Neutrinoless double beta decay, solar neutrinos and mass scales

Per Osland* and Geir Vigdel§

Department of Physics, University of Bergen, Allegt. 55, N-5007 Bergen, Norway

Abstract

We obtain bounds for the neutrino masses by combining atmospheric and solar neutrino data with the phenomenology of neutrinoless double beta decay where hypothetical values of $|\langle m \rangle|$ are envisaged from future $0\nu\beta\beta$-experiments. Different solutions for the solar neutrino data are considered. For the Large-Mixing-Angle solution, a bound $|\langle m \rangle| \leq 0.01$ eV would strongly disfavour an inverted hierarchy of the neutrino masses.

*E-mail: per.osland@fi.uib.no
§E-mail: geir.vigdel@fi.uib.no
1 Introduction

While the two well-known neutrino anomalies, the atmospheric and solar neutrino problems, can be explained in terms of mixing angles and mass-squared differences, the absolute neutrino masses remain largely unknown. There are theoretical models which relate the mixing angles to the masses, and thereby for neutrino oscillation data provide fits in terms of the masses. Since we in this article are trying to deduce mass-bounds based on as few assumptions as possible, such models will not be considered.

There are several (more or less direct) methods for measuring the absolute neutrino masses, but so far none of them has given unequivocal lower bounds. The present most stringent bound from direct measurements of neutrino masses is derived from measuring the end-point energy of electrons in tritium decay. The bound is $m_{\nu_e} < 2.5$ eV [1], and since this is well above the results from the other methods, it will not be discussed here. The relevance for the neutrino to be part of the Dark Matter has declined in the last couple of decades. Once, the individual neutrino mass was assumed to be $m_j \sim 10$ eV, but the current upper limit is $m_j \simeq 1.8$ eV [2], based on cosmological models of galaxy structure formation and the cosmic microwave background radiation. Moreover, we have the fascinating proposal that the cosmic ray spectrum beyond the GZK cutoff [3] could be due to Z-bursts [4, 5] induced by ultra-high-energy neutrinos interacting with relic cosmic neutrinos, and that a study of this spectrum could provide bounds on the neutrino masses.

Here, we will mostly be concerned with neutrinoless double beta decay, which would occur only for Majorana neutrinos. Some relevant numbers derived from the different methods are given in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Mass bounds [eV]</th>
<th>Future bounds [eV]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tritium</td>
<td>$m_{\nu_e} &lt; 2.5$ [1]</td>
<td>$m_{\nu_e} &lt; 0.3$ [6]</td>
<td></td>
</tr>
<tr>
<td>Cosmological</td>
<td>$\sum_j m_j \lesssim 5.5$ [2]</td>
<td>$\sum_j m_j \lesssim 0.3$ [2]</td>
<td></td>
</tr>
<tr>
<td>Z-burst</td>
<td>$0.1 \lesssim m_3 \lesssim 1$ [4]</td>
<td>Speculative</td>
<td></td>
</tr>
<tr>
<td>$0\nu\beta\beta$</td>
<td>$</td>
<td>\langle m \rangle</td>
<td>&lt; 0.26$ [7]</td>
</tr>
</tbody>
</table>

Table 1: Four methods to explore small neutrino masses. The first and last measure linear combinations of the masses; the Z-burst model is sensitive to the heaviest mass state. The parameters are defined below.

As is well known, the solar and atmospheric neutrino oscillation phenomena depend on the masses and mixings in a way which is rather different from the corresponding dependency in neutrinoless double beta decay. The relationship between these phenomena has been studied in several articles, e.g. [9]. Our aim in this paper is to further elucidate this connection, and discuss how a possible future signal from a $0\nu\beta\beta$-experiment, combined with increased precision in the oscillation data, can provide constraints on the absolute values of the neutrino masses.
2 Oscillation parameters and $0\nu\beta\beta$

We assume the mass-squared differences, $\Delta m_{kj}^2 = m_k^2 - m_j^2$, are fixed by the values relevant for the atmospheric and solar neutrino data. In a plane spanned by mass-squared difference and mixing angle, there are four main regions which provide good fits to the solar neutrino problem, see Table 2. When both the atmospheric and solar mass-squared differences are stretched to their limits, they can have the same value; but in such a case the fit between theory and data is quite poor. The best fits are obtained with $\Delta m_{\text{atm}}^2 \simeq 3 \times 10^{-3}$ eV$^2$ and $\Delta m_{\odot}^2 \simeq 4 \times 10^{-5}$ eV$^2$. Thus we consider two possible arrangements of the relative mass-squared differences,

\begin{equation}
\text{Spectrum 1: } \Delta m_{21}^2 = \Delta m_{\odot}^2, \quad \text{Spectrum 2: } \Delta m_{21}^2 = \Delta m_{\text{atm}}^2. \quad (1)
\end{equation}

where, for both spectra, the mass states are denoted such that their respective masses satisfy $m_1 < m_2 < m_3$. In both cases we necessarily have $\Delta m_{31}^2 \simeq \Delta m_{\text{atm}}^2$. Measured in terms of masses, two of the states will be close together and one more apart. A good fit requires a weak coupling between the electron neutrino and the lone mass state which is responsible for the largest mass-squared difference, $\Delta m_{\text{atm}}^2$.

In much of the literature one assumes Spectrum 1 and $\nu_\alpha = \sum_j U_{\alpha j} \nu_j$ as the connection between flavour and mass states. Then, the coupling between the electron neutrino and the most isolated mass state will naturally be denoted as $U_{e3}$. Because of this historical fact, we will use $U_{e3}$ as the strength between the electron neutrino and the lone mass state also for Spectrum 2. Accordingly, shifting from one spectrum to the other corresponds to a cyclic permutation. With two $\Delta m_{kj}^2$ fixed, all three neutrino masses can be expressed in terms of one mass, which we take to be the lightest one, $m_1$. There are three main

<table>
<thead>
<tr>
<th>$\tan^2 \theta$</th>
<th>$\Delta m^2 [\text{eV}^2]$</th>
<th>g.o.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>Max.</td>
<td>Min.</td>
</tr>
<tr>
<td>ATM</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>LMA</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>LOW</td>
<td>0.4</td>
<td>3</td>
</tr>
<tr>
<td>VO</td>
<td>0.2</td>
<td>5</td>
</tr>
<tr>
<td>SMA</td>
<td>$2 \times 10^{-4}$</td>
<td>$9 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 2: The ranges of the observed neutrino parameters within $\sim 99\%$ C.L., as read off from figures (their exact values are not very important for the present discussion). Data for the Large-Mixing-Angle, LOW, Vacuum-Oscillation, and Small-Mixing-Angle solutions are from [10]; and the data for the ATMospheric neutrino observation is from [11].
hierarchy types:

Normal hierarchy: \[ m_1 \ll \sqrt{\Delta m^2_{\odot}} = \sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{\text{atm}}} = \sqrt{\Delta m^2_{32}}, \]
gives \( m_1 \ll m_2 \ll m_3 \).

Degenerated masses: \[ m_1 \gg \sqrt{\Delta m^2_{\odot}} = \sqrt{\Delta m^2_{32}} \gg \sqrt{\Delta m^2_{\text{atm}}} = \sqrt{\Delta m^2_{21}}, \]
gives \( m_1 \simeq m_2 \simeq m_3 \).

Inverted hierarchy: \[ m_1 \ll \sqrt{\Delta m^2_{\odot}} = \sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{\text{atm}}} = \sqrt{\Delta m^2_{32}}, \]
gives \( m_1 \ll m_2 \ll m_3 \).

These are the “extreme” cases, of course \( m_1 \) can be close (or equal) to the square root of some mass-squared difference. It is convenient to introduce a quantitative criterion for when the neutrinos can be called degenerated. If we define the criterion for degeneracy to be \( m_1/m_3 > 0.99 \), then, with \( \Delta m^2_{31} = 3.3 \times 10^{-3} \text{ eV}^2 \), degeneracy is achieved when \( m_1 \gtrsim 0.4 \text{ eV} \). If the criterion reads \( m_1/m_3 > 0.90 \), then the neutrinos are degenerated for \( m_1 \gtrsim 0.1 \text{ eV} \). As we will see, among the methods listed in Table 1, it is only 0νββ-experiments that for non-degenerated masses can give a positive signal in the foreseeable future.

For some unstable elements normal beta disintegration is forbidden by energetic reasons, but double beta decay may be allowed. This is a higher order process in which two nucleons decay at the same time, most of these reactions are of the form

\[
^A ZX \rightarrow ^{A+2} Z + 2e^- + 2\bar{\nu}_e. \tag{3}
\]

If the electron neutrino emitted from a nucleon is a Majorana particle with non-zero mass, then it has a non-zero probability to be right-handed and thereby it can be absorbed as an antineutrino by a nucleon of the same type as the one from which it originated. Thus, the final state of this reaction contains no neutrino,

\[
^A ZX \rightarrow ^{A+2} Z + 2e^- \tag{4}
\]

There are several other ways for 0νββ to occur. In this article we consider as small an extension of the Standard Model as possible, therefore we assume that only left-handed charged currents are involved and that the above mechanism takes place by the exchange of a light Majorana neutrino\(^1\).

The rate for the process (4) depends on the \( M_{ee} \) element (which we hereafter denote as \( \langle m \rangle \)) of the mass matrix

\[
M = U^* D U^\dagger, \tag{5}
\]

where \( D \) is a diagonal matrix whose entries are the neutrino mass eigenvalues. Depending on whether we assume Spectrum 1 or 2 (see Eq. (1)) \( D \) will respectively be\(^2\)

\[
D = \text{diag}(m_1, m_2, m_3) \quad \text{or} \quad D = \text{diag}(m_2, m_3, m_1). \tag{6}
\]

\(^{1}\)This could be due to a Higgs triplet or heavy Majorana partners [12].

\(^{2}\)Note that Spectrum 2 is obtained by a cyclic permutation.
To allow for the possibility of neutrinoless double beta decay, we have to assume the neutrinos are of Majorana type. Then the mixing matrix can be expressed as

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1/2} & 0 \\ 0 & 0 & e^{-i\alpha_2/2} \end{pmatrix},$$

(7)

where $\alpha_1$ and $\alpha_2$ are Majorana phases, their ranges are $0 \leq \alpha_1, \alpha_2 < 2\pi$. The “universal” phase is included in the left mixing matrix. This phase can be rotated to an arbitrary $2 \times 2$ sub matrix of $U$, and because the observable parameter in $0\nu\beta\beta$ contains mixing elements only from the first row of $U$ (see Eqs. (8) and (9)), this phase is of no physical consequence for this kind of phenomenon.

For the two spectra we get for the electron-neutrino state and the $\langle m \rangle$-element:

**Spectrum 1:**

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle,$$

$$\langle m \rangle = U_{e1}^2 m_1 + U_{e2}^2 m_2 e^{i\alpha_1} + U_{e3}^2 m_3 e^{i\alpha_2};$$

(8)

**Spectrum 2:**

$$|\nu_e\rangle = U_{e1}|\nu_2\rangle + U_{e2}|\nu_3\rangle + U_{e3}|\nu_1\rangle,$$

$$\langle m \rangle = U_{e1}^2 m_2 + U_{e2}^2 m_3 e^{i\alpha_1} + U_{e3}^2 m_1 e^{i\alpha_2}.$$

(9)

We will focus somewhat more on Spectrum 1 than the other because the first one seems more natural, and Spectrum 2 is disfavoured for SN 1987A-neutrinos [13]. We note that, in contrast to neutrino oscillations, the Majorana phases have to be accounted for in analysing results of $0\nu\beta\beta$-experiments. Further, we see that $\langle m \rangle$ is $CP$ invariant for $\alpha_1, \alpha_2 = 0$ or $\pi$, and it is useful to note that

$$|\langle m \rangle| \leq m_3.$$

(10)

If the future brings not only an upper bound, but a definite value for $|\langle m \rangle|$, then Eq. (10) yields a lower bound on the heaviest mass.

At present, the strongest bound on $|\langle m \rangle|$ from $0\nu\beta\beta$-measurements is [7]

$$|\langle m \rangle| < 0.26 \text{ eV at 68\% C.L.} \quad \text{and} \quad |\langle m \rangle| < 0.34 \text{ eV at 90\% C.L.}$$

(11)

It should be noted that the exact values of the above limits depend on the nuclear matrix elements, which have a considerably uncertainty [14]. However, for our phenomenological study that will not be taken account of.

For neutrinoless double beta decay the difference between the two spectra diminishes as the degree of degeneracy increases. Therefore, in deriving mass bounds from today’s upper bound on $|\langle m \rangle|$, we get essentially the same result whether we assume Spectrum 1 or 2 because the large masses required correspond to near-degenerated mass states. However, as we shall see, with the foreseen reach of GENIUS [8], we get spectrum-dependent bounds on the individual masses.

### 3 Limiting cases

In order to develop some intuition for the expressions in Eqs. (8) and (9), we will study two realistic limiting cases. It is generally believed that the neutrinos are quite light, and
there is no compelling theory which imposes a lower bound for the lightest mass state. Therefore, one limit of interest is \( m_1 = 0 \). The second limiting case is small \( U_{e3} \), as is indicated by several experiments. In either of these two limits, the expressions for \( \langle m \rangle \) reduce to two terms, where its smallest and largest value (for given values of the masses and the mixing) are obtained with the remaining phase equal to \( \pi \) and 0, respectively.

Some of the figures in this article will be expressed in terms of mixing angles. Our convention for these angles is the one advocated by the Particle Data Group [15],

\[
U_{e1} = \cos \theta_{12} \cos \theta_{13}, \quad U_{e2} = \sin \theta_{12} \cos \theta_{13}, \quad U_{e3} = \sin \theta_{13}.
\]  

(12)

### 3.1 Negligible \( m_1 \)

When \( m_1 \) is small, Eqs. (8) and (9) can be approximated as

\[
|\langle m \rangle| \simeq |U_{e2}^2 \sqrt{\Delta m^2_\odot} + U_{e3}^2 \sqrt{\Delta m^2_{\text{atm}}} e^{i\alpha_1}|, \quad \text{Spectrum 1},
\]

(13)

\[
|\langle m \rangle| \simeq \sqrt{\Delta m^2_{\text{atm}}} |U_{e1}^2 + U_{e2}^2 e^{i\alpha_1}|, \quad \text{Spectrum 2}.
\]

(14)

Under this assumption of negligible \( m_1 \), we can of course not have degenerated neutrinos, but Spectrum 2 would be an example of inverted hierarchy. The ranges of the effective Majorana mass for the SMA solution are \( 0 \lesssim |\langle m \rangle| \lesssim 0.003 \text{ eV} \) (Spectrum 1) and \( 0.02 \lesssim |\langle m \rangle| \lesssim 0.08 \text{ eV} \) (Spectrum 2). Correspondingly for the LMA region, \( 0 \lesssim |\langle m \rangle| \lesssim 0.005 \text{ eV} \) (Spectrum 1) and \( 0.003 \text{ eV} \lesssim |\langle m \rangle| \lesssim 0.08 \text{ eV} \) (Spectrum 2), and for the LOW solution: \( 0 \lesssim |\langle m \rangle| \lesssim 0.003 \text{ eV} \) (Spectrum 1) and \( 0 \lesssim |\langle m \rangle| \lesssim 0.08 \text{ eV} \) (Spectrum 2). These values are well below current limits, but the Spectrum 2 (and perhaps Spectrum 1) values can be explored with the coming GENIUS experiment [8].

### 3.2 Small \( U_{e3} \)

One important difference between this and the former \( (m_1 = 0) \) limit, is that the present one is compatible with all hierarchy types in Eq. (2). It is well-known that both the atmospheric and solar neutrino data give best fits to the neutrino oscillation hypothesis when \( U_{e3} \ll 1 \). A small value for \( U_{e3} \) is also strongly suggested by the CHOOZ data [16]. Here we will study both the “exact” case of \( U_{e3} = 0 \) and the case \( U_{e3} = 0.2 \). The latter value is motivated by the CHOOZ experiment, which implies (to 90% C.L.) \( U_{e3} \lesssim 0.2 \) for \( \Delta m^2_{\text{atm}} > 3 \times 10^{-3} \text{ eV}^2 \). Whether or not this mixing element is negligible, could be determined at a neutrino factory which would either establish a definite value for \( U_{e3} \), or lower the upper bound to \( U_{e3} \lesssim 0.015 \) [17]. If such a low bound should be established, there could not be more than one effective Majorana phase.

For Spectrum 1 and negligible \( U_{e3} \), the effective Majorana mass in the SMA region can be approximated as \( |\langle m \rangle| \simeq m_1 \) (\( \simeq m_2 \) for Spectrum 2). If we assume Majorana neutrinos and the SMA solution, the present upper limit on \( |\langle m \rangle| \) implies \( \sum_j m_j \lesssim 0.78 \text{ eV} \). The degree of degeneracy in this case is bounded by \( m_1/m_3 < 0.98 \).

When \( U_{e3} = 0 \) we get for Spectrum 1

\[
|\langle m \rangle| = |U_{e1}^2 m_1 + U_{e2}^2 m_2 e^{i\alpha_1}| = |U_{e1}^2 m_1 + e^{i\alpha_1} (1 - U_{e1}^2) \sqrt{m_1^2 + \Delta m^2_{21}}|.
\]

(15)
In Fig. 1 we display the relation between \(m_1\) and \(U_{e1}\) according to Eq. (15). The range of \(m_1\)-values in this figure is deduced from the cosmological limit in Table 1. Note that the maximal allowed \(m_1\)-value for the \(|\langle m \rangle|\)-contour decreases as the mixing decreases.

Figure 1: Left panel: \(|\langle m \rangle|\) from Eq. (15) with \(\alpha_1 = \pi\) and \(\Delta m^2_{21} = 10^{-5}\) eV\(^2\). Right panel: Contour for today’s upper limit \(|\langle m \rangle| = 0.26\) eV. Because of the large value assumed for \(|\langle m \rangle|\), the contour is practically the same for Spectrum 1 and 2.

When we include a non-zero \(U_{e3}\) in the \(\langle m \rangle\)-expressions, we set the corresponding phase factor equal to \(-1\), i.e., \(\alpha_2 = \pi\) in Eqs. (8) and (9). For the relevant bound on \(|\langle m \rangle|\) this choice gives the highest allowed mass values. Fig. 2 shows, for Spectrum 1, \(m_1\) as a function of the \(CP\)-parameter \(\alpha_1\) for two values of \(|\langle m \rangle|\), namely 0.26 eV (current limit) and 0.05 eV (within the sensitivity of GENIUS). The highest and lowest mixing in this figure corresponds to the highest and lowest mixing allowed by the LMA region (95% C.L.) for two generations. As we see, the variation of \(m_1\) with the phases, depends on how strong the mixing is. When \(U_{e3} = 0\), the highest possible mass value is \(m_1 \simeq 1.5\) eV. With \(U_{e3} = 0.2\) the highest value is \(m_1 \simeq 2.0\) eV, which for three neutrino generations corresponds to \(\sum m_j \simeq 6\) eV. This is close to the current upper bound from cosmological observations, given in Table 1. Results from the space probes MAP (under way) and Planck (launch in 2007) can lead to sensitivities of \(\sum m_j \simeq 0.5\) eV and \(\sum m_j \simeq 0.3\) eV, respectively [2].

## 4 Combination of data

As shown in Fig. 2, we get restrictions on the absolute masses of the neutrinos by combining mixing results from solar neutrino observations and bounds on \(|\langle m \rangle|\) from 0\(\nu\beta\beta\)-experiments. For a given \(|\langle m \rangle|\) and given values of the mixing angles, the upper bound is, for degenerated masses:

\[
\max(m_1) = \frac{|\langle m \rangle|}{2U_{e1}^2 - 1}, \quad \text{if } \tan^2 \theta_{12} < 1.
\]
Figure 2: Two sets of $\alpha_1$ and $m_1$ projections, for $|\langle m \rangle| = 0.26$ eV and 0.05 eV. The solid, dotted and dashed curves represent $\tan^2 \theta_{12} = 0.15$, 0.35 and 0.7, respectively. Dash-dotted curve: $\tan^2 \theta_{12} = 0.7$, $U_{e3} = 0.2$, $\alpha_2 = \pi$. The lowest horizontal dotted line shows the (most optimistic) mass bound derived from the Planck space probe. The other horizontal dotted line is well within reach for both spacecrafts.

For example, with $U_{e3} = 0$ and $|\langle m \rangle| = 0.26$ eV, the best-fit angle $\tan^2 \theta_{12} = 0.3$ (see Fig. 3) gives $\max(m_1) \simeq 0.5$ eV and $\tan^2 \theta_{12} = 0.8$ gives $\max(m_1) \simeq 2.3$ eV. For the 90% C.L. bound in Eq. (11), these mixings give respectively $\max(m_1) \simeq 0.65$ eV and $\max(m_1) \simeq 3.0$ eV. (For degenerated masses, the lowest possible mass corresponds to $m_1 = |\langle m \rangle|$.)

If a future positive $0\nu\beta\beta$-signal should allow the expression in Eq. (16) to be larger than the observed mass bound, we get restrictions on the allowed mixing and phases, as illustrated in Fig. 2. If we assume Spectrum 1 and two generations, the relation between the remaining neutrino parameters can be written (from Eq. (8))

$$\cos \alpha_1 = \frac{|\langle m \rangle|^2 - m_2^2(1 - 2U_{e1}^2U_{e2}^2)}{2m_1^2U_{e1}^2U_{e2}^2}, \quad \text{if } \Delta m_{21}^2 \ll m_1^2.$$ (17)

In the upper part of Fig. 3 we compare the most probable solution for the solar neutrino problem, the LMA region, with different hypothetical contours for $|\langle m \rangle|$ under the assumption of $m_1 = 0$. Whether or not the LMA region is the correct one, is likely to be determined in the next few years by the KamLAND experiment [18]. The LOW and VO regions are not included in this part because the corresponding $|\langle m \rangle|$-values are far below the values which can be detected in the foreseeable future. The Majorana phases have been chosen to give the smallest possible $|\langle m \rangle|$-value, which in our examples implies $\alpha_1 = \alpha_2 = \pi$. The $|\langle m \rangle|$-values indicated in this hierarchical case ($m_1 = 0$) are at the border of the sensitivity of the most optimistic GENIUS proposal, see Table 1.

For the near-degenerated case $m_1 = 0.1$ eV, the $|\langle m \rangle|$-values are compared to the two most favoured solar neutrino solutions, the LMA and LOW regions. For clarity we don’t indicate the VO region; its allowed region of mixing largely overlaps with that of the LMA
Figure 3: The closed contours give the LMA and LOW (lower right) regions allowed to 95% C.L. Also shown are pairs of contours of $|\langle m \rangle|$ for $m_1 = 0$ (upper part) and $m_1 = 0.1$ eV (lower part). Solid and dashed curves represent $U_{e3} = 0$ and 0.1, respectively. For these $U_{e3}$-values “×” and “+” mark the best-fit points in the LMA region. We set $\alpha_1 = \alpha_2 = \pi$.

and LOW, and like the last one it includes $\tan^2 \theta_{12} = 1$, i.e., it allows very large masses. The LOW region extends to lower values of the mass-squared difference than that shown in the figure, but this part is covered by the shown range of mixing, and the $|\langle m \rangle|$-values have very weak dependence on such small mass-squared differences. For other phases or higher masses than those considered in the lower part of Fig. 3, the near-vertical contours will be shifted towards larger mixing. In other words, if, for example, the effective Majorana mass should turn out to be $|\langle m \rangle| = 0.05$ eV, the region to the left of the corresponding contours would require $m_1 < 0.1$ eV or $U_{e3} > 0.1$. For that particular $|\langle m \rangle|$-value, we showed in Fig. 2 the range in $m_1$ as a function of the phase $\alpha_1$ for four different mixings. To scale the contours in the lower part of Fig. 3 for larger $m_1$-values, we can use the relation $|\langle m \rangle| \propto m_1$, which in this case is valid because Spectrum 1 and small $U_{e3}$ require a weaker constraint than the general one, which is

$$|\langle m \rangle| \propto m_1, \quad \text{if } \Delta m^2_{\text{atm}} \ll m_1^2.$$  \hfill (18)

For negligible $U_{e3}$, this inequality should read $\Delta m^2_{\odot} \ll m_1^2$. From Eq. (18) and the lower part of Fig. 3 we see that the LMA region, if it is confirmed, should lead to a positive signal in GENIUS if we have Spectrum 1, Majorana neutrinos and $m_1 \gtrsim 0.1$ eV. For LOW there
Figure 4: Same as Fig. 3, but for Spectrum 2. The dashed lines and "+" are for $U_{e3} = 0.2$.

is a slight chance of getting positive results for $m_1 = 0.1$ eV, this probability increases as the masses get bigger.

When $|\langle m \rangle| = 0.01$ eV is assumed as the future threshold for a positive $0\nu\beta\beta$-signal, we see from the upper part of Fig. 4 that for Spectrum 2 the whole LMA region (95% C.L.) can be covered by reachable $|\langle m \rangle|$-values, even for $m_1 = 0$. The width of the allowed solar neutrino regions depends somewhat on how the neutrino data are treated (only rates or also spectral information, errors of cross sections, etc.). Thus the LMA region could extend below the future lower bound for $0\nu\beta\beta$-observations. Spectrum 2 would have to be discarded if the LMA region is confirmed and if an $|\langle m \rangle|$-value will be found lying to the right of the new allowed LMA contour in the upper part of Fig. 4. For Spectrum 2 the proportionality relation in Eq. (18) is not valid unless $m_1 \gtrsim 0.4$ eV. Above this value the two spectra are quite similar.

It should be noted that the closed contours in Figs. 3 and 4 are based only on the total rates measured in solar neutrino detectors. As shown in [19], when CHOOZ data are included in these calculations, the allowed regions decrease as $U_{e3}$ increases.

In Fig. 5 we show how the largest allowed $m_1$-value for Spectrum 1 changes as a function of $\tan^2 \theta_{12}$ over a region covering the 95% C.L. LMA and VO/LOW fits for $|\langle m \rangle| = 0.26$ eV, 0.1 eV and 0.01 eV. (As noted above, the VO and LOW regions are not shown separately because they overlap to some extent and both of them cover the maximal mixing case.) The $m_1$-values for the leftmost part of the curves are practically the same as those for the SMA region. The variation of $|\langle m \rangle|$ within the SMA region is totally negligible. The highest
m_1\)-value in Fig. 5 is close to the cosmological bound, see Table 1. If the allowed mass value for the Z-burst theory is to be confirmed and/or sharpened, we see from this figure that in case of Majorana neutrinos, an observation of 0νββ seems very likely, especially if the LMA region should be confirmed by KamLAND.

It is instructive to compare the data from 0νββ and solar neutrino measurements to determine the allowed regions in a plane spanned by m_1 and |⟨m⟩|. This is shown in Fig. 6, where the allowed regions (shaded) are determined by the LMA fit and bounded by the current limit from neutrinoless double beta decay. The highest and lowest |⟨m⟩|-values are for a given m_1 found with α_1 = 0 and π, respectively. For the atmospheric mass-squared difference we used the best-fit value Δm^2_\text{atm} = 3.3 \times 10^{-3} \text{ eV}^2. (Since the shaded region changes insignificantly within the allowed U_{e3} range, the figure is drawn for U_{e3} = 0.) Again, we see that Spectrum 1, as opposed to Spectrum 2, allows for lower |⟨m⟩|-values than those measurable in the planned experiments. This figure also illustrates the confluence of the two spectra for increasing masses.

If there should be no signal above the claimed future sensitivity, i.e., |⟨m⟩| < 0.01 eV, then we have one or two of the possibilities:

- The neutrinos are of Dirac character.
- Spectrum 1 and m_1 below the |⟨m⟩| = 0.01 eV contour in Fig. 5.
- Spectrum 2 and mixing close to maximal.

For Spectrum 2 the SMA and the 95% C.L. part of the LMA region would be discarded.

---

**Figure 5:** Maximal allowed values of m_1 for Spectrum 1. Solid curves: |⟨m⟩| = 0.26 eV, 0.1 eV and 0.01 eV, all for U_{e3} = 0; dashed curves: U_{e3} = 0.1. Spectrum 2 would have produced practically the same results, except for |⟨m⟩| = 0.01 eV when m_1 \lesssim 0.1 eV. Dash-dotted curve: minimum m_1 for |⟨m⟩| = 0.01 eV and Spectrum 1. The regions between the vertical pairs of lines are the allowed LMA and VO/LOW solutions.
Figure 6: Allowed bands in the LMA region (gray shaded). The solid lines represent the borders for Spectrum 1, the dashed curves give the allowed region for Spectrum 2. We have shown today’s upper limit, $|\langle m \rangle| = 0.26$ eV, on neutrinoless double beta decay and the cosmological bound, $(1/3)(\sum_j m_j \lesssim 5.5$ eV). The lowest horizontal axis and the dotted vertical line show the technical sensitivity of planned experiments for measuring these two kinds of phenomena [2]. The line of crosses represents the value of $\sqrt{\Delta m^2_{\text{atm}}}$. 

5 Summary

We have discussed the interrelation between solar neutrino data and current and future results from both neutrinoless double beta decay experiments and cosmological observations. It is qualitatively shown how a Majorana phase and a mixing angle could be related after eventual future measurements of the effective Majorana mass, $|\langle m \rangle|$ and the neutrino masses, $m_j$. Values of $|\langle m \rangle|$ are compared to the LMA and VO/LOW solutions in terms of $m_1$. It is also shown that the allowed non-zero $U_{e3}$-values hardly affect the conclusion.

The two spectra are indistinguishable for $m_1 \gtrsim 0.4$ eV. The four main solutions to the solar neutrino problem can be related to the $0\nu\beta\beta$-phenomenology as follows:

- SMA: Due to the smallness of $U_{e2}$ and $U_{e3}$, the $|\langle m \rangle|$ expressions are proportional to $m_1$ (Spectrum 1) or to $m_2$ (Spectrum 2), and they have a very weak dependence on the Majorana phases. The current bound ($|\langle m \rangle| < 0.26$ eV) leads to the mass bound $m_1 \lesssim 0.3$ eV, which excludes a high degree of degeneracy. The Spectrum 2 part is, for any $m_1$-value, within the sensitivity of GENIUS.

- LMA: Due to the involved mixing, the $|\langle m \rangle|$-value is quite dependent on one of the Majorana phases. The neutrino masses are bounded by $m_1 \lesssim 1.5$ eV ($U_{e3} = 0$), and a high degree of degeneracy is allowed. For planned experiments, Spectrum 1 is perhaps below observation for truly hierarchical masses. The entire allowed region for Spectrum 2 is within the GENIUS sensitivity, but just barely for its lowest $|\langle m \rangle|$-values. If a bound $|\langle m \rangle| \lesssim 0.01$ eV should be established, then Spectrum 2 would be seriously disfavoured.
- LOW: Not detectable for a normal hierarchy unless $U_{e3} \gtrsim 0.2$. This region allows $U_{e1} = U_{e2}$, which implies $\langle |m| \rangle \simeq 0$ for very large masses.

- VO: Similar to the LOW region.

References


