generated by \( \Omega, \Omega \times (-1)^{G_L} \), where \( G_L \) is the left space-time fermion number, and \( \Omega \times (-1)^{F_L} \), where \( F_L \) is the left world-sheet fermion number. The last model, usually called 0'B in the
literature, is non-tachyonic and contains chiral fermions in the D9 open spectrum [6]. Non-
tachyonic orbifold compactifications of the 0'B orientifold were studied in [14], while the
spectrum of its charged branes and their anomaly cancellation mechanisms were recently
described in [27], where it was shown that the theory contains D9, D7, D5, D3, D1 and
D(−1) charged branes, all with individual non-tachyonic chiral spectra based on unitary
gauge groups \(^2\).

The orientifold projection \( \Omega \times (-1)^{F_L} \) leading to the 0'B theory corresponds to the Klein
bottle amplitude

\[
\mathcal{K} = \frac{1}{2} (-O_8 + V_8 - S_8 + C_8), \\
\tilde{\mathcal{K}} = -2^5 S_8,
\]

(6.1)

and the tachyon, odd under the orientifold projection, is thus removed from the spec-
trum, while a net number of D9 branes is needed to compensate the R-R charge of the
O9 planes. As displayed in (6.1), the O-planes are in this case rather peculiar, since
they have a vanishing tension. The D-brane spectrum can actually be anticipated re-
calling that, as described in Section 3, the parent 0B theory has two types of charged
branes, whose R-R charges have the overall signs (+, +) and (+, −), and two correspond-
ing types of charged antibranes. The orientifold acts collectively on the two sets of R-
R fields \( (A, A') \) in (6.1) according to \( \Omega(A, A') = (-A^T, A'^T) \), and on the different R-R
forms this translates into \( \Omega(A^{(0)}, A'^{(0)}) = (-A^{(0)}, A'^{(0)}), \Omega(A^{(2)}, A'^{(2)}) = (A^{(2)}, -A'^{(2)} \) and

\(^2\)Tachyonic modes, however, are present in Dp-Dq exchange spectra, for \(|p - q| < 4\), as is also the case
for the type IIB string.
Ω(A^{(4)}, A'^{(4)}) = (-A^{(4)}, A'^{(4)}). Hence, in all cases only the combination (+, +) + (−, +), and the corresponding antibrane, are invariant under Ω or, in the language of Section 3, configurations with an arbitrary number d of Dp1-Dp2 pairs, whose members are interchanged by the Ω projection. All this is reminiscent of what we saw for the D3 and D7 branes of the type I models, and one can anticipate the occurrence of unitary gauge groups and of a BPS-like behavior for these charged branes, with no mutual tree-level interaction energy.

Let us briefly review the explicit construction of the charged Dp branes present in this model. To begin with, the D9 branes are described by the amplitudes [6]

\[ \tilde{A}_{99} = \frac{2^{5}}{4} \left[ (n + \bar{n})^{2}(V_{8} - S_{8}) - (n - \bar{n})^{2}(O_{8} - C_{8}) \right], \]
\[ \tilde{M}_{9} = (n + \bar{n}) \hat{S}_{8}, \]
\[ A_{99} = n \bar{n} V_{8} - \frac{n^{2} + \bar{n}^{2}}{2} S_{8}, \]
\[ M_{9} = \frac{n + \bar{n}}{2} \hat{S}_{8}, \]

where \( n \) is a “complex” Chan-Paton multiplicity, and the R-R tadpoles require that \( n = \bar{n} = 32 \), thus fixing the \( U(32) \) gauge group. The massless D9 spectrum is chiral, since it includes Weyl fermions in the antisymmetric representation, precisely as needed to compensate the bulk contribution to irreducible gravitational anomalies. Notice that \( \tilde{A} \) contains an unphysical tachyon coupling in the closed sector, proportional to \( (n - \bar{n}) \), consistently with the fact that the closed-string tachyon was actually removed by the orientifold projection.

The D5 and D1 branes are described by

\[ \tilde{A}_{pp} = \frac{2^{-(p+1)/2}}{4} \left[ (d + \bar{d})^{2}(V_{p-1}O_{9-p} + O_{p-1}V_{9-p} - S_{p-1}S_{9-p} - C_{p-1}C_{9-p}) \\
- (d - \bar{d})^{2}(O_{p-1}O_{9-p} + V_{p-1}V_{9-p} - S_{p-1}C_{9-p} - C_{p-1}S_{9-p}) \right], \]
\[ \tilde{M}_{p} = (d + \bar{d})(\hat{S}_{p-1} \hat{S}_{9-p} - \hat{C}_{p-1} \hat{C}_{9-p}) , \]
\[ A_{pp} = d \bar{d} (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) - \frac{d^{2} + \bar{d}^{2}}{2}(S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) , \]
\[ M_{p} = \epsilon \frac{d + \bar{d}}{2} (\hat{S}_{p-1} \hat{S}_{9-p} - \hat{C}_{p-1} \hat{C}_{9-p}) , \]

where, as in the previous sections, the relative sign between the two contributions to \( M_{p} \) reflects the dimensionality of the branes, while the sign \( \epsilon, +1 (-1) \) for D5(D1) branes, is
dictated by the corresponding $P$ matrices. In both cases the gauge group is $U(d)$, and in both cases the massless spectra contain Weyl fermions of one chirality in the symmetric representation, together with Weyl fermions of the opposite chirality in the antisymmetric representation. In addition, the D9-D$p$ spectrum is described by

$$\mathcal{A}_{p^9} = (n\bar{d} + \bar{n}d)(O_{p-1}S_{9-p} + V_{p-1}C_{9-p}) - (nd + \bar{n}d)(S_{p-1}V_{9-p} + C_{p-1}O_{9-p}) \ , \quad (6.4)$$

and therefore [27] both the D5-D9 and D1-D9 spectra have chiral fermions in bi-fundamental representations and no tachyons.

On the other hand, the D7, D(−1) and D3 branes present a subtlety [27], since they couple to the 0-form and to the 4-form from the R-R sector, described by the $C_8$ character in (6.1). This subtlety is precisely encoded in the $P$ matrix, and indeed, starting again from the closed channel and reverting to the open channel by $S$ and $P$ transformations gives the consistent amplitudes

$$\hat{A}_{pp} = \frac{2^{-(p+1)/2}}{4} [(d + \bar{d})^2(V_{p-1}O_{9-p} + O_{p-1}V_{9-p} - S_{p-1}C_{9-p} - C_{p-1}S_{9-p})$$

$$- (d - \bar{d})^2(O_{p-1}O_{9-p} + V_{p-1}V_{9-p} - S_{p-1}S_{9-p} - C_{p-1}C_{9-p})] \ ,$$

$$\hat{M}_p = -(d - \bar{d})(\hat{S}_{p-1}\hat{S}_{9-p} - \hat{C}_{p-1}\hat{C}_{9-p})$$

$$\check{A}_{pp} = d\bar{d}(V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) - \frac{d^2 + \bar{d}^2}{2}(S_{p-1}C_{9-p} + C_{p-1}S_{9-p}) \ ,$$

$$\check{M}_p = \epsilon \frac{d - \bar{d}}{2}(\hat{S}_{p-1}\hat{C}_{9-p} - \hat{C}_{p-1}\hat{S}_{9-p}) \ , \quad (6.5)$$

where the sign $\epsilon$ is +1 (−1) for the D3 (D7 and D(−1)) branes.

In addition, the D9-D$p$ spectra are described by

$$\mathcal{A}_{p^9} = \bar{n}d(O_{p-1}S_{9-p} + V_{p-1}C_{9-p}) + nd(O_{p-1}C_{9-p} + V_{p-1}S_{9-p})$$

$$- nd(S_{p-1}O_{9-p} + C_{p-1}V_{9-p}) - \bar{n}d(S_{p-1}V_{9-p} + C_{p-1}O_{9-p}) \ , \quad (6.6)$$

and therefore [27] there are tachyonic modes in the D9-D7 case. Massless fermions in bi-fundamental representations are present in both the D9-D7 and D9-D3 mixed spectra. Notice that all these charged branes, whose anomaly cancellation was studied in [27], are BPS-like, i.e. there is no brane-brane interaction to lowest order.
We can now turn to the uncharged branes present in the 0′B orientifold, that exist for \(p = 0, 2, 4, 6, 8\), and whose spectra can again be determined starting from the uncharged branes of the parent 0B model. As we saw in Section 3, in type 0 theories the uncharged branes are of two types, \(D_p^+\) and \(D_p^-\), distinguished by the sign of their coupling to the tachyon. In this case the orientifold projection interchanges them, since it removes the closed string tachyon, and therefore the final invariant combinations contain an arbitrary number \(d\) of \(D_p^+\)-\(D_p^-\) pairs, with corresponding \(U(d)\) gauge groups.

The consistency requirements summarized in Section 2 uniquely determine the \(D_p\)-\(D_p\) amplitude, and therefore the spectrum of a generic stack of these uncharged D-branes can be read from

\[
\tilde{A}_{pp} = 2^{-(p+1)/2} \left[ (d + \bar{d})^2 (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) - (d - \bar{d})^2 (O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) \right],
\]

\[
A_{pp} = d\bar{d} (O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) - 2 \times \frac{(d^2 + \bar{d}^2)}{2} S'_{p-1}S'_{9-p},
\]

where the fermions are now described by the non-chiral characters (A3) appropriate for these odd-dimensional world-volumes. In contrast with the parent 0B theory, in this case there is a single type of uncharged brane, that couples only to the dilaton. A peculiar and interesting feature of these uncharged branes is the lack of \(D_p\)-\(O9\) propagation, since only the R-R fields couple to the \(O9\)-planes while, on the contrary, only the NS-NS dilaton couples to these \(D_p\)-branes. As in Section 4, the lack of a Möbius contribution implies the presence of two (symmetric+antisymmetric) pairs of R sectors. In addition, there are \(D_p\)-\(D9\) string excitations described by

\[
A_{p9} = (n\bar{d} + \bar{n}d)(O_{p-1} + V_{p-1})S'_{9-p} - (nd + \bar{n}\bar{d})S'_{p-1}(O_{9-p} + V_{9-p}),
\]

\[
\tilde{A}_{p9} = \frac{2^{-(p+1)/2}}{\sqrt{2}} [(n + \bar{n})(d + \bar{d})(V_{p-1}O_{9-p} - O_{p-1}V_{9-p}) - (n - \bar{n})(d - \bar{d})(O_{p-1}O_{9-p} - V_{p-1}V_{9-p})],
\]

where \(n\), equal to 32, is the D9 Chan-Paton multiplicity.

We can now turn to the orientifold of the 0B model obtained by the standard \(\Omega\) projection, whose Klein bottle amplitude reads [5]

\[
\mathcal{K} = \frac{1}{2} (O_8 + V_8 - S_8 - C_8),
\]

\[
\tilde{\mathcal{K}} = 2^5 V_8.
\]

(6.9)
In this case there is no consistency condition asking for a D9 sector, aside from a dilaton tadpole, that just signals the need for a non-trivial space-time background. If, as in [5], one includes it, the resulting open spectrum is described by

\begin{align}
A_{99} &= \frac{n_1^2 + n_2^2 + n_3^2 + n_4^2}{2} V_8 + (n_1 n_2 + n_3 n_4) O_8 \\
&\quad - (n_1 n_3 + n_2 n_4) S_8 - (n_1 n_4 + n_2 n_3) C_8,
\end{align}

and the induced R-R tadpole cancellation conditions require that \( n_1 = n_2 \) and \( n_3 = n_4 \), thus determining the family of D9 gauge groups \( \text{SO}(n_1)^2 \times \text{SO}(n_4)^2 \), while the dilaton tadpole is canceled by the unique choice \( n_1 + n_3 = 32 \). In addition, as for type I strings, one has the option of reversing the Möbius projection altogether, thus obtaining symplectic gauge groups but, as for the USp(32) type I string, it is then impossible to cancel the resulting dilaton tadpole. Notice that, in both cases, no choice for the charges \( n_1 \) and \( n_3 \) can eliminate the open-string tachyons, since we are forced to add branes and antibranes in equal numbers.

Since the projected closed spectrum contains two R-R two-forms, the model should also have charged D1 and D5 branes. Moreover, the two types of branes of the parent 0B, \( Dp_1 \) and \( Dp_2 \), separately invariant under \( \Omega \), will both appear in this orientifold, together with their antibranes, albeit with projected gauge groups. The resulting \( Dp-Dp \) annulus amplitudes are simply determined by the dimensional reduction of the D9-D9 amplitude in [5] to the Dp world volume, and read

\begin{align}
A_{pp} &= \frac{d_1^2 + d_2^2 + d_3^2 + d_4^2}{2} (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) + (d_1 d_2 + d_3 d_4) (O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) \\
&\quad - (d_1 d_3 + d_2 d_4) (S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) - (d_1 d_4 + d_2 d_3) (S_{p-1}C_{9-p} + C_{p-1}S_{9-p}),
\end{align}

\begin{align}
\tilde{A}_{pp} &= \frac{2^{-(p+1)/2}}{4} \left[ (d_1 + d_2 + d_3 + d_4)^2 (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) \\
&\quad + (d_1 + d_2 - d_3 - d_4)^2 (O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) \right] \\
&\quad - \frac{2^{-(p+1)/2}}{4} \left[ (d_1 - d_2 + d_3 - d_4)^2 (S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) \\
&\quad + (d_1 - d_2 - d_3 + d_4)^2 (S_{p-1}C_{9-p} + C_{p-1}S_{9-p}) \right].
\end{align}

Eq. (6.11) clearly displays the couplings of these branes to the two sets of R-R fields. The
peculiar properties of the lower-dimensional branes are again fully encoded in the $P$ matrix and, taking into account the proper character decompositions,

$$M_p = \frac{d_1 + d_2 + d_3 + d_4}{2} \epsilon (\hat{V}_{p-1} \hat{O}_{9-p} - \hat{O}_{p-1} \hat{V}_{9-p}) ,$$

(6.12)

where the sign $\epsilon$ is $+1$ for the D5 branes and $-1$ for the D1 branes. The resulting gauge groups are thus $\text{USp}(d_1) \times \text{USp}(d_2) \times \text{USp}(d_3) \times \text{USp}(d_4)$ for the D5 branes and $\text{SO}(d_1) \times \text{SO}(d_2) \times \text{SO}(d_3) \times \text{SO}(d_4)$ for the D1 branes. In a similar fashion, for the ten-dimensional model with symplectic gauge groups, one would find orthogonal groups for D5 branes and symplectic groups for D1 branes. In all these cases, one can obtain tachyon-free configurations considering only branes, i.e. setting $d_2 = d_4 = 0$, with gauge groups of the type $\text{USp}(d_1) \times \text{USp}(d_3)$ for the D5 branes and $\text{SO}(d_1) \times \text{SO}(d_3)$ for the D1 branes, and vice versa for the other class of ten-dimensional models. Moreover, equal numbers of $Dp_1$ and $Dp_2$ branes give gauge groups $\text{USp}(d_1^2) (\text{SO}(d_1)^2)$ for D5(D1) branes, and vice versa for the second class of ten-dimensional models, and these $Dp$-$Dp$ configurations are BPS-like, i.e. there is no net tree-level brane-brane interaction. As we have seen, this property is shared not only by the BPS branes of type II and type I models, but also by the D9 [6] and the other charged $Dp$ branes [27] of the 0'B orientifold. One can similarly obtain the additional spectra related to $Dp$-$D9$ exchanges. For instance, the D5-D9 spectrum is encoded in the amplitude

$$\mathcal{A}_{59} = (n_1d_1 + n_2d_2 + n_3d_3 + n_4d_4) (O_4C_4 + V_4S_4)$$

$$+ (n_1d_2 + n_2d_3 + n_3d_4 + n_4d_1) (O_4S_4 + V_4C_4)$$

$$- (n_1d_4 + n_2d_3 + n_3d_2 + n_4d_1) (C_4O_4 + S_4V_4)$$

$$- (n_1d_3 + n_2d_4 + n_3d_1 + n_4d_2) (S_4O_4 + C_4V_4) ,$$

(6.13)

with massless scalars and fermions in bi-fundamental representations, while the D1-D9 amplitude contains fermions of opposite chiralities, obtained interchanging the $S$ and $C$ sectors.

The chiral spectra of these charged branes imply an anomaly inflow from the D9 branes, and it is instructive to study the corresponding anomaly cancellation. The anomaly polynomial of the ten-dimensional model (6.9) is in this case

$$I_{12} = X_4^{(+)} X_8^{(-)} + X_4^{(-)} X_8^{(+)} ,$$

(6.14)
with

$$X_4^{(+)} = \frac{1}{4} (tr F_1^2 - tr F_3^2), \quad X_4^{(-)} = \frac{1}{4} (tr F_2^2 - tr F_4^2),$$

$$X_8^{(+)} = \frac{1}{24} (tr F_3^4 - tr F_4^4) - \frac{tr R^2}{192} (tr F_3^2 - tr F_4^2),$$

$$X_8^{(-)} = \frac{1}{24} (tr F_4^4 - tr F_3^4) - \frac{tr R^2}{192} (tr F_2^2 - tr F_1^2), \quad (6.15)$$

where $F_1, \cdots, F_4$ denote the four D9-brane gauge groups. In order to derive unambiguously Bianchi identities and field equations of the R-R-forms, one also needs the anomaly polynomials for the D1 and D5 branes,

$$I_4 = d_1 X_4^{(+)} + d_3 X_4^{(-)}, \quad (6.16)$$

$$I_8 = \frac{d_1}{2} X_8^{(+)} + \frac{d_2}{2} X_8^{(-)} + Y_4^{(+)} (X_4^{(-)} + \frac{d_1}{2} \chi(N)) + Y_4^{(+)} (X_4^{(+)} + \frac{d_2}{2} \chi(N)),$$

where $\chi(N)$ denotes the Euler class of the normal bundle. Letting

$$Y_4^{(+)} = -\frac{1}{2} tr G_1^2 + \frac{d_1}{96} tr R^2 - \frac{d_1}{48} tr N^2,$$

$$Y_4^{(-)} = -\frac{1}{2} tr G_3^2 + \frac{d_3}{96} tr R^2 - \frac{d_3}{48} tr N^2, \quad (6.17)$$

where $G_1$ and $G_3$ describe the USp($d_1$) \times USp($d_3$) D5-brane gauge fields, the residual anomalies on the D1 and D5 branes are compensated by the Wess-Zumino terms

$$S_{WZ}(D1) = T_1 \int_{D1} \left[ d_1 (B_2^{(1)} + B_2^{(2)}) + d_3 (B_2^{(1)} - B_2^{(2)}) \right],$$

$$S_{WZ}(D5) = T_5 \int_{D5} \left[ \frac{d_1}{2} (B_6^{(1)} + B_6^{(2)}) + \frac{d_3}{2} (B_6^{(1)} - B_6^{(2)}) \right]$$

$$+ T_1 \int_{D5} \left[ Y_4^{(+)} (B_2^{(1)} + B_2^{(2)}) + Y_4^{(-)} (B_2^{(1)} - B_2^{(2)}) \right]. \quad (6.18)$$

Notice that, since the D5 gauge groups are symplectic, the number of D5 branes is actually $d_3/2$, as neatly reflected in the Wess-Zumino couplings above. In verifying the anomaly cancellation for the D5 brane, one needs again the relation between the $\delta$ function on its world volume and the Euler class of the normal bundle [39]

$$\delta(D5)|_{D5} = \chi(N). \quad (6.19)$$

It is interesting to examine the spectrum of the D string, since this could provide hints for the strong coupling limit of this orientifold. This issue was already considered in [35]. As we
have seen, there are two types of D1 branes, that differ in the spectra of the corresponding D1-D9 states. For the first, there are Majorana-Weyl fermions in the fundamental of SO($n_3$) and Majorana-Weyl fermions of opposite chirality in the fundamental of SO($n_4$), while for the second $n_{3,4}$ are to be replaced by $n_{1,2}$. For the first type the central charge, $c = 10 + n_4/2$, actually becomes critical if $n_4 = 32$, and therefore if $n_1 = n_2 = 0$. One is thus tempted, following [35], to relate the S-dual of the orientifold with the gauge group SO(32)$^2$ to the bosonic string compactified on an SO(32) lattice. An additional argument in favor of this conjecture is the presence of charged D5 branes in the orientifold, such that for a single five-brane the gauge group is SU(2), since in the S-dual theory this could become the NS five-brane of the ten-dimensional bosonic theory with gauge group SO(32) × SO(32).

Compactifications on group lattices of this type play a central role in the scenario proposed by Englert et al to relate all ten-dimensional fermionic strings to the bosonic string [43]. However, the orientifold contains a second set of D5 and D1 stable (charged) branes, whose role in the S-dual theory is not clear.

The orientifold (6.9) also contains uncharged D-branes. The D7, D3 and D(−1) branes are in this case Ω-invariant combinations of the form $(+, +) + (-, -)$ and $(+, -) + (-, +)$. Since Ω interchanges branes and antibranes of the parent 0B theory, one expects unitary gauge groups for each invariant brane-antibrane configuration, and indeed the corresponding open-string amplitudes read

\[
A_{pp} = (d_1 \bar{d}_1 + d_2 \bar{d}_2)(V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) \\
+ (d_1^2 + \bar{d}_1^2 + d_2^2 + \bar{d}_2^2)(O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) \\
- (d_1 \bar{d}_2 + d_2 \bar{d}_1)(S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) \\
- (d_1 \bar{d}_2 + \bar{d}_1 d_2)(S_{p-1}C_{9-p} + C_{p-1}S_{9-p}) ,
\]

\[
M_p = - \frac{d_1 + \bar{d}_1 + d_2 + \bar{d}_2}{2} \epsilon(\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) ,
\]

where the sign $\epsilon$, determined again by the $P$ matrix, is +1 for D7 and D(−1) branes and −1 for D3 branes. The resulting gauge groups are therefore $U(d_1) \times U(d_2)$, and the open tachyon can be eliminated for D7 branes with gauge group $U(1) \times U(1)$, obtained if $d_1 = d_2 = 1$, with a non-chiral fermion spectrum, or for a single D7 brane with a U(1) gauge group and no fermions in the spectrum, but not for the D3 brane, that is thus unstable.
The D7-D9 spectrum can be derived from the amplitude

\[ A_{79} = (n_1 d_1 + n_2 d_1 + n_3 d_2 + n_4 d_2)(O_6 S_2 + V_6 C_2) \]
\[ + (n_1 \bar{d}_1 + n_2 d_1 + n_3 \bar{d}_2 + n_4 d_2)(O_6 C_2 + V_6 S_2) \]
\[ - (n_1 \bar{d}_2 + n_2 d_2 + n_3 \bar{d}_1 + n_4 d_1) (C_6 O_2 + S_6 V_2) \]
\[ - (n_1 d_2 + n_2 \bar{d}_2 + n_3 d_1 + n_4 \bar{d}_1) (S_6 O_2 + C_6 V_2) , \] (6.21)

that contains tachyonic modes, while the D3-D9 amplitude could be obtained from (6.21) interchanging the two fermion chiralities \( S \) and \( C \). For the symplectic orientifolds, the roles of D3 and D7 are interchanged, and a single D3 is now completely free of tachyons, that are also absent in the D3-D9 sector.

The model contains additional uncharged \( D_p \) branes, with \( p = 0, 2, 4, 6, 8 \). The orientifold projection acts directly on the Chan-Paton factors of the parent 0B uncharged branes \( D_{p+} \) and \( D_{p-} \), and therefore one can anticipate the presence of orthogonal or symplectic gauge groups in all these cases. If the Chan-Paton multiplicities of the invariant \( D_{p+} \) and \( D_{p-} \) combinations are denoted by \( d_1 \) and \( d_2 \), the resulting annulus and Möbius amplitudes read

\[ A_{pp} = \frac{d_1^2 + d_2^2}{2}(O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) - 2d_1d_2 S'_{p-1} S'_9 , \] (6.22)

\[ M_p = -\frac{d_1 + d_2}{\sqrt{2}} \left[ \sin \left( \frac{(p-5)\pi}{4} \right) (\hat{O}_{p-1} \hat{O}_{9-p} + \hat{V}_{p-1} \hat{V}_{9-p}) + \cos \left( \frac{(p-5)\pi}{4} \right) (\hat{O}_{p-1} \hat{V}_{9-p} - \hat{V}_{p-1} \hat{O}_{9-p}) \right] , \]

in terms of the non-chiral \( S' \) characters defined in (A3). The D6 and D4 branes have gauge groups \( \text{SO}(d_1) \times \text{SO}(d_2) \), and the open tachyons can be eliminated for the D6 brane if \( d_1 = d_2 = 1 \), leaving a spectrum without a residual gauge symmetry. For D0, D2 and D8 branes, the gauge groups are \( \text{USp}(d_1) \times \text{USp}(d_2) \), but these branes are unstable, since no configuration can eliminate their open-string tachyons. As usual, for the symplectic orientifolds the orthogonal and symplectic groups are interchanged for all these lower-dimensional branes.

Finally, the \( D_p \)-D9 spectrum manifests itself in the amplitude

\[ A_{p9} = [(n_1 + n_2)d_1 + (n_3 + n_4)d_2] (O_{p-1} + V_{p-1}) S'_{9-p} \]
\[ - [(n_3 + n_4)d_1 + (n_1 + n_2)d_2] S'_{9-p} (O_{9-p} + V_{9-p}) , \] (6.23)
that, as usual, contains tachyons for \( p > 5 \).

The third 0B orientifold of [6] is determined by the projection \( \Omega \times (-1)^{G_L} \), so that its Klein bottle amplitude reads

\[
\mathcal{K} = \frac{1}{2} (O_8 + V_8 + S_8 + C_8),
\]

\[
\tilde{\mathcal{K}} = 2^5 O_8.
\]

As in the previous case, \( \tilde{\mathcal{K}} \) does not introduce any R-R tadpoles, and therefore the model is consistent even without introducing D9 branes. The orientifold projection now keeps the two R-R 0-forms and an unconstrained R-R four-form, and therefore the model contains two types of charged D3, D7 and D\((-1)\) branes. For \( p = -1, 3, 7 \), the charged branes of the parent 0B are separately invariant, and therefore the projection acts diagonally on their Chan-Paton factors, generating orthogonal or symplectic gauge groups. On the other hand, for \( p = 5, 9 \) the invariant combinations are \((+, +) + (-, -)\) and \((+, -) + (-, +)\), and thus one can again anticipate the occurrence uncharged branes with unitary gauge groups. Finally, the two type 0B uncharged branes, D\(_p^+\), D\(_p^-\), present for even \( p \), are left invariant by the orientifold projection, and therefore in these cases one expects orthogonal or symplectic gauge groups. The most general uncharged D9 brane spectra are based on the gauge group \( U(n) \times U(m) \), have tachyons in the antisymmetric representation of the first unitary factor and in the symmetric representation of the second, and the relevant D9 amplitudes are [6]

\[
\mathcal{A}_{99} = (n\bar{n} + m\bar{m})V_8 + \frac{n^2 + \bar{n}^2 + m^2 + \bar{m}^2}{2}O_8
\]

\[- (n\bar{n} + \bar{n}m)S_8 - (nm + \bar{n}\bar{m})C_8,\]

\[
\mathcal{M}_9 = \frac{1}{2} (n + \bar{n} - m - \bar{m})\hat{O}_8.\]

Therefore, as in the previous orientifold, there is no configuration without D9 open-string tachyons. The open string amplitudes for the charged D3 and D7 or D\((-1)\) branes read

\[
\mathcal{A}_{pp} = \frac{d_1^2 + d_2^2 + d_3^2 + d_4^2}{2} (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) + (d_1d_2 + d_3d_4)(O_{p-1}O_{9-p} + V_{p-1}V_{9-p})
\]

\[- (d_1d_3 + d_2d_4)(S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) - (d_1d_4 + d_2d_3)(S_{p-1}C_{9-p} + C_{p-1}S_{9-p}),\]

\[
\mathcal{M}_p = - \frac{d_1 + d_2 - d_3 - d_4}{2} \epsilon (V_{p-1}\hat{O}_{9-p} - \hat{O}_{p-1}V_{9-p}),\]

\[(6.26)\]
where $\epsilon$ is +1 for D7 and D(−1) branes and −1 for D3 branes, so that the resulting gauge groups are $SO(d_1) \times SO(d_2) \times USp(d_3) \times USp(d_4)$ and $USp(d_1) \times USp(d_2) \times SO(d_3) \times SO(d_4)$ in the two cases. As in the parent 0B model, there are tachyon-free configurations, obtained including the two types of charged branes but not their antibranes, i.e. for $d_2 = d_4 = 0$, with gauge groups $SO(d_1) \times USp(d_3)$ for the D7 and D(−1) branes and $USp(d_1) \times SO(d_3)$ for the D3 branes. Moreover, configurations with equal numbers $d$ of $(+, +)$ and $(+, −)$ branes, with gauge groups $SO(d) \times USp(d)$, have no brane-brane interactions at the classical level. The D3 branes are particularly interesting: as for the 0′B model, their spectra become conformal in the large-$d$ limit, and the AdS/CFT correspondence [44] can thus be extended to them even in the absence of supersymmetry, proceeding as in [34]. The D3-D9 spectrum can be derived from the amplitude

$$A_{39} = (nd_1 + \bar{n}d_2 + md_3 + \bar{m}d_4) (O_2S_6 + V_2C_6)$$

$$+ (nd_2 + \bar{n}d_1 + md_4 + \bar{m}d_3) (O_2C_6 + V_2S_6)$$

$$- (nd_3 + \bar{n}d_4 + md_1 + \bar{m}d_2) (S_2O_6 + C_2V_6)$$

$$- (nd_4 + \bar{n}d_3 + md_2 + \bar{m}d_1) (S_2V_6 + C_2O_6) ,$$

(6.27)

and contains massless fermions in bi-fundamental representations, while the D7-D9 amplitude is very similar, aside from the interchange of the two spinor chiralities.

As in the previous orientifold, the lower-dimensional charged (D3 and D7) branes have an anomaly inflow from the D9 branes. In this case, the anomaly polynomial is given by

$$I_{12} = X_2^{(+)}X_1^{(-)} + X_2^{(-)}X_1^{(+)} + X_6^{(+)}X_6^{(-)} ,$$

(6.28)

with

$$X_2^{(+)} = -i \, tr F_2 , \quad X_2^{(-)} = 2i \, tr F_1 ,$$

$$X_6^{(+)} = -\frac{i}{3} \, tr F_2^3 , \quad X_6^{(-)} = \frac{i}{6} \, tr F_1^3 ,$$

$$X_{10}^{(+)} = -\frac{i}{120} \, tr F_2^5 + \frac{i}{288} \, tr R^2 \, tr F_2^3 - i \, \frac{7p_1^2 - 4p_2}{2 \times 5760} \, tr F_2 ,$$

$$X_{10}^{(-)} = \frac{i}{60} \, tr F_1^5 - \frac{i}{144} \, tr R^2 \, tr F_1^3 + i \, \frac{7p_1^2 - 4p_2}{5760} \, tr F_1 ,$$

(6.29)
where $F_1$, $F_2$ describe the D9 $U(m) \times U(n)$ field strengths. The anomaly polynomials for the D3 and D7 branes are then

\begin{align*}
I_6 &= d_3(-X_6^{(-)} + \frac{d_1}{4} \chi(N)) + \frac{d_1}{2} (X_6^{(+)} - \frac{d_3}{2} \chi(N)) + X_2^{(+)} Y_4^{(+)} + X_2^{(-)} Y_4^{(-)}, \\
I_{10} &= \frac{d_3}{2}(-X_{10}^{(-)} + \frac{N}{2} Z_8^{(+)} ) + d_1(X_{10}^{(+)} - \frac{N}{2} Z_8^{(-)} ) + (X_6^{(+)} - \frac{N}{2} Z_4^{(-)} ) Z_4^{(+)} \\
&\quad + (X_6^{(-)} - \frac{N}{2} Z_4^{(-)} ) Z_4^{(-)} + (X_2^{(+)} + \frac{d_3}{4} N ) Z_8^{(+)} + (X_2^{(-)} - \frac{d_1}{2} N ) Z_8^{(-)}, \quad (6.30)
\end{align*}

where

\begin{align*}
Y_4^{(+)} &= \frac{1}{2} tr G_1^2 - \frac{d_1}{48} (tr R^2 - tr N^2), \\
Y_4^{(-)} &= \frac{1}{4} tr G_3^2 + \frac{d_3}{96} (tr R^2 - tr N^2), \\
Z_4^{(+)} &= \frac{1}{4} tr G_1^2 - \frac{d_1}{48} N^2, \quad Z_4^{(-)} = -\frac{1}{2} tr G_3^2 + \frac{d_3}{24} N^2, \\
Z_8^{(+)} &= \frac{1}{24} tr G_1^4 - \frac{1}{48} tr G_3^2 N^2 + \frac{d_1}{1920} N^4 - \frac{1}{96} tr R^2 ( tr G_1^2 - \frac{d_1}{12} N^2 ) + d_1 \frac{7 p_1^2 - 4 p_2}{2 \times 5760}, \\
Z_8^{(-)} &= -\frac{1}{48} tr G_3^4 + \frac{1}{96} tr G_3^2 N^2 - \frac{d_3}{3840} N^4 + \frac{1}{192} tr R^2 ( tr G_3^2 - \frac{d_3}{12} N^2 ) - \frac{d_3}{4} \frac{7 p_1^2 - 4 p_2}{4 \times 5760}.
\end{align*}

In (6.30), $\chi(N)$ denotes the Euler class of the normal bundle for the D3 brane, and $N$ denotes the curvature of the U(1) normal bundle for the D7 brane.

The resulting Wess-Zumino couplings are then

\begin{align*}
S_{WZ}(D3) &= T_3 \int_{D3} \left[ \frac{d_1}{2} (B_4^{(1)} + B_4^{(2)}) + d_3 (B_4^{(1)} - B_4^{(2)}) \right] \\
&\quad + T_{-1} \int_{D3} \left[ (B_0^{(1)} + B_0^{(2)}) Y_4^{(+)} + (B_0^{(1)} - B_0^{(2)}) Y_4^{(-)} \right], \\
S_{WZ}(D7) &= T_7 \int_{D7} \left[ d_1 (B_8^{(1)} + B_8^{(2)}) + \frac{d_3}{2} (B_8^{(1)} - B_8^{(2)}) \right] \\
&\quad + T_3 \int_{D7} \left[ (B_4^{(1)} + B_4^{(2)}) Z_4^{(+)} + (B_4^{(1)} - B_4^{(2)}) Z_4^{(-)} \right] \\
&\quad + T_{-1} \int_{D7} \left[ (B_0^{(1)} + B_0^{(2)}) Z_8^{(+)} + (B_0^{(1)} - B_0^{(2)}) Z_8^{(-)} \right]. \quad (6.32)
\end{align*}

Notice that, again, the number of symplectic-like branes is actually $d_1/2$ in the D3 case and $d_3/2$ in the D7 case. In (6.32), $B_0^{(1,2)}$ are the two R-R zero-forms, $B_8^{(1,2)}$ are their duals,
$B_4^{(1)}$ is the self-dual four-form and $B_4^{(2)}$ the anti self-dual four-form. Once more, in verifying the anomaly cancellation induced by the Wess-Zumino terms, it is crucial to use [39]

$$\delta(D3)|_{D3} = \chi(N), \quad \delta(D7)|_{D7} = N.$$  \hspace{1cm} (6.33)

There are also uncharged D5 and D1 branes, with open string amplitudes

$$A_{pp} = (d\bar{d} + q\bar{q})(V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) + \frac{d^2 + \bar{d}^2 + q^2 + \bar{q}^2}{2} (O_{p-1}O_{9-p} + V_{p-1}V_{9-p})$$

$$- (d\bar{q} + \bar{d}q)(S_{p-1}S_{9-p} + C_{p-1}C_{9-p}) - (d\bar{q} + \bar{d}q)(S_{p-1}C_{9-p} + C_{p-1}S_{9-p}),$$

$$M_p = - \epsilon \frac{d + \bar{d} - q - \bar{q}}{2} (\dot{O}_{p-1}\dot{O}_{9-p} + \dot{V}_{p-1}\dot{V}_{9-p}),$$  \hspace{1cm} (6.34)

where $\epsilon$ is +1 for the D5 branes and −1 for the D1 branes, with gauge groups $U(d) \times U(q)$, as previously anticipated. The open-string tachyon can be eliminated for a single D1 brane or for a single D5 brane, with a U(1) gauge group and no massless space-time fermions. The D5-D9 amplitude is

$$A_{59} = (n\bar{d} + \bar{n}d + mq + \bar{m}q) (O_4S_4 + V_4C_4) + (nd + \bar{n}d + mq + \bar{m}q) (O_4C_4 + V_4S_4)$$

$$- (nq + \bar{n}q + md + \bar{m}d) (S_4O_4 + C_4V_4) - (n\bar{q} + \bar{n}q + m\bar{d} + \bar{m}d) (S_4V_4 + C_4O_4),$$  \hspace{1cm} (6.35)

while the D1-D9 amplitude could be obtained from it interchanging the two spinor chiralities.

Finally, there are uncharged $D_p$ branes with $p = 0, 2, 4, 6, 8$, whose open string amplitudes are

$$A_{pp} = \frac{d_2^2 + d_1^2}{2} (O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) - 2d_1d_2 S'_{p-1}S'_{9-p},$$  \hspace{1cm} (6.36)

$$M_p = \frac{d_2 - d_1}{\sqrt{2}} \left[ \cos \left(\frac{(p-5)\pi}{4}\right) (\dot{O}_{p-1}\dot{O}_{9-p} + \dot{V}_{p-1}\dot{V}_{9-p}) + \sin \left(\frac{(p-5)\pi}{4}\right) (\dot{V}_{p-1}\dot{O}_{9-p} - \dot{O}_{p-1}\dot{V}_{9-p}) \right],$$

so that the resulting gauge groups for the $D_+$ and $D_-$ branes are $SO(d_1)$ and $USp(d_2)$. The open-string tachyons can be eliminated for the D2 and D6 branes, leaving no residual gauge symmetry. On the other hand, the remaining D0, D4 and D8 branes are all unstable.

7. The D-branes of the 0A orientifold

One can also derive the charged and uncharged brane spectrum of the 0A orientifold, whose D9 structure was first displayed in [5]. As we shall see, this model contains non-
tachyonic charged branes of lower dimensionalities with full-fledged non-Abelian gauge groups.

The 0A orientifold is obtained supplementing the torus amplitude of [7]

$$\mathcal{T} = |O_8|^2 + |V_8|^2 + S_8 \tilde{C}_8 + C_8 \tilde{S}_8 ,$$  \hspace{1cm} (7.1)

with the Klein-bottle amplitudes [5]

$$\tilde{\mathcal{K}} = \frac{2^5}{2}(O_8 + V_8) ,$$

$$\mathcal{K} = \frac{1}{2}(O_8 + V_8) ,$$  \hspace{1cm} (7.2)

related as usual by an $S$ transformation. In this case the orientifold projection $\Omega$ can not eliminate the closed-string tachyon, that will consequently couple to the open sector. It interchanges the two types of R-R $n$-forms ($n = 1, 3, 5, 7, 9$) in the parent 0A model, here denoted by $A_n$ and $A'_n$, according to

$$\Omega A_n = (-1)^{(n+1)(n+2)/2} A'_n ,$$  \hspace{1cm} (7.3)

as can be deduced from the corresponding $\gamma$ matrices. This novel feature compared to the 0B or IIB cases, where $\Omega$ acts diagonally on the R-R fields, reflects the nature of the 0A model, that is described by a non-diagonal modular invariant. Notice that the orientifold projection of this model is not connected by T-duality to any of the three 0B orientifold projections discussed in the previous section.

As one can see from $\tilde{\mathcal{K}}$, there is no induced R-R tadpole, and therefore also in this case one could refrain from including a D9 open sector. One has, however, the option of including it, as in [5], and this contains precisely the two types of uncharged branes already discussed in Section 3, albeit with suitably projected Chan-Paton assignments. The D9-D9 amplitude is thus described by

$$\tilde{\mathcal{A}}_{99} = \frac{2^{-5}}{2} \left[ (n_1 + n_2)^2 V_8 + (n_1 - n_2)^2 O_8 \right] ,$$

$$\mathcal{A}_{99} = \frac{n_1^2 + n_2^2}{2} (O_8 + V_8) - n_1 n_2 (S_8 + C_8) ,$$  \hspace{1cm} (7.4)

where the branes of the first type, with Chan-Paton multiplicity $n_1$, have a positive coupling to the closed-string tachyon, while those of the second type, with Chan-Paton multiplicity

$n_2$, have a negative coupling to it. The corresponding D9-O9 Möbius amplitude is then

$$\mathcal{M}_9 = - (n_1 - n_2) \hat{O}_8 - (n_1 + n_2) \hat{V}_8 ,$$

$$\mathcal{M}_9 = - \frac{n_1 + n_2}{2} \hat{V}_8 + \frac{n_1 - n_2}{2} \hat{O}_8 ,$$

(7.5)

and the D9 gauge group is therefore $SO(n_1) \times SO(n_2)$, with tachyons in the $(n_1(n_1 + 1)/2, 1) + (1, n_2(n_2 - 1)/2)$ representations and Majorana fermions in the bi-fundamental representation. Since the dilaton tadpole condition $n_1 + n_2 = 32$ only eliminates the need for a background redefinition [42] of the type discussed explicitly in [40], also in this case one has the additional option of reversing both $n_1$ and $n_2$, thus replacing the O9$_+$ planes with O9$_-$ ones, with opposite tachyon and dilaton couplings. In both cases the D9 branes are unstable and can decay into the vacuum.

This model has also charged Dp branes, with $p=0,2,4,6,8$, that can be deduced from the brane configurations of the parent 0A that are invariant under $\Omega$. Here the signs in eq. (7.3) play a crucial role, and indeed when $\Omega A = A'$, the configurations invariant under $\Omega$ are of the type $d_1 Dp_1 + d_2 \overline{Dp}_1$ or $m(Dp_2 + \overline{Dp}_2)$, while when $\Omega A = -A'$ they are of the type $d_1 Dp_2 + d_2 \overline{Dp}_2$ and $m(Dp_1 + \overline{Dp}_1)$. For $p = 0, 2, 4, 6, 8$, the Dp-Dp annulus amplitudes are thus

$$\tilde{A}_{pp} = \frac{2^{-(p+1)/2}}{4} [(d_1 + d_2 + m + \tilde{m})^2 (V_{p-1}O_{9-p} + O_{p-1}V_{9-p})$$

$$+ (d_1 + d_2 - m - \tilde{m})^2 (O_{p-1}O_{9-p} + V_{p-1}V_{9-p})$$

$$- |d_1 - d_2 - m + \tilde{m} |^2 (S_{p-1} + C_{p-1}) (S_{9-p} + C_{9-p})] ,$$

$$A_{pp} = \frac{(d_1^2 + d_2^2 + m \tilde{m}) (O_{p-1}V_{9-p} + V_{p-1}O_{9-p})}{2}$$

$$+ (d_1 d_2 + \frac{m^2 + \tilde{m}^2}{2}) (O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) - (d_1 + d_2)(m + \tilde{m}) S'_{p-1} S'_{9-p} ,$$

where we have expressed the open spinor content, $1/2(S_8 + C_8)$, in terms of the single odd-dimensional $S'$ spinors. For $p = 0, 4$ and 8 the Möbius amplitudes are

$$\tilde{M}_p = - \frac{d_1 + d_2 + m + \tilde{m}}{\sqrt{2}} (\hat{V}_{p-1}\hat{O}_{9-p} - \hat{O}_{p-1}\hat{V}_{9-p})$$

$$- \frac{d_1 + d_2 - m - \tilde{m}}{\sqrt{2}} (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) ,$$

(7.7)

$$M_p = \frac{d_1 + d_2}{2} \epsilon (\hat{O}_{p-1}\hat{V}_{9-p} - \hat{V}_{p-1}\hat{O}_{9-p}) - \frac{m + \tilde{m}}{2} \epsilon (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) ,$$

$$\tilde{M}_p = \frac{d_1 + d_2}{2} \epsilon (\hat{V}_{p-1}\hat{O}_{9-p} - \hat{O}_{p-1}\hat{V}_{9-p}) - \frac{m + \tilde{m}}{2} \epsilon (\hat{V}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) .$$
where $\epsilon$ is $+1$ for $p = 0, 8$ and $-1$ for $p = 4$. The resulting gauge groups are therefore $\text{SO}(d_1) \times \text{SO}(d_2) \times \text{U}(m)$ for $p = 0, 8$ and $\text{USp}(d_1) \times \text{USp}(d_2) \times \text{U}(m)$ for $p = 4$ in the orthogonal 0A orientifolds, and \textit{vice versa} in the symplectic ones. Notice that for $m = 0$ and $d_1 = 0$ (or $d_2 = 0$) these spectra contain \textit{no open-string tachyons}. In a similar fashion, for $p = 2, 6$ the M"obius amplitudes are

$$
\tilde{M}_p = -\frac{d_1 + d_2 + m + \bar{m}}{\sqrt{2}} (\hat{V}_{p-1}\hat{O}_{9-p} - \hat{O}_{p-1}\hat{V}_{9-p}) \\
+ \frac{d_1 + d_2 - m - \bar{m}}{\sqrt{2}} (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}),$

$$
M_p = \frac{d_1 + d_2}{2} \epsilon (\hat{O}_{p-1}\hat{V}_{9-p} - \hat{V}_{p-1}\hat{O}_{9-p}) \\
- \frac{m + \bar{m}}{2} \epsilon (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}),
$$

(7.8)

where $\epsilon$ is $+1$ for $p = 2$ and $-1$ for $p = 6$. Notice that the closed-string tachyon appears in the transverse M"obius amplitudes (7.7) and (7.8) with opposite signs, consistently with the fact that the branes involved in the two cases, of types $D_p$ and $D_p$, have opposite tachyonic couplings. The gauge groups for $p = 2$ ($p = 6$) are analogous to those for $p = 0, 8$ ($p = 4$). The 0A orientifold has thus a rather unusual feature: \textit{for the same} (even) $p$, there are both charged branes, with orthogonal or symplectic gauge groups, and uncharged ones, with unitary gauge groups. As usual, there are additional states in the $D_p$-D9 sector, if D9 branes are present, whose annulus amplitudes are

$$
\tilde{A}_{p9} = \frac{2^{-5}}{\sqrt{2}} \{(n_1-n_2)(d_1 + d_2 - m - \bar{m})(O_{p-1}O_{9-p} - V_{p-1}V_{9-p}) \\
+ (n_1+n_2)(d_1 + d_2 + m + \bar{m})(V_{p-1}O_{9-p} - O_{p-1}V_{9-p})\},

A_{p9} = [n_1(d_1 + d_2) + n_1\bar{m} + n_2m] (O_{p-1} + V_{p-1}) S_{9-p}^p \\
- [n_2(d_1 + d_2) + n_1m + n_2\bar{m}] S_{p-1}(O_{9-p} + V_{9-p}).
$$

(7.9)

There are therefore massless fermions in the bi-fundamental representation of $\text{SO}(n_2)$ and of the $D_p$ gauge group and scalars (tachyonic for $p > 5$) in the bi-fundamental of $\text{SO}(n_1)$ and the $D_p$ gauge group.

This model also contains uncharged $D_p$ branes for $p = 1, 3, 5, 7$, that can be obtained by an orientifold projection from the $D_p \pm$ 0A branes discussed in Section 3, and are therefore
of two types, depending on the sign of their coupling to the closed-string tachyon. The D$p$-D$p$ amplitudes
\[
\tilde{A}_{pp} = \frac{2^{-(p+1)/2}}{2} \left[ (d_1 + d_2)^2 (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) + (d_1 - d_2)^2 (O_{p-1}O_{9-p} + V_{p-1}V_{9-p}) \right],
\]
\[
A_{pp} = \frac{d_1^2 + d_2^2}{2} (O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) - d_1 d_2 (S_{p-1} + C_{p-1})(S_{9-p} + C_{9-p})
\]
are obtained introducing real Chan-Paton charges $d_1, d_2$ and dimensionally reducing on the D$p$ world-volume the D9-D9 amplitudes (7.4), while the Möbius amplitudes are
\[
\tilde{M}_p = -(d_1 - d_2) (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) - (d_1 + d_2) (\hat{V}_{p-1}\hat{O}_{9-p} - \hat{O}_{p-1}\hat{V}_{9-p})
\]
\[
M_p = \frac{d_1 - \epsilon' d_2}{2} \epsilon (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) + \frac{d_1 + \epsilon' d_2}{2} \epsilon (\hat{O}_{p-1}\hat{V}_{9-p} - \hat{V}_{p-1}\hat{O}_{9-p}).
\]
The signs $(\epsilon, \epsilon')$ are $(+1, +1)$ for the D1 branes, $(+1, -1)$ for the D3 branes, $(-1, +1)$ for the D5 branes and $(-1, -1)$ for the D7 and D($-1$) branes. In the orthogonal orientifolds, the resulting gauge groups are therefore SO($d_1$) $\times$ SO($d_2$) for the D1 branes, SO($d_1$) $\times$ USp($d_2$) for the D3 branes, USp($d_1$) $\times$ USp($d_2$) for the D5 branes and USp($d_1$) $\times$ SO($d_2$) for the D7 and D($-1$) branes. In the symplectic orientifolds, the overall sign of $\mathcal{M}_p$ would be reverted, and therefore the roles of the D3 and D7 (or D($-1$)) and D1 and D5 branes would be interchanged.

The tachyons can be eliminated for one D1 brane or for one D7 brane, i.e. if $d_1 = 0$, $d_2 = 1$. Finally, the D$p$-D9 spectrum is described by
\[
\tilde{A}_{p9} = 2^{-5} \left[ (n_1 - n_2)(d_1 - d_2)(O_{p-1}O_{9-p} - V_{p-1}V_{9-p}) + (n_1 + n_2)(d_1 + d_2)(V_{p-1}O_{9-p} - O_{p-1}V_{9-p}) \right],
\]
\[
A_{p9} = (n_1 d_1 + n_2 d_2) (O_{p-1} + V_{p-1})(S_{9-p} + C_{9-p}) - (n_1 d_2 + n_2 d_1)(S_{p-1} + C_{p-1})(O_{9-p} + V_{9-p}),
\]
where, as usual, new tachyons appear if $p > 5$. All the D$p$-brane spectra in the 0A orientifold are non-chiral.

As already mentioned, a single D1 brane ($d_1 = 0$, $d_2 = 1$) is stable and contains two-dimensional Majorana fermions in the fundamental representation of the SO($n_1$) D9 gauge
The resulting central charge $c_{L,R} = 10 + n_1/2$ reaches its critical value 26 for $n_1 = 32$, when the D9 gauge group becomes SO(32). It is interesting to notice that this value is singled out in ten dimensions if the tachyon coupling is also eliminated in $\tilde{A}$. All this suggests that the strong coupling limit of the 0A orientifold with SO(32) gauge group can be related to the quantization of this critical ten-dimensional closed string, with a gauge group SO(32) $\times$ SO(32). On the other hand, the 0A orientifold tells us that the left and right-moving world-sheet fermions should have the same gauge transformations, and it is then natural to conjecture that the S-dual of this particular 0A orientifold is actually an orientifold of the above-mentioned critical closed string theory, where the two gauge factors are identified, so that one finally obtains an SO(32) gauge group. Its modular invariant torus amplitude is

$$T = |O_{32}|^2 + |V_{32}|^2 + |S_{32}|^2 + |C_{32}|^2,$$

(7.13)

to be combined with the Klein-bottle projection

$$K = \frac{1}{2}(O_{32} + V_{32} - S_{32} - C_{32}).$$

(7.14)

The resulting massless spectrum comprises the graviton, the dilaton and the SO(32) gauge bosons, together with scalars in a 4-index representation of the gauge group. The low-lying spectra of the two theories do not exactly coincide, since the 0A orientifold has a one-form and a three-form not present in the orientifold (7.14) with no open sector. In addition, the charged massless scalars present in (7.14) have no counterpart in the 0A orientifold. However, (7.14) contains a singlet tachyon of mass $m^2 = -4/\alpha'$ and additional tachyons in the symmetric representation of SO(32), of mass $m^2 = -1/\alpha'$ that, interestingly enough, match the spectrum of corresponding closed and open tachyons of the 0A orientifold.

8. Comments on fractional branes

Orbifold projections can also lead to the appearance, at orbifold fixed points, of fractional branes, that carry new types of (twisted) R-R charges, lack the moduli associated to their displacements, and have tensions smaller than those of the branes that can move to the bulk. In addition, the O-planes themselves can acquire twisted R-R charges as a result of $Z_N$ projections with $N \neq 2$. Fractional branes present themselves in a number of orbifold
instances, and in particular in orientifolds, where twisted R-R forms play a central role in the generalized Green-Schwarz mechanism [29]. For instance, overall neutral combinations of branes with twisted R-R couplings of this type are present in the supersymmetric models of [5], that are $Z_2$ orbifold reductions with a quantized $B_{ab}$ [3], as in [48], albeit with rational geometric moduli, and in the “brane supersymmetry breaking” $T^4/Z_2$ model of [24]. Further, non-neutral combinations of fractional branes are present in any supersymmetric $Z_N$ orbifold with $N \neq 2$, and for instance in [49], where they absorb the twisted R-R charge that the O-planes acquire in these cases.

It is instructive to display an explicit example of such branes, in the spirit of the preceding sections. To this end, let us consider the 0’B D3-branes [27] of (6.5) and (6.6) at an orbifold singularity in $R^{1,5} \times R^4/Z_2^3$. The orbifold action breaks the D3 gauge group to $U(d_1) \times U(d_2)$, while new twisted tadpoles, inadmissible for the space-filling background D9 branes, demand that their $U(32)$ gauge group break to $U(16) \times U(16)$. The direct-channel amplitudes

$$A_{33} = \frac{1}{2} \left[ |d_1 + d_2|^2 (V_4 O_4 + O_4 V_4) - \frac{(d_1 + d_2)^2 + (\bar{d}_1 + \bar{d}_2)^2}{2} (C_4 S_4 + S_4 C_4) \right]$$

$$+ |d_1 - d_2|^2 (V_4 O_4 - O_4 V_4) + \frac{(d_1 - d_2)^2 + (\bar{d}_1 - \bar{d}_2)^2}{2} (C_4 S_4 - S_4 C_4) \right]$$

$$M_{3} = \frac{1}{4} \left[ (d_1 + d_2 - \bar{d}_1 - \bar{d}_2) (\hat{S}_2 \hat{C}_2 \hat{S}_4 + \hat{S}_2 \hat{S}_2 \hat{C}_4 - \hat{C}_2 \hat{S}_2 \hat{S}_4 - \hat{C}_2 \hat{C}_2 \hat{C}_4) \right]$$

$$- (d_1 + d_2 - \bar{d}_1 - \bar{d}_2) (\hat{S}_2 \hat{C}_2 \hat{S}_4 - \hat{S}_2 \hat{S}_2 \hat{C}_4 - \hat{C}_2 \hat{S}_2 \hat{S}_4 + \hat{C}_2 \hat{C}_2 \hat{C}_4) \right]$$

$$A_{93} = (\bar{n}_1 d_1 + \bar{n}_2 d_2) (O_2 C_2 C_4 + V_2 S_2 C_4) + (n_1 \bar{d}_1 + n_2 \bar{d}_2) (O_2 S_2 C_4 + V_2 C_2 C_4)$$

$$+(\bar{n}_1 d_2 + \bar{n}_2 d_1) (O_2 S_2 S_4 + V_2 C_2 C_4) + (n_1 \bar{d}_2 + n_2 \bar{d}_1) (O_2 C_2 S_4 + V_2 S_2 S_4)$$

$$- (n_1 d_1 + n_2 d_2) (S_2 O_2 O_4 + C_2 V_2 O_4) - (\bar{n}_1 \bar{d}_1 + \bar{n}_2 \bar{d}_2) (C_2 O_2 O_4 + S_2 V_2 O_4)$$

$$- (n_1 d_2 + n_2 d_1) (S_2 V_2 V_4 + C_2 O_2 V_4) - (\bar{n}_1 \bar{d}_2 + \bar{n}_2 \bar{d}_1) (C_2 V_2 V_4 + S_2 O_2 V_4)$$

can be obtained enforcing the $Z_2$ orbifold projection in (6.5) and (6.6), while the D3-D3 closed-channel amplitude

$$\hat{A}_{33} = \frac{2^{-2}}{8} \left[ (d_1 + d_2 + \bar{d}_1 + \bar{d}_2)^2 (V_4 O_4 + O_4 V_4 - S_4 C_4 - C_4 S_4) \right]$$

---

3Fractional IIB branes on this orbifold were previously considered in [50].
them can not move off the fixed points, and define the fractional branes. Moreover, the D9-D3 spectrum contains pairs of complex scalars in the $(n, \bar{n})$ real scalars in the adjoint of the D3 gauge group, four complex scalars in the $(d_1 - d_2, \bar{d}_1 - \bar{d}_2)$, two Dirac fermions in the $(1, \bar{1})$, two Weyl fermions of positive chirality in antisymmetric representations and two Weyl fermions of negative chirality in symmetric representations. Moreover, the D9-D3 spectrum contains pairs of complex scalars in the $(n_1, \bar{n}_2)$, $(n_2, \bar{n}_1)$ and $(n_1, \bar{d}_2)$ and Weyl fermions of positive chirality in the $(n_1, d_1)$ and $(n_2, d_2)$. The

\[
\begin{align*}
- (d_1 + d_2 - \bar{d}_1 - \bar{d}_2)^2 (O_4 O_4 + V_4 V_4 - S_4 S_4 - C_4 C_4) \\
+ (d_1 - d_2 + \bar{d}_1 - \bar{d}_2)^2 (O_4 C_4 + V_4 S_4 - S_4 V_4 - C_4 O_4) \\
- (d_1 - d_2 - \bar{d}_1 + \bar{d}_2)^2 (O_4 S_4 + V_4 C_4 - S_4 O_4 - C_4 V_4)
\end{align*}
\]

encodes the D3 brane couplings to the closed sector, and it is transparent that this brane configuration indeed contains $N$ regular branes and $M$ fractional branes with half-tension, where $N$ and $M$ are defined by

\[
d_1 = N + M \quad , \quad d_2 = N .
\]

In addition, the last two lines in (8.2) display the couplings of fractional branes to twisted closed fields, that are clearly reminiscent of similar patterns found in orientifold vacua

The resulting massless modes are described by

\[
A_{33,0} + \mathcal{M}_{33,0} + A_{93,0} = (d_1 \bar{d}_1 + d_2 \bar{d}_2)(V_2 O_2 O_4 + O_2 V_2 O_4) + (d_1 \bar{d}_2 + d_2 \bar{d}_1)O_2 O_2 V_4 - (d_1 d_2 + \bar{d}_1 \bar{d}_2)(S_2 C_2 S_4 + C_2 S_2 S_4) - \frac{d_1(d_1 - 1) + d_2(d_2 - 1) + \bar{d}_1(\bar{d}_1 + 1) + \bar{d}_2(\bar{d}_2 + 1)}{2} S_2 S_2 C_4 - \frac{d_1(d_1 + 1) + d_2(d_2 + 1) + \bar{d}_1(\bar{d}_1 - 1) + \bar{d}_2(\bar{d}_2 - 1)}{2} C_2 C_2 C_4 + (\bar{n}_1 d_1 + \bar{n}_2 d_2)O_2 C_2 C_4 + (n_1 \bar{d}_1 + n_2 \bar{d}_2)O_2 S_2 C_4 + (n_1 \bar{d}_2 + n_2 \bar{d}_1)O_2 C_2 S_4 - (n_1 d_1 + n_2 d_2)S_2 O_2 O_4 - (\bar{n}_1 d_1 + \bar{n}_2 d_2)C_2 O_2 O_4 ,
\]

and the resulting spectrum is chiral but free of irreducible gauge anomalies for any $d_1$ and $d_2$. In addition to the $U(d_1) \times U(d_2)$ gauge vectors, the D3-D3 sector contains two real scalars in the adjoint of the D3 gauge group, four complex scalars in the $(d_1, \bar{d}_2)$, two Dirac fermions in the $(d_1, d_2)$, two Weyl fermions of positive chirality in antisymmetric representations and two Weyl fermions of negative chirality in symmetric representations.

\footnote{Actually, $d_1(d_2)$ simply count the numbers of (anti)branes carrying twisted R-R charges. $|d_1 - d_2|$ of them can not move off the fixed points, and define the fractional branes.}
third 0B orientifold model of (6.24) allows a similar construction containing \( m \) fractional branes, with a resulting gauge group \( \text{SO}(n + m) \times \text{SO}(n) \), and/or \( s \) fractional branes, with a resulting gauge group \( \text{USp}(r + s) \times \text{USp}(r) \). In particular, the force-free configurations, with \( 2n + m = 2r + s \), are suitable for applications to the gauge-gravity correspondence.

Fractional branes are actually a generic feature of orbifolds of conformal theories, a phenomenon neatly illustrated by the SU(2) WZW models. For the diagonal \( A \)-series, a nice geometrical interpretation has recently emerged, in terms of D2 branes on special SU(2) conjugacy classes [51], and the level-\( k \) model has \( (k + 1) \) such branes, corresponding to its \( (k + 1) \) Cardy states, whose couplings to states of half-odd-integer isospin mimic the usual R-R charges. There are, however, additional non-diagonal modular invariants, with more peculiar properties. This is the case, in particular, for the whole \( D \) odd series of [52], that presents an amusing extension of the boundary symmetry first noticed in [30]. The simplest example of this type, the \( D_5 \) model, occurs at level \( k = 6 \) and can be obtained from the \( A_6 \) model

\[
T_{A_6} = |\chi_1|^2 + |\chi_3|^2 + |\chi_5|^2 + |\chi_7|^2 + |\chi_4|^2 + |\chi_2|^2 + |\chi_6|^2 ,
\]

where \( \chi_i \) corresponds to isospin \( (i - 1)/2 \), by an orbifold projection under which the sectors with integer isospin \( (1,3,5,7) \) are even, while those with half-odd-integer isospin, \( (2,4,6) \), are odd [53]. This operation reverses all the R-R charges, in a way reminiscent of what in Section 2 led from the charged to the uncharged type II branes [12]. It is important to note, however, that in the corresponding partition function

\[
T_{D_5} = |\chi_1|^2 + |\chi_3|^2 + |\chi_5|^2 + |\chi_7|^2 + |\chi_4|^2 + \chi_2 \bar{\chi}_6 + \chi_6 \bar{\chi}_2 ,
\]

all the last three terms, including the diagonal one, come from the twisted sector of the orbifold, that defines a WZW model on the SO(3) group manifold. An untwisted charge is thus transmuted into a twisted one by this orbifold.

The diagonal \( A_6 \) model has three types of branes, three corresponding types of anti-branes and a self-conjugate brane, with R-R charges identified by their couplings to \( \chi_2, \chi_4 \) and \( \chi_6 \) in the transverse annulus amplitude

\[
\tilde{A}_6 \sim \sum_a \frac{\chi_a}{\sin \left( \frac{\pi a}{8} \right)} \left| \sum_b \sin \left( \frac{\pi ab}{8} \right) n_b \right|^2 ,
\]

where

\[
\chi_i = \frac{\chi_i - i \chi_i}{2}, \quad 1 \leq i \leq 7
\]
determined by

\[ \mathcal{A}_{A_6} = \sum_{a,b,k} A^k_a n_a \bar{n}_b \chi_k , \]  

(8.8)

where, for the diagonal model, the Cardy ansatz [32] identifies the ranges of \{a, b\} with that of \( k \), and the annulus coefficients \( A_{ka}^b \) with the fusion-rule coefficients \( N_{ki}^j \). The branes with (complex) multiplicities \( n_i \) and \( n_{8-i} \) have opposite R-R couplings and, differently from what we saw in Sen’s construction [12] for type II theories, one of them, identified by the multiplicity \( n_4 \), is self-conjugate. This would seem to lead to only four types of invariant combinations, but actually new twisted R-R couplings emerge, that distinguish between two branes with opposite twisted charges. This provides a geometric picture for the splitting in [30], while recovering the correct brane spectrum, as can be seen identifying in (8.8) the pairs of multiplicities \( n_i \) and \( n_{8-i} \). The resulting annulus amplitude, obtained after an overall rescaling that accounts for the different tensions of the resulting branes,

\[ \mathcal{A} = \left[ n_1 \bar{n}_1 + n_2 \bar{n}_2 + n_3 \bar{n}_3 + \frac{n_4 \bar{n}_4}{2} \right] (\chi_1 + \chi_7) + \left[ n_1 \bar{n}_2 + n_2 \bar{n}_3 + n_3 \bar{n}_4 + h.c. \right] (\chi_2 + \chi_6) \]

\[ + \left[ n_2 \bar{n}_2 + 2n_3 \bar{n}_3 + \frac{n_4 \bar{n}_4}{2} + (n_4 \bar{n}_4 + h.c. \right] (\chi_3 + \chi_5) \]

\[ + [n_1 \bar{n}_4 + 2n_2 \bar{n}_3 + n_3 \bar{n}_4 + h.c.] \chi_4 , \]

(8.9)

acquires a proper particle interpretation after letting \( n_4 \rightarrow n_4 + n_5 \), while also adding a new twisted \( \chi_4 \) coupling in the transverse amplitude, that contributes

\[ \frac{|n_4 - n_5|^2}{2} (\chi_1 - \chi_3 + \chi_5 - \chi_7) \]  

(8.10)

to the direct channel. The end result,

\[ \mathcal{A}_{D_6} = [n_1 \bar{n}_1 + n_2 \bar{n}_2 + n_3 \bar{n}_3 + n_4 \bar{n}_4 + n_5 \bar{n}_5] \chi_1 \]

\[ + [n_1 \bar{n}_2 + n_2 \bar{n}_3 + n_3 \bar{n}_4 + n_5 \bar{n}_5 + h.c.] (\chi_2 + \chi_6) \]

\[ + [n_2 \bar{n}_2 + 2n_3 \bar{n}_3 + (n_1 \bar{n}_3 + n_2 \bar{n}_4 + n_5 \bar{n}_5 + h.c.)] \chi_3 \]

\[ + [n_1 \bar{n}_4 + n_1 \bar{n}_5 + 2n_2 \bar{n}_3 + n_4 \bar{n}_4 + n_5 \bar{n}_5 + h.c.] \chi_4 \]

\[ + [n_2 \bar{n}_2 + 2n_3 \bar{n}_3 + n_4 \bar{n}_4 + n_5 \bar{n}_5 + (n_1 \bar{n}_3 + n_2 \bar{n}_4 + n_5 \bar{n}_5 + h.c.)] \chi_5 \]

\[ + [n_3 \bar{n}_3 + (n_4 \bar{n}_5 + h.c.)] \chi_7 , \]

(8.11)
should be compared with the annulus amplitude in [30], that follows from it after the orientifold restriction to real charges, after the redefinitions $n_i \rightarrow l_i$, $l_1 \leftrightarrow l_2$ and $l_3 \leftrightarrow l_5$.

In conclusion, while no brane in the original $A_6$ model was charged with respect to $\chi_4$, in the orbifold this becomes a twisted sector, and a fractional brane charged with respect to it (together with its antibrane) appears. The same type of construction applies to all the other $D_{odd}$ models, and provides a handy geometric derivation of their brane spectra. This adds to the original derivation in [30], obtained starting from the polynomial equations for the boundary couplings extracted from [54], and to the one in [55], obtained starting from the completeness conditions of [30],

$$A_{ia}^{\hat{b}} A_{jbc} = \sum_k N_{ij}^k A_{k}^{ac} .$$

These state that the annulus coefficients satisfy the fusion algebra or, in equivalent geometrical terms, indeed imply the completeness of the given brane spectrum.

**Acknowledgments**

We are grateful to C. Angelantonj, C. Bachas, B. Gato-Rivera, L. Huiszoon, A.N. Schellekens, Ya.S. Stanev and N. Sousa for interesting discussions and comments. This research was supported in part by the EEC contract HPRN-CT-2000-00122, in part by the EEC contract HPRN-CT-2000-00148, and in part by the INTAS contract 99-1-590. A.S. would like to thank LPT-Orsay and NIKHEF for the kind hospitality during the course of this work and at its concluding stages and CNRS for financial support.

**A Notation and conventions**

It is often convenient to express the partition functions in the NS and R sectors of string models in terms of the $SO(p)$ characters ($p$ even)

$$O_p = \frac{1}{2\eta_{p/2}} (\vartheta_3^{p/2} + \vartheta_4^{p/2}) , \quad V_p = \frac{1}{2\eta_{p/2}} (\vartheta_3^{p/2} - \vartheta_4^{p/2}) ,$$

$$S_p = \frac{1}{2\eta_{p/2}} (\vartheta_2^{p/2} + i\vartheta_1^{p/2}) , \quad C_p = \frac{1}{2\eta_{p/2}} (\vartheta_2^{p/2} - i\vartheta_1^{p/2}) . \quad (A1)$$
where the $\vartheta_i$ are the four Jacobi theta-functions with half-integer characteristics. These characters make the space-time interpretation of the amplitudes rather transparent, since $O_p$ begins with a scalar, $V_p$ with a vector and $S_p$ and $C_p$ with the two spinors of opposite chiralities. Their decompositions with respect to lower-dimensional even orthogonal groups,

$$
O_8 = O_{p-1}O_{9-p} + V_{p-1}V_{9-p}, \quad V_8 = V_{p-1}O_{9-p} + O_{p-1}V_{9-p},
$$

$$
S_8 = S_{p-1}S_{9-p} + C_{p-1}C_{9-p}, \quad C_8 = S_{p-1}C_{9-p} + C_{p-1}S_{9-p}
$$

(A2)

reflect the simple class properties of the corresponding Lie algebras. On the other hand, for D$p$ branes with odd-dimensional world volumes one needs $O_p$, $V_p$ and the single spinor character

$$
S'_p = \frac{1}{\sqrt{2}} \left( \frac{\vartheta_2}{\eta} \right)^{p/2},
$$

(A3)

that involves a rescaling, needed to give the ground state its proper degeneracy. Aside from the torus $T$ amplitude, the spectra of orientifold models [3] require the Klein-bottle amplitude $K$, the annulus amplitude $A$ and the M"obius amplitude $M$, whose direct-channel modular parameters are as follows:

$$
\text{Klein} : \tau = 2it, \quad \text{Annulus} : \tau = \frac{it}{2}, \quad \text{Moebius} : \tau = \frac{it}{2} + \frac{1}{2}.
$$

(A4)

The first two amplitudes are related to the corresponding closed-channel amplitudes $\tilde{K}$ and $\tilde{A}$ by an $S$ transformation (corresponding to the redefinition $\tau \rightarrow -1/\tau$ of the modular parameter), while the third is related to the closed-channel amplitude $\tilde{M}$ by a $P$ transformation (corresponding to the redefinition $it/2 + 1/2 \rightarrow i/2t + 1/2$ of the modular parameter). In a CFT with central charge $c$, the action of $T$ (corresponding to the redefinition $\tau \rightarrow \tau + 1$ of the modular parameter) on a generic basis of characters $\chi_i$ with conformal weights $h_i$ is described by a diagonal unitary matrix

$$
T_{ij} = e^{2\pi i(h_i - c/24)} \delta_{ij},
$$

(A5)

while, when all distinct sectors are described by different characters, $S$ and $P$ act as symmetric unitary matrices.

In writing the M"obius amplitude, for which $\tau$ is not purely imaginary, it is convenient to work with the real basis of characters

$$
\hat{\chi}_r = e^{-i\pi h_r} \chi_r.
$$

(A6)
While on the \( \{\chi_i\} \) \( P \) would act as the sequence \( TST^2S \), on the real \( \{\hat{\chi}_i\} \) basis

\[
P = T^{1/2}ST^2ST^{1/2} ,
\]

where \( T^{1/2} \) denotes a diagonal unitary matrix with eigenvalues \( e^{i\pi(h_i-c/24)} \). The effect of these transformations on the characters \( O_p, V_p, S_p \) and \( C_p \) can be deduced from the corresponding transformations of the Jacobi theta-functions. For even \( p \) the \( S \) and \( P \) matrices are thus [5]

\[
S_p = \frac{1}{2} \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & e^{-ip\pi/4} & -e^{-ip\pi/4} \\
1 & -1 & -e^{-ip\pi/4} & e^{-ip\pi/4}
\end{pmatrix},

P_p = \begin{pmatrix}
c & s & 0 & 0 \\
s & -c & 0 & 0 \\
0 & 0 & \zeta_c & i\zeta_s \\
0 & 0 & i\zeta_s & \zeta_c
\end{pmatrix},
\]

\[
(A7)
\]

where \( c = \cos(p\pi/8) \), \( s = \sin(p\pi/8) \) and \( \zeta = e^{-ip\pi/8} \) [5], and where a phase ambiguity is fixed uniquely demanding that

\[
S^2 = (ST)^3 .
\]

\[
(A9)
\]

For odd \( p \), when a single spinor class is present, the spinorial character is to be rescaled as in (A3). The three resulting characters have the Ising-like \( S \) and \( P \) matrices

\[
S_p = \frac{1}{2} \begin{pmatrix}
1 & 1 & \sqrt{2} \\
1 & 1 & -\sqrt{2} \\
\sqrt{2} & -\sqrt{2} & 0
\end{pmatrix},

P_p = \begin{pmatrix}
c & s & 0 \\
s & -c & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

\[
(A10)
\]

where \( c = \cos(\pi p/8) \) and \( s = \sin(\pi p/8) \).

Notice that, since all these \( S \) and \( P \) matrices have an analytic dependence on \( p \), one can also formally continue them to zero or negative values. This suffices to describe the charge assignments for D0 and D(−1) branes, without the need to revert to the covariant formulation.

All the Chan-Paton assignments described in the text rest on the peculiar fusion rules of the space-time characters [47], for which the vector class \( V_{p-1} \) plays the role of the identity. For instance, for \( p-1 = 4k \), the two spinor classes are self-conjugate, and these would read

\[
[S_{p-1}] [S_{p-1}] = [C_{p-1}] [C_{p-1}] = [O_{p-1}] [O_{p-1}] = [V_{p-1}] [V_{p-1}] = [V_{p-1}] ,
\]

\[
[S_{p-1}] [C_{p-1}] = [O_{p-1}] [V_{p-1}] = [O_{p-1}] ,
\]

\[
(A11)
\]
to be compared with the standard fusion rules for the internal characters, that would read
\[
[S_{9-p}] [S_{9-p}] = [O_{9-p}] [O_{9-p}] = [V_{9-p}] [V_{9-p}] = [O_{9-p}],
\]
\[
[S_{9-p}] [C_{9-p}] = [O_{9-p}] [V_{9-p}] = [V_{9-p}],
\]
so that $V$, rather than $O$, plays the role of identity in the fusion of space-time characters.

For $p−1=4k+2$, similar results hold, but the two spinor classes are mutually conjugate, so that
\[
[S_{p-1}] [S_{p-1}] = [O_{p-1}] [C_{p-1}] = [S_{p-1}] [C_{p-1}] = [V_{p-1}]
\]
and
\[
[S_{9-p}] [S_{9-p}] = [C_{9-p}] [C_{9-p}] = [V_{9-p}] [S_{9-p}] [C_{9-p}] = [O_{9-p}]
\]
for the two cases of space-time and internal characters. This unusual feature, first elucidated in [47], can be related to the behavior of world-sheet ghosts, and played a key role in [5] and in the following work on open-string spectra. The odd-dimensional Ising-like fusion
\[
[S'_{p-1}] [S'_{p-1}] = [O_{p-1}] + [V_{p-1}]
\]
also plays a role for some of the branes with odd-dimensional world volumes.

For the sake of brevity, all amplitudes are presented in the text omitting modular integrals, contributions of space-time bosons and some overall factors that reflect the brane tensions. Thus, for $Dp$ branes, with $p + 1$ longitudinal dimensions and $9 − p$ transverse dimensions, the complete string amplitudes would be
\[
\frac{1}{(4\pi^2\alpha')^5} \int \frac{d^2\tau}{\tau_6^6} \frac{T}{|\eta(\tau)|^{16}}, \quad \frac{1}{(4\pi^2\alpha')^5} \int \frac{d\tau_2}{\tau_2^6} \frac{K}{\eta(2i\tau_2)^8},
\]
\[
\frac{1}{(8\pi^2\alpha')^{(p+1)/2}} \int \frac{dt}{t^{(p+3)/2}} \frac{A_{pp}}{\eta(it/2)^8},
\]
\[
\frac{1}{(8\pi^2\alpha')^{(p+1)/2}} \int \frac{dt}{t^{(p+3)/2}} \frac{\mathcal{M}_p}{\eta(it/2)^{1/2}} \left(\frac{2\eta}{\eta_2}\right)^{(9-p)/2},
\]
while the full additional $p-q$ amplitudes ($p > q$) are
\[
\frac{1}{(8\pi^2\alpha')^{(q+1)/2}} \int \frac{dt}{t^{(q+3)/2}} \frac{A_{pq}}{\eta(it/2)^{8-p+q}} \left(\frac{\eta}{\eta_4}\right)^{(p-q)/2}.
\]
The contributions of space-time fermions and internal bosons and fermions in the “amputated” amplitudes are sufficient to encode the corresponding GSO projections.
The one-loop amplitudes have dual interpretations as tree-level closed-string exchanges between boundaries and crosscaps, or equivalently between D-branes and O-planes. The corresponding closed-string modulus, $l$, is related to the parameters in (A4) by

$$\text{Klein} : l = \frac{1}{2\tau_2}, \quad \text{Annulus} : l = \frac{2}{t}, \quad \text{Moebius} : l = \frac{1}{2t}. \quad (A18)$$

In the closed-string channel, the complete amplitudes (A16) would thus become

$$\frac{1}{(4\pi^2\alpha')^5} \int dl \frac{\tilde{K}}{\eta(il)^8}, \quad \frac{1}{(8\pi^2\alpha'(p+1)/2} \int dl \frac{\tilde{A}_{pp}}{l^{(9-p)/2} \eta(il)^8},$$

$$\frac{1}{(8\pi^2\alpha'(p+1)/2} \int dl \frac{\tilde{M}_p}{\eta(il+1/2)^{p-1}} \left(\frac{2\eta}{\vartheta_2}\right)^{(9-p)/2},$$

$$\frac{1}{(8\pi^2\alpha'(q+1)/2} \int dl \frac{\tilde{A}_{pq}}{l^{(9-p)/2} \eta(il)^8-p+q} \left(\frac{2\eta}{\vartheta_2}\right)^{(p-q)/2}, \quad (A19)$$

where, as we have seen, $\tilde{K}, \tilde{A} (\tilde{M})$ are related to the loop amplitudes $K, A (M)$ by the $S (P)$ transformation defined in (A8).

Notice that, with lower-dimensional branes, some transverse amplitudes contain powers of $l$, that signal momentum flow across them. As a result, their factorization properties are slightly obscured, but they are nonetheless neatly revealed if these powers are traded for corresponding momentum integrals, exposing the propagation of the individual closed-string excitations.

References


