Doublet channel neutron-deuteron scattering in leading order effective field theory

B. Blankleider
SoCPES, The Flinders University of South Australia, Bedford Park, SA 5042, Australia

J. Gegelia
INFN - Sezione di Ferrara, via Paradiso 12, 44100 Ferrara, Italy
and High Energy Physics Institute of TSU, University str. 9, Tbilisi 380086, Georgia

Abstract

The doublet channel neutron-deuteron scattering amplitude is calculated in leading order effective field theory (EFT). It is shown that this amplitude does not depend on a constant contact interaction three-body force. Satisfactory agreement with available data is obtained when only two-body forces are included.

I. INTRODUCTION

An intriguing difficulty arises in the application of leading order EFT to the three-body problem. One finds that the full amplitude describing three-boson scattering, or nucleon-deuteron (nd) scattering in the $J = 1/2$ channel, is sensitive to the cutoff used to solve the scattering equations - even though each perturbation diagram, with resummed two-body interactions, is individually finite. In [1] it was argued that the addition of a one-parameter three-body force counter-term is necessary and sufficient to eliminate this cutoff dependence. On the other hand, in refs. [3] and [4] we have shown, on the example of three bosons, that the cutoff dependence is just a natural consequence of the existence of infinitely many solutions to the given scattering equation; moreover, by carefully identifying the physical amplitude from amongst the infinitely many non-physical solutions, we have shown that the cutoff problem can be solved without the introduction of a three-body force.

In the present contribution we show that the physical three-body scattering amplitude in fact does not depend on the constant contact interaction three-body force at all. Further, we demonstrate that for doublet channel nd scattering, good agreement with experiment is obtained with the inclusion of two-body forces only.
II. WHY THERE IS NO THREE-BODY FORCE DEPENDENCE

To show that the leading order EFT three-body amplitude is independent of constant three-body forces, it is sufficient to restrict the discussion to the case of three bosons.

A. Scattering amplitude without three-body forces

In the three-boson case without three-body forces, the s-wave particle-bound-state scattering amplitude \( a(p, k) \) satisfies the equation [5]

\[
a(p, k) = M(p, k) + \frac{2\lambda}{\pi} \int_0^\infty dq \, M(p, q) \frac{q^2}{q^2 - k^2 - i\epsilon} a(q, k),
\]

where

\[
M(p, q) = \frac{8}{3} \left( \frac{1}{a_2} + \sqrt{\frac{3}{4} p^2 - mE} \right) \left[ \frac{1}{2pq} \ln \left( \frac{q^2 + pq + p^2 - mE}{q^2 - pq + p^2 - mE} \right) \right].
\]

In this equation \( k(p) \) is the incoming (outgoing) momentum magnitude, \( E = 3k^2/4m - 1/ma_2^2 \) is the total energy, and \( a_2 \) is the two-body scattering length. Here it is assumed that the summation of perturbation theory diagrams and loop integration can be interchanged in the sense that the difference is of higher order and hence negligible in given leading order calculations. In general such assumptions have to be investigated very carefully as they may lead to fictitious fundamental problems [2].

Eq. (1) is known as the S-TM equation [6], and in the three-boson case has \( \lambda = 1 \). Three nucleons in the spin \( J = 1/2 \) channel obey a pair of integral equations with similar properties to this bosonic equation, while the \( J = 3/2 \) channel corresponds to \( \lambda = -1/2 \). For \( \lambda > 0 \) Danilov’s work [7] shows that the homogeneous equation corresponding to Eq. (1) has a solution for arbitrary \( E \); in particular, there exists a solution for every energy corresponding to the scattering of a projectile off a two-body bound state.

The existence of these solutions implies that Eq. (1) has an infinite number of solutions. In fact the homogeneous equation has more than one solution for any given \( E \). Writing these solutions as \( a_i^h \) where \( i = 1, 2, 3, \ldots \), the most general solution of Eq. (1) can be written as

\[ a = a_p + \sum_i C_i a_i^h \]

where \( a_p \) is any particular solution. It is useful to examine the asymptotic behaviour of \( a(p, k) \) for large \( p \). Because the inhomogeneous term \( M \) behaves asymptotically as \( 1/p \), it follows that either (i) \( a \to 0 \) faster than \( 1/p \), or (ii) the asymptotic behaviour of \( a \) is determined by the asymptotic behaviour of the homogeneous solution \( a^h \). In the latter case the asymptotic behaviour has the form [7]

\[
a(p, k) = \sum_i A_i (k) p^{s_i} + O(1/p)
\]

where \( s_i \) are roots of the equation

\[
1 - \frac{8\lambda}{\sqrt{3}} \frac{\sin \pi s/6}{\sin \pi s/2} = 0.
\]
The summation in Eq. (3) goes over all solutions of Eq. (4) for which $|\text{Res}| < 1$. For $\lambda = 1$ Eq. (4) has two roots for which $|\text{Res}| < 1$: $s = \pm is_0$, where $s_0 \approx 1.00624$, so that Eq. (3) gives the asymptotic behaviour of the amplitude as

$$a(p, k) \sim A_1(k)p^{is_0} + A_2(k)p^{-is_0}.$$  \hspace{1cm}(5)

By contrast, for $\lambda < 0$ the homogeneous equation has no non-trivial solution and the solution of Eq. (1) is unique; in this case the physical amplitude $a$ must vanish asymptotically faster than $1/p$.

In refs. [3,4] we have shown that the oscillatory behaviour of the general amplitude $a$ for $\lambda = 1$ is simply an artifact of the homogeneous equation (corresponding to Eq. (1)) having non-zero solutions, and that the physical amplitude does not display this spurious behaviour; instead, it behaves just like the solution for $\lambda < 0$, namely, it vanishes asymptotically faster than $1/p$. Thus amongst the solutions given by Eq. (5), the physical solution is the one with $A_1(k) = A_2(k) = 0$. More generally, for any $\lambda > 0$ the physical amplitude is the one that has no admixtures of homogeneous equation solutions, and by the above argument, it must therefore vanish asymptotically faster than $1/p$.

Considering Eq. (1) for the case where $a$ is the physical amplitude, since the free term of Eq. (1) behaves like $1/p$ for large $p$, the coefficient of $1/p$ coming from this inhomogeneous term should cancel the coefficient of a similar term coming from the integral part (we note that this argument is valid for both $\lambda = 1$ and $\lambda = -1/2$). Hence

$$0 = \frac{4}{\sqrt{3}} + \frac{8\lambda}{\sqrt{3}\pi} \int_0^{\infty} dq \frac{q^2}{q^2 - k^2 - i\epsilon} a(q, k).$$  \hspace{1cm}(6)

B. Identical scattering amplitude with a constant three-body force

Multiplying Eq. (6) by $2/\sqrt{3}(1/a_2 + \sqrt{3}/4p^2 - mE)H$ and adding the result to Eq. (1), we obtain the scattering equation where the constant three-body force $H$ is included to all orders (an $H$ is simply added to the term in the square bracket of Eq. (2)). Hence the physical non-oscillating solution of Eq. (1) with no three-body force also satisfies the modified Eq. (1) where an arbitrary $H$ is included. Hence the inclusion of a constant three-body force has no effect on the physical scattering amplitude.

In a recent paper [8] it has been shown that doublet channel neutron-deuteron scattering amplitude in EFT with effective range parameters taken into account, does not exhibit any dependence on a constant three-body force. This result is therefore in agreement with the observations of the present work.

III. ND SCATTERING WITH TWO-BODY FORCES ONLY

Doublet channel neutron-deuteron scattering in leading order EFT is analogous to the scalar case and does not involve any additional problems. Starting from the EFT Lagrangian, one can obtain the following system of doublet channel neutron-deuteron scattering equations [6,9]:

\begin{align*}
0 &= \frac{4}{\sqrt{3}} + \frac{8\lambda}{\sqrt{3}\pi} \int_0^{\infty} dq \frac{q^2}{q^2 - k^2 - i\epsilon} a(q, k). \\
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\end{align*}
where $a$ and $b$ are the neutron-$^3S_1$ (nd) and nucleon-$^1S_0$ amplitudes, respectively,

$$G_1(q) = \frac{q^2}{q^2 - k^2 - i\epsilon}, \quad G_2 = \frac{3/4}{\sqrt{3/4q^2 - mE + 1/a_2^s}} \left( \sqrt{3/4q^2 - mE - 1/a_2^s - i\epsilon} \right),$$

$a_2^{s,t}$ are the two-particle scattering lengths in singlet and triplet channels, and $E = 3k^2/4m - 1/m(a_2^s)^2$ is the total energy. Apart from a factor of $1/4$, the driving term $M(p,q)$ differs from Eq. (2) only in that $a_2$ is replaced by $a_2^s$.

Solving the system (7)-(8) and isolating the non-oscillating solution for $a(p, k)$ we obtain the physical amplitude for $nd$ scattering. The results are shown in Fig. 1. In order to isolate the physical amplitude we had to solve the corresponding homogeneous system of equations. This task is especially difficult due to the existence of a continuum of solutions corresponding to the continuous spectrum of scattering energies. The method employed to achieve this numerical solution does not allow us to obtain high accuracy for low momenta [3,4] - that is why our curve does not extend to the origin. We note that while our calculations fit the experimental data quite well, the accuracy of these data is open to question. Ref. [10] does not contain error estimates and ref. [11] claims that at least the scattering length calculated in ref. [10] may be incorrect.

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