Semileptonic decays of double heavy baryons

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We study the semileptonic decays of the lowest lying double heavy baryons using the relativistic three-quark model. We do not employ a heavy quark mass expansion but keep the masses of the heavy quarks and baryons finite. We calculate all relevant form factors and decay rates.

Key words: Heavy baryons, semileptonic decays, relativistic quark models.


The semileptonic decays of heavy mesons and baryons are ideally suited to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The heaviest flavored bottom-charm $B_c$-meson was observed by the CDF Collaboration [1] in the analysis of the decay mode $B_c \rightarrow J/\psi \bar{\nu}$. The discovery of the $B_c$-meson raises hopes that double heavy flavored baryons will also be discovered in the near future. The theoretical treatment of the systems with two heavy quarks is complicated by the fact that one cannot make use of an expansion in terms of the inverse heavy quark masses. Previously, nonrelativistic potential models, diquark approximation, QCD sum rules and nonrelativistic QCD have been used to describe the spectroscopy of double heavy baryons and to estimate the inclusive and some exclusive decay modes of such systems (for review, see [2]-[4] and references therein).

In [5] we have studied the semileptonic decays of the double heavy $B_c$-meson within a relativistic constituent quark model. The relativistic constituent qu-
ark model [6] can be viewed as an effective quantum field theory approach based on an interaction Lagrangian of hadrons interacting with their constituent quarks. Universal and reliable predictions for exclusive processes involving both mesons composed from a quark and antiquark and baryons composed from three quarks result from this approach. The coupling strength of hadrons \( H \) to their constituent quarks is determined by the compositeness condition \( Z_H = 0 \) [7,8] where \( Z_H \) is the wave function renormalization constant of the hadron. The quantity \( Z_H^{1/2} \) is the matrix element between a physical particle state and the corresponding bare state. The compositeness condition \( Z_H = 0 \) enables us to represent a bound state by introducing a hadronic field interacting with its constituents so that the renormalization factor is equal to zero. This does not mean that we can solve the QCD bound state equations but we are able to show that the condition \( Z_H = 0 \) provides an effective and self-consistent way to describe the coupling of the particle to its constituents. One starts with an effective interaction Lagrangian written down in terms of quark and hadron variables. Then, by using Feynman rules, the \( S \)-matrix elements describing hadron-hadron interactions are given in terms of a set of quark diagrams. In particular, the compositeness condition enables one to avoid the double counting of quark and hadron degrees of freedom. This approach is self-consistent and all calculations of physical observables are straightforward.

There is a small set of model parameters: the values of the constituent quark masses and the scale parameters that define the size of the distribution of the constituent quarks inside a given hadron. The shapes of the vertex functions and the quark propagators can in principle be determined from an analysis of the Bethe-Salpeter (Faddeev) and Dyson-Schwinger equations, respectively, as done e.g. in [9,10]. In the present paper we, however, choose a more phenomenological approach where the vertex functions are modelled by Gaussian forms and the quark propagators are given by local representations. We have demonstrated in our papers [6,11] that the relativistic constituent model is consistent with the heavy quark symmetry in the limit of infinite quark masses. We mention that the authors of [12] have developed a relativistic quark model approach for meson transitions which shows some similarities to our approach. They also use an effective heavy meson Lagrangian to describe the couplings of mesons to quarks. They use, however, point-like meson-quark interactions. Loop momenta are explicitly cut off at around 1 GeV in their approach [12]. In our approach we use momentum dependent meson-quark interactions which provide for an effective cut-off of the loop integration.

We have elaborated the so-called relativistic three-quark model (RTQM) to study the properties of heavy baryons containing a single heavy quark (bottom or charm). For the heavy quarks we have used propagators appropriate for the heavy quark limit. Various observables describing semileptonic and nonleptonic decays as well as one-pion and one-photon transitions have been successfully described in this approach [11]. Recently, the RTQM was extended to include the effects of finite quark masses [13].
In this paper we employ the RTQM \cite{13} to calculate the form factors and widths of the semileptonic decays of the lowest lying $\Xi_{bc}$ and $\Xi_{cc}$ baryons. We follow the strategy adopted in Ref. \cite{5} where we have studied leptonic and semileptonic decays of the $B_c$-meson. We employ the impulse approximation in calculating the matrix elements which previously has been widely used in phenomenological Dyson-Schwinger equation studies (see, e.g. Ref. \cite{10}). In the impulse approximation one assumes that the vertex functions depend only on the loop momentum flowing through the vertex.

We start with a brief description of our approach. As was mentioned above, baryons are described as bound states of constituent quarks in the RTQM. The general form of the SU(5)-invariant lagrangian describing the interaction of three low-lying SU(5)-multiplets with their three-quark currents are written as

$$\mathcal{L}_{\text{int}}(x) = \mathcal{L}_{\text{int}}^{1/2^-}(x) + \mathcal{L}_{\text{int}}^{1/2^+}(x) + \mathcal{L}_{\text{int}}^{3/2^+}(x)$$

(1)

where

$$\mathcal{L}_{\text{int}}^{1/2^-}(x) = g_F \mathcal{F}^{[m_1 m_2 m_3]}(x) J_F^{m_1 m_2 m_3}(x) + \text{h.c.},$$

$$\mathcal{L}_{\text{int}}^{1/2^+}(x) = g_B \mathcal{B}^{[m_1 m_2 m_3]}(x) J_B^{m_1 m_2 m_3}(x) + \text{h.c.},$$

$$\mathcal{L}_{\text{int}}^{3/2^+}(x) = g_D \mathcal{D}^{[m_1 m_2 m_3]}(x) J_D^{m_1 m_2 m_3}(x) + \text{h.c.}.$$

Here $m_i = u, d, c, s, b$ are flavor indeces. According to the SU(5)-classification

$$5 \otimes 5 \otimes 5 = 10_A \oplus 40_M \oplus 40_M \oplus 35_S$$

there is the antisymmetric decuplet $\mathcal{F}^{[m_1 m_2 m_3]}$ with $J^P = \frac{1}{2}^-$, two 40-plets $\mathcal{B}^{[m_2 m_1 m_3]}$ with mixed symmetry and $J^P = \frac{1}{2}^+$ and a symmetric 35-plet $\mathcal{D}^{[m_1 m_2 m_3]}$ with $J^P = \frac{3}{2}^+$. The three-quark currents are written as

$$J_F^{m_1 m_2 m_3}(x) = \int dx_1 \int dx_2 \int dx_3 \Phi_F(x; x_1, x_2, x_3)$$

$$\times \gamma^\mu \gamma^5 q_{a_1}^{m_1}(x_1) \left( q_{a_2}^{m_2}(x_2) C \gamma^\mu q_{a_3}^{m_3}(x_3) \right) \epsilon^{a_1 a_2 a_3},$$

$$J_B^{m_1 m_2 m_3}(x) = \int dx_1 \int dx_2 \int dx_3 \Phi_B(x; x_1, x_2, x_3)$$

$$\times \gamma^\mu \gamma^5 q_{a_1}^{m_1}(x_1) \left( q_{a_2}^{m_2}(x_2) C \gamma^\mu q_{a_3}^{m_3}(x_3) \right) \epsilon^{a_1 a_2 a_3},$$

$$J_D^{m_1 m_2 m_3}(x) = \int dx_1 \int dx_2 \int dx_3 \Phi_D(x; x_1, x_2, x_3)$$

$$\times q_{a_1}^{m_1}(x_1) \left( q_{a_2}^{m_2}(x_2) C \gamma^\mu q_{a_3}^{m_3}(x_3) \right) \epsilon^{a_1 a_2 a_3}.$$
Table 1
The lowest-lying states of double heavy Ξ-baryons with diquarks in the symmetric state. The light quark q denotes u or d.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Interpolating current</th>
<th>Mass (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ξ_{cs}</td>
<td>(-\sqrt{2}\gamma^\mu\gamma^5 c_u (s_b C\gamma^\mu q_c)\varepsilon^{abc})</td>
<td>2.47</td>
</tr>
<tr>
<td>Ξ_{cc}</td>
<td>(\gamma^\mu\gamma^5 q_u (c_b C\gamma^\mu c_c)\varepsilon^{abc})</td>
<td>3.61</td>
</tr>
<tr>
<td>Ξ_{bs}</td>
<td>(-\sqrt{2}\gamma^\mu\gamma^5 b_u (s_b C\gamma^\mu q_c)\varepsilon^{abc})</td>
<td>5.80</td>
</tr>
<tr>
<td>Ξ_{bc}</td>
<td>(-\sqrt{2}\gamma^\mu\gamma^5 b_u (c_b C\gamma^\mu q_c)\varepsilon^{abc})</td>
<td>7.00</td>
</tr>
</tbody>
</table>

where \(a_i = 1, 2, 3\) are color indices. \(F, B\) and \(D\) denote the above three multiplets. Note that the function \(\Phi_H\) is taken to be invariant under the translation \(x \rightarrow x + a\) which guarantees Lorentz invariance for the interaction Lagrangian Eq. (1). SU(5)-symmetry is broken by employing explicit baryon and quark mass values when calculating matrix elements. In this paper we limit our attention to the basic semileptonic decay modes of the lowest lying \(1/2^+\) double heavy baryons: \(\Xi_{bcq} \rightarrow \Xi_{bsq} + \bar{l}\nu\), \(\Xi_{bcq} \rightarrow \Xi_{ccq} + \bar{l}\nu\) and \(\Xi_{ccq} \rightarrow \Xi_{csq} + \bar{l}\nu\) (q=u or d). The appropriate interpolating currents with diquarks in the symmetric state and masses are shown in Table 1. The values of the masses are taken from potential models (see, for example, [2,14]).

The semileptonic transition amplitude is defined as

\[
A\left(\Xi_i(p) \rightarrow \Xi_f(p') \bar{l}\nu\right) = V_{ij} \frac{G_F}{\sqrt{2}} \left(\bar{u}_f(p') \Lambda_{i \rightarrow f}^\mu(p, p') u_i(p)\right),
\]

where \(O^\mu = \gamma^\mu (1 - \gamma^5)\). \(V_{ij}\) is the relevant element of the CKM-matrix where we use \(V_{bc} = 0.04\) and \(V_{cs} = 0.97\). The amplitude \(\Lambda^\mu\) is decomposed into a set of six invariant form factors which are functions of the momentum transfer squared \(q^2 = (p - p')^2\) only:

\[
\Lambda_{i \rightarrow f}^\mu(p, p') = \gamma^\mu \left(F_1^V - F_1^A \gamma^5\right) + i\sigma^{\mu\nu} q^\nu \left(F_2^V - F_2^A \gamma^5\right) + q^\mu \left(F_3^V - F_3^A \gamma^5\right).
\]

We shall not write down rate expressions in terms of these form factors since these have been worked out in great detail in Ref. [3].

In the impulse approximation which is being employed in our approach, the matrix element \(\Lambda^\mu\) is calculated according to

\[
\Lambda_{i \rightarrow f}^\mu(p, p') = -12 g_i g_f \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \hat{\phi}_i(-k_1^2) \hat{\phi}_f(-k_2^2) C_{i \rightarrow f}^\mu,
\]
where

\[ C_{bc\rightarrow cc}^\mu = -\sqrt{2}\gamma^\alpha\gamma^5 S_q(k_2)\gamma^\beta S_c(k_1 + k_2)\gamma^\alpha S_c(k_1 + p') O^\mu S_b(k_1 + p) \gamma^\beta\gamma^5, \]
\[ C_{bc\rightarrow bs}^\mu = \gamma^\alpha\gamma^5 S_b(k_1 + p) \gamma^\beta\gamma^5 \operatorname{tr} \left( S_s(k_2 - q) O^\mu S_c(k_2) \gamma^\beta S_q(k_1 + k_2) \gamma^\alpha \right), \]
\[ C_{cc\rightarrow cs}^\mu = -\sqrt{2}\gamma^\beta\gamma^5 S_c(k_1 + p) \gamma^\alpha S_c(k_2) O^\mu S_s(k_2 + q) \gamma^\beta S_q(k_2 - k_1) \gamma^\alpha\gamma^5, \]

and where \( O^\mu = \gamma^\mu (1 + \gamma^5) \) and \( k^2 \equiv k_1^2 + (k_1 + k_2)^2 + k_2^2 \). The quark propagator is chosen to have a local form

\[ S_i(k) = \frac{1}{m_i - \not{k}} \quad (i = u, d, s, c, b) \quad (5) \]

with \( m_i \) being a constituent quark mass. The vertex function \( \phi_H \) is directly related to the Fourier-transform of the function \( \Phi_H \)

\[ \Phi_H(p_1, \ldots, p_4) = \int dx_1 \ldots \int dx_4 e^{i \sum_{i=1}^4 x_i p_i} \Phi_H(x_1, \ldots, x_4) \]
\[ = (2\pi)^4 \delta^{(4)}(\sum_{i=1}^4 p_i) \phi_H(p_1, p_2, p_3). \]

Generally, \( \phi_H \) is a function of three momentum variables. However, in the impulse approximation employed in our approach, we assume that it only depends on the sum of relative momentum squared as indicated in Eq. (4).

The compositeness condition reads

\[ Z_H = 1 - g_H^2 \Sigma'(m_H) = 0 \quad (6) \]

where \( \Sigma'(m_H) \) is the derivative of baryon mass operator taken on its mass-shell. In the impulse approximation Eq. (6) may be rewritten in a form suitable for the determination of the coupling constants:

\[ -12 g_{q_1 q_2 q_3}^2 \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \phi_{q_1 q_2 q_3}^2(-k^2) D_{q_1 q_2 q_3}^\mu \big|_{\gamma^\mu = m_H} = \gamma^\mu, \quad (7) \]

\[ D_{q_1 q_2 q_3}^\mu = \gamma^\alpha\gamma^5 S_{q_1}(k_1 + p) \gamma^\mu S_{q_1}(k_1 + p) \gamma^\beta\gamma^5 \operatorname{tr} \left( S_{q_2}(k_1 + k_2) \gamma^\alpha S_{q_3}(k_2) \gamma^\beta \right), \]
\[ (q_1 q_2 q_3) = (bcq), (bsq), (csq), \]

\[ D_{ccq}^\mu = \gamma^\alpha\gamma^5 S_c(k_2) \gamma^\beta\gamma^5 \operatorname{tr} \left( \gamma^\alpha S_c(k_1 + p) \gamma^\mu S_c(k_1 + p) \gamma^\beta S_c(k_1 + k_2) \right). \]

For the coupling constants we obtain \( g_{bcq} = 0.96, \quad g_{bsq} = 3.33, \quad g_{ccq} = 2.63, \quad g_{csq} = 3.75. \)
Next we turn to the calculation of the transition form factor. The calculational techniques are outlined in Ref. [13]. The three main ingredients are

- use of the Laplace transform of the vertex function

\[ \Phi(z) = \int_0^\infty ds \Phi_L(s) e^{-sz} \]

- the \( \alpha \)-transform of the denominator

\[ \frac{1}{m^2 - (k + p)^2} = \int_0^\infty d\alpha e^{-\alpha(m^2-(k+p)^2)} \]

- differential representation of the numerator

\[ (m+k+p) e^{kq} = (m + \gamma^\mu \frac{\partial}{\partial q^\mu} + p) e^{kq} \]

The calculation of the transition form factors amounts to a two-loop integrations. Four of the eight two-loop integrations are done analytically. One ends up with 4-fold integrals which are not difficult to evaluate numerically. All calculations are done by using computer programs written in FORM for the manipulations of Dirac matrices and in FORTRAN for numerical evaluations.

The common structure of the expressions for the form factors may be written as

\[ v(q^2) = \int_0^\infty dt t^2 \int \frac{d^4\alpha}{|A|^2} \delta \left( 1 - \sum_{i=1}^4 \alpha_i \right) \left( F(z)W_0 - \frac{1}{2} F_1(z)W_1 + \frac{1}{4} F_2(z)W_2 \right) \] (8)

where

\[ F(z) = \phi_{in}(z)\phi_{out}(z), \quad F_i(z) = \int_0^\infty d\tau \tau^i F(z + \tau), \]

\[ z = t \left( \sum_{i=1}^4 \alpha_i m_{q_i}^2 - \alpha_1 p^2 - \alpha_2 p'^2 \right) + t^2 A_{11}^{-1} (\alpha_1 p - \alpha_2 p')^2. \]

and where the matrix \( A \) is defined as

\[ A = \begin{pmatrix} 2 + t(\alpha_1 + \alpha_2 + \alpha_3) & 1 + t\alpha_3 \\ 1 + t\alpha_3 & 2 + t(\alpha_3 + \alpha_4) \end{pmatrix} \]

The functions \( W_i \) contain integration variables and masses. One has to emphasize that the above expressions are valid for any vertex functions decreasing
rapidly enough in the Euclidean region. Since the quark masses satisfy the confinement constraint $m_H < \sum_{i=1}^{3} m_{q_i}$ all form factors are real.

Before presenting our numerical results we need to specify our values for the constituent quark masses and shapes of the vertex functions. As concerns the vertex functions, we found a good description of various physical quantities [5,6,11,13] adopting a Gaussian form. Here we apply the same procedure using $\phi_H(k_E^2) = \exp\{-k_E^2/\Lambda_H^2\}$ in the Euclidean region. The magnitude of $\Lambda_H$ characterizes the size of the vertex function and is an adjustable parameter in our model. The $\Lambda_H$ parameters in the meson sector were determined [6] by a least-squares fit to experimental data and lattice determinations. The nucleon $\Lambda_N$ parameter was determined from the best description of the electromagnetic properties of the nucleon [6]. The $\Lambda_H$ parameters for baryons with one heavy quark (bottom or charm) were determined by analyzing available experimental data on bottom and charm baryon decays. Since there is no experimental information on the properties of double heavy baryons yet we use the simple observation that the magnitude of $\Lambda_H$ is increasing with the mass value of the hadron whose shape it determines. Keeping in mind that $\Lambda_N = 1.25$ GeV, $\Lambda_{Qq} = 1.8$ GeV and $\Lambda_{B_c} = 2.43$ GeV, we simply choose the value of $\Lambda_{QQq} = 2.5$ GeV for the time being. We found that variations of this value by 10% does not much affect the values of form factors. We employ the same values for the quark masses (see, Eq.(9)) as have been used previously for the description of light and heavy baryons [6,11]. We thus use

$$\begin{array}{cccc}
m_u & m_s & m_c & m_b \\
0.420 & 0.570 & 1.67 & 5.06 \text{ (GeV)}
\end{array} \tag{9}$$

The resulting form factors are approximated by the interpolating form

$$f(q^2) = \frac{f(0)}{1 - a_1 q^2 + a_2 q^4} \tag{10}$$

It is interesting that for most of the form factors the numerical fit values of $a_1$ and $a_2$ obtained from the interpolating form (10) are such that the form factors can be represented by dipole formula

$$f(q^2) \approx d(q^2) = \frac{f(0)}{(1 - q^2/m_V^2)^2} \tag{11}$$

The values of $m_V$ in the dipole representation are very close to the values of the appropriate lower-lying $(\bar{q}q')$ vector mesons ($m_{D_s^*} = 2.11$ GeV for (c-s)-transitions and $m_{B_c^*} \approx m_{B_c} = 6.4$ GeV for (b-c)-transitions). In Fig.1 we show
two representative form factors and their dipole approximations. We have shown in Ref. [5] that the form factors of the CKM-enhanced semileptonic $B_c$-decays may be approximated by a monopole function. It is gratifying to see that our relativistic quark model with the Gaussian vertex function and free quark propagators reproduces the monopole in the meson case and the dipole in the baryon case for most of the form factors.

Finally, in Table 2 we present our predictions for the decay rates and compare them with the free quark decay width which is the leading contribution to the semileptonic inclusive width ($x = m_i^2/m_f^2$)

$$
\Gamma_0(i \to f) = |V_{ij}|^2 \frac{G_F^2 m_i^5}{192 \pi^3} \left(1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \right) .
$$

The nonleading corrections to the leading order rate Eq.(12) lower the inclusive rate by approximately 1% and 15% in the $b \to c$ and $c \to s$ case, respectively (see e.g. [15]). In the numerical evaluation of the inclusive rate Eq.(12) we used current pole mass values $m_b = 4.8$ GeV and $m_c = 1.325$ GeV [16]. There are many values quoted in the literature for the current pole mass of the strange quark. For the sake of definiteness we take $m_s = 0.15$ GeV. Note that in both $cc \to cs$ and $bc \to cc$ decays there is an additional factor of 2 due to the fact that there are two c-quarks in the double charmed baryon in the initial and final state, respectively [2,4]. One notes that the rates for the exclusive modes $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ and $\Xi_{bc} \to \Xi_{bs} + l\bar{\nu}$ are rather small when compared to the total semileptonic inclusive rate estimated by Eq.(12). The remaining part of the inclusive rate would have to be filled in by decays into excited or multi-body baryonic states. Note that the smallness of the exclusive/inclusive ratio of the above exclusive modes markedly differs from that of the mesonic semileptonic $b \to c$ transitions, where the exclusive transitions to the ground state S-wave mesons $B \to D, D^*$ make up approximately 66% of the total semileptonic $B \to X_c$ rate [17]. For $\Lambda_b \to \Lambda_c$ transitions one expects even higher semileptonic exclusive-inclusive ratio of amount 80% [15]. Note that the rate for $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ is of the same order of magnitude as the rates calculated for the corresponding double heavy mesonic decays $B_c \to \eta_c + l\bar{\nu}$ and $B_c \to J/\Psi + l\bar{\nu}$ [5]. The QCD sum rule and potential model predictions for the rates of $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ and $\Xi_{bc} \to \Xi_{cs} + l\bar{\nu}$ given in [2] exceed our rate predictions by factors of 10 and 3 respectively. In fact, the exclusive semileptonic rates given in [2] tend to saturate the inclusive semileptonic rates calculated from Eq.(12) as given in Table 2.

In Table 3 we present values for the invariant form factors at $q^2_{\min} = 0$ and $q^2_{\max} = (m_i - m_f)^2$. Note that the values of the axial vector form factor $F_1^A$ are rather small for the two decays $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ and $\Xi_{bc} \to \Xi_{bs} + l\bar{\nu}$. This provides for a partial explanation of why the rates of these two modes are small compared to the inclusive semileptonic rate. Also the zero recoil values of the
vector form factors are significantly below the value of one which one would
expect from a naive application of the heavy quark limit. The smallness of
the vector form factors provide for the remaining explanation of the smallness
of the predicted respective rates. We mention that the QCD sum rule and
potential model estimates of the zero recoil values of both the vector and
axial vector form factors \(F_V^1\) and \(F_A^1\) given in [2] are close to one. In
the model of [2] the form factors \(F_V^2\) and \(F_A^2\) are set to zero. In our approach
we find that the numerical values of \(F_V^2\) and \(F_A^2\) are quite small compared to
those of \(F_V^1\) and \(F_A^1\) in all cases when expressed in terms of the mass scale
\((m_i + m_f)\).

It is interesting to compare our full zero recoil results with those of a naive
spectator quark model calculation where the zero recoil values of the form
factors result from a simple overlap calculation. For the spin-flavor wave func-
tions we use the \(SU(12)\) spin-flavor wave functions of the naive spectator
quark model (6 flavors \(\times 2\) spin projections) as extended from its original
\(SU(6)\) version to include the heavy flavors. As mentioned before this does not
imply that one is assuming \(SU(12)\) symmetry to hold for the transitions since
one is using physical quark and baryon masses which badly break the \(SU(12)
\) symmetry. Instead one uses \(SU(12)\) symmetry only to construct the spin-flavor
wave functions of the heavy and double heavy baryons. In the \(SU(12)\) naive
spectator quark model the spin-flavor wave functions \(|B>\) of \(\Xi_{bcq}, \Xi_{bsq}\) and
\(\Xi_{ccq}\) baryons with diquarks in the symmetric state and positive (+1/2) spin
projection are given by

\[
|\Xi_{bcq}; \uparrow> \equiv \frac{1}{\sqrt{6}} [2qcb + 2cqb - qbc - cbq - bqc - bcq + 2cqb - qbc - cbq - bqc - bcq - qcb]|\uparrow\uparrow + \uparrow\uparrow - \downarrow\uparrow >
\]

\[
|\Xi_{bsq}; \uparrow> \equiv \frac{1}{\sqrt{6}} [2qsb + 2sqb - qbs - sbq - bqs - bsq + 2sqb - qbs - sbq - bqs - bsq - qsb]|\uparrow\uparrow + \uparrow\uparrow - \downarrow\uparrow >
\]

\[
|\Xi_{ccq}; \uparrow> \equiv \frac{1}{2\sqrt{2}} [qcc + cqc - 2ccq + qcc + cqc - 2ccq]|\uparrow\uparrow + \uparrow\uparrow - \downarrow\uparrow >
\]

The values of \(F_V^1\) and \(F_A^1\) form factors at zero recoil can be calculated from
the matrix elements

\[
F_V^1 = <B | \sum_{i=1}^{3} [I_{fl}]^{(i)} |B > \quad \text{and} \quad F_A^1 = <B | \sum_{i=1}^{3} [\sigma_3 I_{fl}]^{(i)} |B >
\]

where \(\sigma_3\) is the Pauli (third component) spin matrix and \(I_{fl}\) is the flavor
matrix responsible for the s.l. transitions.

In the Table 4 we present the results for the \(F_V^1\) and \(F_A^1\) form factors at
zero recoil calculated in the naive spectator quark model. It is evident that
the spectator quark model prediction of a vanishing axial vector coupling
$F_A^1(0) = 0$ for the decay $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ is close to the suppressed value of $F_A^1(0) = -0.091$ in the full calculation. The ratios of the axial and vector couplings in the decay $\Xi_{bc} \to \Xi_{bs} + l\bar{\nu}$ are $F_A^1(0)/F_V^1(0) = 1/2$ (naive quark model) and $F_A^1(0)/F_V^1(0) \approx 1/6$ (our approach). For $\Xi_{cc} \to \Xi_{cs} + l\bar{\nu}$ decay one has $F_A^1(0)/F_V^1(0) = 1$ (naive spectator quark model) and $F_A^1(0)/F_V^1(0) \approx 1.3$ (our approach). The different normalizations of axial and vector constants obtained in our approach compared to the naive spectator quark model result and the suppression of the ratio $F_A^1(0)/F_V^1(0)$ in the decay $\Xi_{bc} \to \Xi_{bs} + l\bar{\nu}$ in the full calculation can be explained by relativistic effects and nontrivial heavy quark/baryon mass dependence of the relevant matrix elements.

In order to test the sensitivity of our results on various choices of the bottom and charm quark masses, we have varied their values within a reasonable range. From the confinement constraints one obtains lower permissible values for heavy quark masses: $m_c \geq (m_{\Xi_{cc}} - m_u)/2 = 1.60$ GeV and $m_b \geq (m_{\Xi_{bc}} - m_c - m_u) = 4.98$ GeV. The upper values were found from the experimental bounds for the $\Lambda_b \to \Lambda_c e^- \bar{\nu}$ and $\Lambda_c \to \Lambda_s e^+ \nu$ decay rates: $m_c \leq 1.72$ GeV and $m_b \leq 5.25$ GeV [13]. We have calculated the values of decay rates of double baryons for three set of quark masses in Table 5. The decay rates do not change significantly in the chosen regions of the heavy quark masses.

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Table 2
Calculated decay widths of lowest lying $J^P = 1/2^+$ double heavy $\Xi$-baryons. Inclusive widths are calculated using the current quark pole masses.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Decay widths, ps$^{-1}$</th>
<th>RTQM</th>
<th>Inclusive width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi_{bc} \rightarrow \Xi_{cc} + l \bar{\nu}$</td>
<td>0.012</td>
<td>$2 \cdot \Gamma_0(b \rightarrow c) = 0.162$</td>
<td></td>
</tr>
<tr>
<td>$\Xi_{bc} \rightarrow \Xi_{bs} + l \bar{\nu}$</td>
<td>0.043</td>
<td>$\Gamma_0(c \rightarrow s) = 0.122$</td>
<td></td>
</tr>
<tr>
<td>$\Xi_{cc} \rightarrow \Xi_{cs} + l \bar{\nu}$</td>
<td>0.224</td>
<td>$2 \cdot \Gamma_0(c \rightarrow s) = 0.244$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3
Values of $F_V^1$ and $F_A^1$ form factors at maximum and zero recoil.

<table>
<thead>
<tr>
<th></th>
<th>$\Xi_{bc} \rightarrow \Xi_{cc}$</th>
<th>$\Xi_{bc} \rightarrow \Xi_{bs}$</th>
<th>$\Xi_{cc} \rightarrow \Xi_{cs}$</th>
<th>$F_V^1$</th>
<th>$F_A^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2 = 0$</td>
<td>0.46</td>
<td>-0.091</td>
<td>0.39</td>
<td>0.061</td>
<td>0.47</td>
</tr>
<tr>
<td>$q^2 = q^2_{\text{max}}$</td>
<td>0.83</td>
<td>-0.086</td>
<td>0.58</td>
<td>0.065</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table 4
Values of $F_V^1$ and $F_V^2$ form factors at zero recoil in the naive spectator quark model.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$\Xi_{bcq} \rightarrow \Xi_{ccq}$</th>
<th>$\Xi_{bcq} \rightarrow \Xi_{bsq}$</th>
<th>$\Xi_{ccq} \rightarrow \Xi_{csq}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_V^1$</td>
<td>$1/\sqrt{2}$</td>
<td>1</td>
<td>$1/\sqrt{2}$</td>
</tr>
<tr>
<td>$F_V^2$</td>
<td>0</td>
<td>$1/2$</td>
<td>$1/\sqrt{2}$</td>
</tr>
</tbody>
</table>

Table 5
Calculated decay widths of double heavy $\Xi$-baryons for three sets of quark masses.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$m_c = 1.60$ GeV</th>
<th>$m_c = 1.67$ GeV</th>
<th>$m_c = 1.72$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m_b = 4.98$ GeV</td>
<td>$m_b = 5.06$ GeV</td>
<td>$m_b = 5.25$ GeV</td>
</tr>
<tr>
<td>$\Gamma (\Xi_{bc} \rightarrow \Xi_{cc} + l \bar{\nu})$, ps$^{-1}$</td>
<td>0.0102</td>
<td>0.0117</td>
<td>0.0123</td>
</tr>
<tr>
<td>$\Gamma (\Xi_{bc} \rightarrow \Xi_{bs} + l \bar{\nu})$, ps$^{-1}$</td>
<td>0.0498</td>
<td>0.0432</td>
<td>0.0371</td>
</tr>
<tr>
<td>$\Gamma (\Xi_{cc} \rightarrow \Xi_{cs} + l \bar{\nu})$, ps$^{-1}$</td>
<td>0.258</td>
<td>0.224</td>
<td>0.208</td>
</tr>
</tbody>
</table>
References


Fig. 1. Upper panel: Form factor $F_1^V(q^2)$ (solid-dotted line) for $bc \to bs$ transitions and its dipole approximation (solid line) with $m_{cs} = 2.88$ GeV; Lower panel: Form factor $F_1^V(q^2)$ (solid-dotted line) for $bc \to cc$ transitions and its dipole approximation (solid line) with $m_{bc} = 6.81$ GeV.