Illuminating Dense Quark Matter

Cristina Manuel¹ and Krishna Rajagopal²

¹*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

²*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

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We imagine shining light on a lump of cold dense quark matter, in the color-flavor locked phase and therefore a transparent insulator. We calculate the angles of reflection and refraction, and the intensity of the reflected and refracted light. Although the only potentially observable context for this phenomenon (reflection of light from and refraction of light through an illuminated quark star) is unlikely to be realized, our calculation casts new light on the old idea that confinement makes the QCD vacuum behave as if filled with a condensate of color-magnetic monopoles.

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At high enough baryon density and low temperature, the ground state of QCD with three flavors of quarks is the color-flavor locked (CFL) phase [1,2]. In this phase, quarks of all three colors and all three flavors form Cooper pairs, meaning that all fermionic quasiparticles are gapped. The gap Δ is likely of order tens to 100 MeV at astrophysically accessible densities, with quark chemical potential $\mu \sim (350-500)$ MeV [2]. The condensate is charged with respect to eight of the nine massless gauge bosons (eight gluons, one photon) of the ordinary vacuum, meaning that eight gauge bosons get a mass. Chiral symmetry is spontaneously broken, and so is baryon number (i.e., the material is a superfluid). At asymptotic densities, the effective coupling is weak and the properties of the ground state and its low-energy excitations can be determined quantitatively by adapting methods used in the theory of superconductivity (BCS theory).

The CFL phase persists for finite masses and even for unequal masses, so long as the differences are not too large [3]. It is very likely the ground state for real QCD, assumed to be in equilibrium with respect to the weak interactions, over a substantial range of densities. Throughout the range of parameters over which the CFL phase exists as a bulk (and therefore electrically neutral) phase, it consists of equal numbers of u , d , and s quarks and is therefore electrically neutral in the absence of any electrons [4]. The equality of the three quark number densities is enforced in the CFL phase by the fact that this equality maximizes the pairing energy associated with the formation of ud , us , and ds Cooper pairs. This equality is enforced even though the strange quark, with mass m_s , is heavier than the light quarks [5].

In the CFL phase, there is an unbroken $U(1)_{\tilde{Q}}$ gauge symmetry and a corresponding massless photon given by a linear combination of the ordinary photon and one of the gluons [1,2,6]. $U(1)_{\tilde{Q}}$ is generated by $\tilde{Q} = Q + T_8/\sqrt{3}$, where Q is the conventional electromagnetic charge generator and the color hypercharge generator T_8 is normalized such that, in the representation of the quarks, $T_8/\sqrt{3} = \text{diag}(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$ in color space. The CFL con-

densate is $\tilde{Q}$ neutral, the $U(1)$ symmetry generated by $\tilde{Q}$ is therefore unbroken, the associated $\tilde{Q}$ photon remains massless, and within the CFL phase the $\tilde{Q}$ -electric and $\tilde{Q}$ -magnetic fields satisfy Maxwell's equations. The massless combination of the photon and the eighth gluon, $A_\mu^{\tilde{Q}}$, and the orthogonal massive combination which experiences the Meissner effect, A_μ^X , are given by

$$A_\mu^{\tilde{Q}} = \cos\theta A_\mu + \sin\theta G_\mu^8, \quad (1)$$

$$A_\mu^X = -\sin\theta A_\mu + \cos\theta G_\mu^8. \quad (2)$$

The mixing angle θ (called α_0 in Ref. [6]) which specifies the unbroken $U(1)$ is given by

$$\cos\theta = \frac{g}{\sqrt{g^2 + e^2/3}}. \quad (3)$$

θ is the analog of the Weinberg angle in electroweak theory. At accessible densities, the gluons are strongly coupled ($g^2/4\pi \sim 1$) and the photons are weakly coupled ($e^2/4\pi \approx 1/137$), so θ is small, perhaps of order $1/20$. The "rotated photon" consists mostly of the usual photon, with only a small admixture of the G^8 gluon.

All elementary excitations in the CFL phase are either $\tilde{Q}$ neutral or couple to $A_\mu^{\tilde{Q}}$ with charges which are integer multiples of the $\tilde{Q}$ charge of the electron $\tilde{e} = e \cos\theta$, which is less than e because the electron couples only to the A_μ component of $A_\mu^{\tilde{Q}}$. The only massless excitation (the superfluid mode) is $\tilde{Q}$ neutral. Because all charged excitations have nonzero mass and there are no electrons present, the CFL phase in bulk is a transparent insulator at low temperatures: $\tilde{Q}$ -magnetic and $\tilde{Q}$ -electric fields within it evolve simply according to Maxwell's equations, and $\tilde{Q}$ light traverses it without scattering. This analysis is limited to light with ω and k both much less than the energy needed to create a charged excitation in the CFL phase. This means $\omega, k \ll \Delta$, where Δ is the fermionic gap, to avoid the breaking of pairs and the creation of quasiparticles. It also means $\omega, k \ll m_{\pi^\pm}, m_{K^\pm}$, where $\pi^\pm$ and $K^\pm$ are the charged pions and kaons of the CFL phase.

Their masses are of the order of $\tilde{z}\Delta$ in the chiral limit [7], and the contribution to their masses due to finite quark masses has also been evaluated [8].

Imagine shining light on a chunk of dense quark matter in the CFL phase. If CFL matter occurs only within the cores of neutron stars, cloaked under kilometers of hadronic matter [9], the thought experiment we describe here in which light waves traveling in vacuum strike CFL matter can never arise in nature. If, however, the fact that quark matter features many more strange quarks than ordinary nuclear matter renders it stable even at zero pressure, then one may imagine quark stars in nature [10]. Such a quark star may be made of CFL quark matter throughout, or may have an outer layer in which a less symmetric pattern of pairing occurs. For example, quarks of only two colors and flavors may pair, yielding the 2SC phase which was the first color superconducting phase studied [11]. Some of the remaining quarks with differing Fermi momenta may also form a crystalline color superconductor [12]. As in the CFL phase, the 2SC condensate leaves a (slightly different) $\tilde{Q}$ photon massless. However, the 2SC phase is a good $\tilde{Q}$ conductor because of the presence of unpaired quarks and electrons. Thus, 2SC matter is opaque and metallic rather than transparent and insulating. Illuminating it would result in absorption and reflection, but no refraction. We shall assume that the quark matter we illuminate is in the transparent CFL phase all the way out to its surface.

Consider, then, an enormously dense, but transparent, illuminated quark star. Some light falling on its surface will reflect, and some will refract into the star in the form of $\tilde{Q}$ light. We shall calculate the reflection and refraction angles and the intensity of the reflected light and refracted $\tilde{Q}$ light. The partial Meissner effect induced by a static magnetic field has been analyzed previously [6]. We analyze a time-varying electromagnetic field. As a bonus, our analysis allows us to use well-understood properties of dense quark matter in the CFL phase to learn about the (less well understood) QCD vacuum.

In vacuum the electromagnetic fields obey the free Maxwell's equations

$$\nabla \cdot \mathbf{D} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (4a)$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}, \quad (4b)$$

where $\mathbf{D} = \epsilon_0 \mathbf{E}$ and $\mathbf{B} = \mu_0 \mathbf{H}$, and ϵ_0 and μ_0 are the vacuum dielectric constant and magnetic permeability, respectively, such that the velocity of light $c = 1/\sqrt{\mu_0 \epsilon_0}$. Deep in the CFL phase, the rotated fields $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ obey the same field equations, but with dielectric constant [13]

$$\tilde{\epsilon} = \epsilon_0 \left(1 + \frac{8\alpha}{9\pi} \cos^2 \theta \frac{\mu^2}{\Delta^2} \right), \quad (5)$$

where α is the electromagnetic fine structure constant and μ is the chemical potential. This expression for $\tilde{\epsilon}$ is valid to leading order in α , and for $\omega, k \ll \Delta$. The dependence of $\tilde{\epsilon}$ on ω arises only in corrections to (5) which are suppressed by ω^2/Δ^2 , and we therefore neglect dispersion in this Letter. The magnetic permeability in the CFL phase remains unchanged to leading order, $\tilde{\mu} = \mu_0$. The index of refraction of CFL quark matter thus reduces to $\tilde{n} = \sqrt{\tilde{\mu}\tilde{\epsilon}/\mu_0\epsilon_0} = \sqrt{\tilde{\epsilon}/\epsilon_0}$. If we apply (5) for $\mu/\Delta \sim 4 - 10$, we obtain $\tilde{n} \sim 1.02 - 1.1$.

We take the surface of the CFL matter to be planar, with the CFL phase at $z > 0$ and vacuum at $z < 0$. (That is, we assume any curvature of the surface is on length scales long compared to the wavelength of the light.) For an ordinary dielectric, the analogous problem is solved in Ref. [14]. The complication here is that we must match the ordinary electric and magnetic fields in vacuum onto $\tilde{Q}$ -electric and $\tilde{Q}$ -magnetic fields within the CFL phase. The properties of the reflected and refracted waves will therefore depend upon both the dielectric constant $\tilde{\epsilon}$ and the mixing angle θ .

We are interested only in the reflected and refracted waves, and not in the detailed field configurations very close to the interface. This means that we can follow the strategy of Ref. [6] and encapsulate the physics of the interface into boundary conditions relating $\mathbf{E}$ and $\mathbf{B}$ on the vacuum side of the interface to $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ on the CFL side. On the CFL side, the massive X fields can be neglected as long as z is greater than some λ^{CFL} , while on the vacuum side, the confined gluon fields can be neglected as long as $|z|$ is greater than some λ^{QCD} . λ^{QCD} is a length scale characteristic of confinement. For the nonstatic fields of interest, and in the weak coupling regime, λ^{CFL} is of the order of $1/\Delta$, longer than the inverse Meissner mass $\sim g\mu$ [15]. In order to describe light whose wavelength is long compared to λ^{QCD} and λ^{CFL} , we need boundary conditions relating $\mathbf{E}$ and $\mathbf{B}$ at $z = -\lambda^{\text{QCD}}$ to $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ at $z = +\lambda^{\text{CFL}}$.

X -magnetic fields experience a Meissner effect in the CFL phase, meaning that supercurrents in the CFL matter within λ^{CFL} of the interface screen the X component of any ordinary magnetic field parallel to the interface on the vacuum side, yielding the boundary condition

$$\tilde{\mathbf{H}}_{\parallel}(t, x, y, \lambda^{\text{CFL}}) = \cos \theta \mathbf{H}_{\parallel}(t, x, y, -\lambda^{\text{QCD}}). \quad (6)$$

The CFL condensate is charged with respect to the X gauge boson, meaning that if there is an ordinary electric field perpendicular to the interface on the vacuum side, the X component of the electric flux will terminate in the CFL phase within λ^{CFL} of the interface, yielding

$$\tilde{\mathbf{D}}_{\perp}(t, x, y, \lambda^{\text{CFL}}) = \cos \theta \mathbf{D}_{\perp}(t, x, y, -\lambda^{\text{QCD}}). \quad (7)$$

We expect that the confined QCD vacuum should behave as if it is a condensate of color-magnetic monopoles [16].

That is, in the vacuum color magnetic field lines end: if there is a $\tilde{Q}$ -magnetic field perpendicular to the interface on the CFL side, the vacuum will ensure that only the ordinary magnetic field is admitted. Thus,

$$\mathbf{B}_\perp(t, x, y, -\lambda^{\text{QCD}}) = \cos\theta \tilde{\mathbf{B}}_\perp(t, x, y, \lambda^{\text{CFL}}). \quad (8)$$

Finally, color magnetic currents on the vacuum side of the interface should exclude the color component of any $\tilde{Q}$ -electric field parallel to the interface on the CFL side, ensuring that

$$\mathbf{E}_\parallel(t, x, y, -\lambda^{\text{QCD}}) = \cos\theta \tilde{\mathbf{E}}_\parallel(t, x, y, \lambda^{\text{CFL}}). \quad (9)$$

At sufficiently high density, the property of CFL matter from which (6) and (7) follow, namely the Meissner effect for X bosons, is a weak-coupling phenomenon which can be understood analytically. The properties of the QCD vacuum used to deduce (8) and (9) follow from a reasonable and familiar description of confinement as a dual Meissner effect, but confinement is not yet understood analytically. It is therefore of interest that our analysis below provides a *derivation* of (8) and (9) from (6).

Consider an incident wave, with wave vector $\mathbf{k} = \frac{\omega}{c}(\sin i, 0, \cos i)$, so that

$$\mathbf{E} = \mathcal{E} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}, \quad \mathbf{B} = \sqrt{\mu_0 \epsilon_0} \frac{\mathbf{k}}{k} \times \mathbf{E}. \quad (10)$$

There are two orthogonal linear polarizations, shown in Fig. 1, which we will treat separately. In the first, the vector $\mathcal{E}$ is parallel to the interface while in the second, $\mathcal{E}$ lies in the plane of incidence. The reflected wave is

$$\mathbf{E}' = \mathcal{E}' e^{i(\mathbf{k}' \cdot \mathbf{r} - \omega t)}, \quad \mathbf{B}' = \sqrt{\mu_0 \epsilon_0} \frac{\mathbf{k}'}{k'} \times \mathbf{E}', \quad (11)$$

with wave vector $\mathbf{k}' = \frac{\omega}{c}(\sin i', 0, -\cos i')$, while the refracted wave is

$$\tilde{\mathbf{E}}_r = \tilde{\mathcal{E}}_r e^{i(\mathbf{k}_r \cdot \mathbf{r} - \omega t)}, \quad \tilde{\mathbf{B}}_r = \sqrt{\tilde{\mu} \tilde{\epsilon}} \frac{\mathbf{k}_r}{k_r} \times \tilde{\mathbf{E}}_r, \quad (12)$$

with wave vector $\mathbf{k}_r = \omega \sqrt{\tilde{\mu} \tilde{\epsilon}} (\sin r, 0, \cos r)$.

The boundary conditions must be obeyed at all times, which immediately implies that the frequency of all the waves is the same, as above. The boundary conditions must be obeyed at all points on the planar interface. For $1/k \gg \lambda^{\text{CFL}}, \lambda^{\text{QCD}}$ this implies that $\mathbf{k} \cdot \mathbf{r} = \mathbf{k}' \cdot \mathbf{r} = \mathbf{k}_r \cdot \mathbf{r}$ at $z = 0$, independent of details of the boundary conditions. To satisfy this kinematic constraint, all three wave vectors must lie in a plane and $k \sin i = k' \sin i' = k_r \sin r$. Since $k = k'$, we must have $i = i'$: that is, the angle of incidence is the same as the angle of reflection. Since $k_r = \tilde{n}k$, we also reproduce Snell's law

$$\sin i = \tilde{n} \sin r. \quad (13)$$

The kinematics of the reflection and refraction of light on CFL quark matter are unaffected by the mixing angle θ .

We now use the boundary conditions to find the intensities of the reflected and refracted radiation. For the first polarization in Fig. 1, (6) and (9) yield

$$\begin{aligned} \cos\theta(\mathcal{E} - \mathcal{E}') \sqrt{\frac{\epsilon_0}{\mu_0}} \cos i &= \tilde{\mathcal{E}}_r \sqrt{\frac{\tilde{\epsilon}}{\tilde{\mu}}} \cos r, \\ \mathcal{E} + \mathcal{E}' &= \cos\theta \tilde{\mathcal{E}}_r, \end{aligned} \quad (14)$$

and, using Snell's law, (8) is equivalent to (9) in this case. Solving, we find

$$\frac{\tilde{\mathcal{E}}_r}{\mathcal{E}} = \frac{2 \cos\theta \cos i}{\cos^2\theta \cos i + \frac{\mu_0}{\mu} \tilde{n} \cos r}, \quad (15a)$$

$$\frac{\mathcal{E}'}{\mathcal{E}} = \frac{\cos^2\theta \cos i - \frac{\mu_0}{\mu} \tilde{n} \cos r}{\cos^2\theta \cos i + \frac{\mu_0}{\mu} \tilde{n} \cos r}, \quad (15b)$$

where r is easily eliminated using Snell's law in the form $\tilde{n} \cos r = \sqrt{\tilde{n}^2 - \sin^2 i}$. To the order we are working,

FIG. 1. Incident wave $\mathbf{k}$ strikes a planar interface between vacuum and CFL quark matter, giving a reflected wave $\mathbf{k}'$ and a refracted wave $\mathbf{k}_r$. We use boundary conditions to relate electromagnetic fields just below the interface to $\tilde{Q}$ -electromagnetic fields just above, assuming that the wavelength of the light is long compared to the screening length in the CFL phase and the confinement length in vacuum, symbolized by gray shading. Top panel: polarization perpendicular to the plane of incidence and thus parallel to the interface. Bottom panel: polarization parallel to the plane of incidence.

$\tilde{\mu} = \mu_0$. For the second polarization of Fig. 1, (9) and either (6) or (7) yield

$$\begin{aligned} (\mathcal{E} - \mathcal{E}') \cos i &= \cos \theta \tilde{\mathcal{E}}_r \cos r, \\ \cos \theta \sqrt{\frac{\epsilon_0}{\mu_0}} (\mathcal{E} + \mathcal{E}') &= \sqrt{\frac{\tilde{\epsilon}}{\tilde{\mu}}} \tilde{\mathcal{E}}_r, \end{aligned} \quad (16)$$

and hence

$$\frac{\tilde{\mathcal{E}}_r}{\mathcal{E}} = \frac{2\tilde{n} \cos \theta \cos i}{\frac{\mu_0}{\tilde{\mu}} \tilde{n}^2 \cos i + \tilde{n} \cos r \cos^2 \theta}, \quad (17a)$$

$$\frac{\mathcal{E}'}{\mathcal{E}} = \frac{\frac{\mu_0}{\tilde{\mu}} \tilde{n}^2 \cos i - \tilde{n} \cos r \cos^2 \theta}{\frac{\mu_0}{\tilde{\mu}} \tilde{n}^2 \cos i + \tilde{n} \cos r \cos^2 \theta}. \quad (17b)$$

Upon setting $\cos \theta = 1$, the relations (15) and (17) reproduce results for reflection and refraction off standard dielectric media (see Ref. [14]). Decreasing $\cos \theta$ decreases the A_μ component of A_μ^Q , and thus favors reflection over refraction. For θ as small as in nature, the changes introduced by $\theta \neq 0$ are small. In the (unphysical) limit in which $\cos \theta = 0$, A_μ^Q would be orthogonal to A_μ making the CFL phase a superconductor with respect to ordinary electromagnetism. In this limit, we expect and find zero refraction and perfect reflection for both polarizations.

The value of Brewster's angle is modified by a nonvanishing θ . This is the incident angle for which there is no reflected wave if the polarization is parallel to the plane of incidence. We find

$$i_B = \arctan\left(\frac{\tilde{n}}{\cos^2 \theta} \sqrt{\frac{\tilde{n}^2 - \cos^4 \theta}{\tilde{n}^2 - 1}}\right). \quad (18)$$

We can also imagine sending a $\tilde{Q}$ electromagnetic wave from CFL matter into vacuum. Doing this calculation yields the same results, but with $\tilde{n}$ replaced by $1/\tilde{n}$. As usual, Snell's law then implies that for incident angles bigger than $\arcsin(1/\tilde{n})$, the $\tilde{Q}$ light cannot escape from the CFL matter.

Are the solutions (15) and (17) consistent with energy conservation? The Poynting vector $\mathbf{S} = \frac{1}{2}(\mathbf{E} \times \mathbf{H})$ measures the energy flow per unit area and time. Continuity of the z component of the Poynting vector requires

$$\mathcal{E}^2 \sqrt{\frac{\epsilon_0}{\mu_0}} \cos i = (\mathcal{E}')^2 \sqrt{\frac{\epsilon_0}{\mu_0}} \cos i + (\tilde{\mathcal{E}}_r)^2 \sqrt{\frac{\tilde{\epsilon}}{\tilde{\mu}}} \cos r, \quad (19)$$

a relation which is indeed satisfied by both (15) and (17).

Notice that in our analysis of each of the two polarizations, one boundary condition was irrelevant and Snell's law could be used to eliminate a second. If we had used energy conservation in the form (19) in our derivation instead of just as a check, we could have derived all our results from the single boundary condition (6). That is, given only the boundary condition (6) which is easily derived, Snell's law (13) which is kinematic, and energy conservation (19), we can derive our solutions describing the reflection and refraction of light of both polarizations and, from these electromagnetic fields, we can then derive the remaining boundary conditions (7), (8), and (9). This

means that we have *derived* the boundary conditions motivated above by the idea that the QCD vacuum behaves like a dual superconductor filled with a condensate of color-magnetic monopoles [16]. Having analyzed the illumination of dense quark matter, we find that in addition we have illuminated our understanding of the QCD vacuum.

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