Constraining the quintessence equation of state with SnIa data and CMB peaks

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ABSTRACT

Quintessence has been introduced as an alternative to the cosmological constant scenario to account for the current acceleration of the universe. This new dark energy component allows values of the equation of state parameter $w \geq -1$, and in principle measurements of cosmological distances to Type Ia supernovae can be used to distinguish between these two types of models. Assuming a flat universe, we use the supernovae data and measurements of the position of the acoustic peaks in the Cosmic Microwave Background (CMB) spectra to constrain a rather general class of quintessence potentials, including inverse power law models and recently proposed Supergravity inspired potentials. The likelihood analysis gives an upper limit on the present value of the equation of state parameter, $-1 \leq w^0_Q \leq -0.96$ at 2$\sigma$, a result that appears to rule out a class of recently proposed potentials.

Subject headings: Cosmic Microwave Background Anisotropy, Cosmology

1. Introduction

Observations of distant type Ia supernovae (Perlmutter et al. 1999; Riess et al. 1999) and small angular scale anisotropies in the Cosmic Microwave Background (CMB) (De Bernardis et al. 2000; Balbi et al. 2000; Netterfield et al. 2001; Pryke et al. 2001) suggest that the universe is dominated by a large amount of dark energy with a negative equation of state parameter $w$. One obvious explanation would be the presence for all time of a cosmological constant with $w = -1$, although there is no satisfactory reason known why it should be so close to the critical energy density (for a general review see Sahni & Starobinsky 2000). An alternative proposal introduces a new type of matter and is called 'Quintessence' (Caldwell et al. 1998). Assuming that some unknown mechanism cancels the true cosmological constant, this dark energy is associated with a light scalar field $Q$ evolving in a potential $V(Q)$. The equation of state parameter of the $Q$ component is given by

$$w_Q = \frac{\dot{Q}^2 - V(Q)}{\dot{Q}^2 + V(Q)} \quad (1)$$

and it is a function of time. According to the form of $V(Q)$ the present value of $w_Q$ is in the range $w^0_Q \geq -1$. The temporal dependence of $w_Q$ implies that high red-shift observations could in principle distinguish between $\Lambda CDM$ and $QCDM$ models (Maor et al. 2000; Huterer & Turner 2000; Alcaniz & Lima 2001; Benabed & Bernardeau 2001; Cappi 2001, Weller & Albrecht 2001). Moreover, a number of authors have recently pointed out that the position of the CMB peaks could provide an efficient way to constrain quintessence models (Kamionkowski & Buchalter 2000; Crooks et al. 2000; Doran et al. 2001).

In this paper we use the supernovae sample of Perlmutter et al. (1999) and the recent measurements of the location of the CMB peaks (De Bernardis et al. 2001) to determine new limits on the quintessence equation of state. Our study is similar in approach to an earlier analysis by Efstathiou (1999). We consider a general class of...
potentials parametrized in such a way that we can control their shape, and apply a likelihood analysis to find the confidence regions for the parameters of the potential and the best value for the fractional quintessence energy density $\Omega_Q$. The constraints which emerge for the prior $\Omega_Q = 0.65$ give an upper value for the equation of state, $w_Q^0 \leq -0.96$ at 2$\sigma$ for these class of models. This limit is stronger than those previously obtained in Efstathiou (1999), $w_Q^0 \leq -0.6$ at 2$\sigma$ for the same value of $\Omega_Q$, simply because we are making use of the new improved CMB data. The key result is that in these class of models, for them to succeed the scalar field dynamics has to produce effects similar to pure vacuum energy and in this case it is unlikely that Quintessence can be distinguished from a cosmological constant (see also Maor et al. 2000).

2. Quintessence equation of state

The scalar field dynamics is described by the Klein-Gordon equation

$$\ddot{Q} + 3H\dot{Q} + \frac{dV}{dQ} = 0, \quad (2)$$

with

$$H^2 = \frac{8\pi G}{3} \left[ \rho_m + \rho_r + \frac{\dot{Q}^2}{2} + V(Q) \right], \quad (3)$$

where $\rho_m$ and $\rho_r$ are the matter and radiation energy densities respectively. It is well known that for a wide class of potentials, Eq. (2) possesses attractor solutions (Steinhardt et al. 1999). In this regime the kinetic energy of the field is subdominant allowing $w_Q$ to become negative. The present value of $w_Q^0$ depends on the slope of the potential in the region reached by the field. Actually if the quintessence field rolls down a very flat region (Barreiro et al. 2000) or if it evolves close to a minimum (Albrecht & Skordis 1999, Brax & Martin 1999, Copeland et al. 2000) the equation of state parameter varies in the range $-1 \leq w_Q^0 < -0.8$. On the other hand models like the inverse power law potential (Ratra & Peebles 1988; Zlatev, Wang & Steinhardt 1999) require larger values of $w_Q^0$. A general potential which can accomodate a large class of scenarios is:

$$V(Q) = \frac{M^{4+\alpha}}{Q^\alpha} e^{\frac{1}{2}(\kappa Q)^\beta}, \quad (4)$$

where $\kappa = \sqrt{8\pi G}$ and $M$ is fixed in such a way that today $\rho Q = \rho_c\Omega_Q$. For $\beta = 0$ Eq. (4) becomes an inverse power law, while for $\beta = 2$ we have the SUGRA potential proposed by (Brax & Martin 1999). For $\alpha, \beta \neq 0$ the potential has a minimum, the dynamics can be summarized as the following. For small values of $\beta$ and for a large range of initial conditions, the field does not reach the minimum by the present time and hence $w_Q^0 > -1$. For example, if the quintessence energy density initially dominates over the radiation, the $Q$ field quickly rolls down the inverse power law part of the potential eventually resting in the minimum with $w_Q \sim -1$ after a series of damped oscillations (Riazuelo & Uzan 2000). This behaviour however requires fine tuning the initial value of $Q$ to be small. On the other hand, this can be avoided if we consider large values of $\alpha$ and $\beta$ (Fig.1a). In these models the fractional energy density of the Quintessence field, $\Omega_Q$, is always negligible during both radiation and matter dominated eras. In fact, for small initial values of $Q$, $V(Q)$ acts like an inverse power law potential, hence as $Q$ enters the scaling regime its energy density is subdominant compared to that of the background component. Therefore nucleosynthesis constraints (Bean et al. 2001) are always satisfied and there are no physical effects on the evolution of the density perturbations. The main consequence is that for a different value of $w_Q^0$ the Universe starts to accelerate at a different red-shift (Fig.1b). This implies that different values of $\alpha$ and $\beta$ lead to a different luminosity distance and angular diameter distance. Consequently by making use of the observed distances we may in principle determine an upper limit on $w_Q^0$, potentially constraining the allowed shape of the quintessence potential (Huterer & Turner 2000).
models with $\Omega_{Q} \approx 0.01$, whereas the late ISW is the only effect in $\Omega_{Q} \approx 0$ or in non-minimally coupled models (Amenard & Albrecht 2000, Barreiro, Copeland & Nunes 2001). Of crucial importance is the observation that the position of the third peak remains insensitive to other cosmological quantities, hence we can make use of this fact to test dark energy models (Doran et al 2001).

3. CMB peaks

The CMB power spectrum provides information on combinations of the fundamental cosmological quantities. The position of the Doppler peaks depends on the geometry of the Universe through the angular diameter distance, although the amplitude of the peaks are sensitive to many different parameters. The important point for us is that in general the Quintessence field can contribute to the shape of the spectrum through both the early integrated Sachs-Wolfe effect (ISW) and the late one (Hu et al. 1997). The former is important if the dark energy contribution at the last scattering surface (LSS) is not negligible (Skovordis & Albrecht 2000, Barreiro, Copeland & Nunes 2000) or in non-minimally coupled models (Amenard 2000, Perrotta et al. 2000, Baccigalupi et al. 2000), whereas the late ISW is the only effect in models with $\Omega_{Q} \sim 0$ at LSS (Brax et al. 2000).

A potential problem that we face however is that the amplitude of the CMB spectrum is degenerate in the cosmological parameter space, which means that a Quintessence imprint could remain hidden if we simply concentrate on the amplitudes of the peaks. Fortunately, as has recently been demonstrated an accurate determination of the position of the Doppler peaks is more sensitive to the actual amount of dark energy (Doran et al 2000). To be more precise, the multipole of the $m$-th peak is $l_{m} = m\delta l_{h}$, where $\delta l_{h}$ is proportional to the angular scale of the sound horizon at LSS. In a flat universe $\delta l_{h}$ is given by:

$$l_{m} = l_{sh} (m - \delta l - \delta l_{m}),$$  

where $\delta l_{sh}$ is the mean sound velocity and $\tau_{0}$, $\tau_{ls}$ are the conformal time today and at last scattering respectively. However, physical effects before recombination can shift the scale of the sound horizon at different multipoles, resulting in a better estimate for the peak positions being given by:

$$l_{m} = l_{sh}(m - \delta l - \delta l_{m}),$$

where $\delta l$ is an overall shift (W. Hu et al. 2000) and $\delta l_{m}$ is the shift of the $m$-th peak. These corrections depend on the amount of baryons $\Omega_{bh} h^{2}$, on $\Omega_{Q}^{2}/\Omega_{Q}$ and on the scalar spectral index $n$. Recently, analytic formulae, valid over a large range of the cosmological parameters, have been provided to good accuracy for $\delta l$ and $\delta l_{m}$ (Doran & Lilley 2001). Of crucial importance is the observation that the position of the third peak remains insensitive to other cosmological quantities, hence we can make use of this fact to test dark energy models (Doran et al. 2001).

4. Likelihood analysis and results

4.1. Constraints from supernovae

We want to constrain the set of parameters $\alpha$, $\beta$ and $\Omega_{Q}$ confined in the range: $\alpha \in (1, 10)$, $\beta \in (0, 10)$ and $\Omega_{Q} \in (0.1)$, subject to the assumption of a flat universe. We use the SNeIa data fit C of Perlmutter et al. (1999), that excludes 4 high redshift data points. The magnitude-redshift relation is given by:

$$m(z) = M + 5 \log D_{L}(z, \alpha, \beta, \Omega_{Q}),$$

where $M$ is the ‘Hubble constant free’ absolute magnitude and $D_{L} = H_{0}d_{L}(z)$ is the free-Hubble constant luminosity distance. In a flat universe

$$d_{L}(z) = (\tau_{0} - \tau(z))(1 + z),$$

where $\tau_{0}$ is the conformal time today and $\tau(z)$ is the conformal time at the red-shift $z$ of the observed supernova. Both of these quantities are
calculated solving numerically Eq.(2) and Eq.(3) for each value of $\alpha$, $\beta$ and $\Omega_Q$. In $\mathcal{M}$ we neglect the dependence on a fifth parameter ($\alpha$ in Perlmutter et al. 1999) and assume it to be 0.6, the Perlmutter et al. (1999) best value. We then obtain a gaussian likelihood function $\mathcal{L}^{\text{Sn}}(\alpha, \beta, \Omega_Q)$, by marginalizing over $\mathcal{M}$. In Fig.2a we present the one-dimensional likelihood function normalized to its maximum value for $\Omega_Q$. There is a maximum at $\Omega_Q = 1$, in agreement with Efstathiou (1999). In Fig.3a we present the likelihood contours in the $\alpha - \beta$ parameter space, obtained after marginalizing over $\Omega_Q$. Note that all values are allowed at the $2\sigma$ level. The confidence regions for the SnIa data correspond to quintessence models with $w_0^Q < -0.4$ for $\Omega_Q = 0.6$, an upper limit that agrees with both Perlmutter et al. (1999) and Efstathiou (1999).

Fig. 2.— Fractional Quintessence energy density likelihoods, (a) for SnIa, (b) for CMB Doppler peaks, (c) combined peaks and (d) the combined data sets.

4.2. Constraints from Doppler peaks and SnIa

We now compute the position of the first three Doppler peaks $l_1, l_2$ and $l_3$ using Eq. (6), and considering the same parameter space as used in the supernovae analysis. We assume $\Omega_b h^2 = 0.020$ in agreement with BBN constraints (Burles et al. 2001) and $n = 1$, but we have also checked that our final result on $w_0^Q$ does not change by assuming a lower value of $n$. The Hubble constant is set to $h = 0.70$ in agreement with the recent HST observations (Freedman et al. 2000). The predicted peak multipoles in the CMB are then compared with those measured in the BOOMERANG and DASI spectra (De Bernardis et al. 2001). Note, that the third peak has been detected in the BOOMERANG data but not in the DASI data. Furthermore the authors of De Bernardis et al. (2001), with a model independent analysis, estimated the position of the peaks accurately at $1\sigma$. However because the errors associated with the data are non Gaussian, to be conservative we take our $1\sigma$ errors on the data to be larger than those reported in De Bernardis et al. (2001), so that our analysis is significant up to $2\sigma$. We then evaluate a gaussian likelihood function $\mathcal{L}^{\text{Peaks}}(\alpha, \beta, \Omega_Q)$, and in Fig.2b, we have plotted the normalized one-dimensional likelihoods for the three peaks, as a function of $\Omega_Q$. They overlap over the same values of $\Omega_Q$. For $l_2$ and $l_3$ the variance is larger due to the uncertainty on the peak positions at $2\sigma$. The combined likelihood for the peaks is shown in Fig.2c, we find $\Omega_Q = 0.56 \pm 0.13$. The likelihood

![Fig. 3.— Likelihood contour plots for SnIa, I, II and III acoustic peaks. The blue region is the 68% confidence region while the 90% is the light blue one. For the SnIa the white region correspond to $2\sigma$. The position of the third CMB acoustic peak strongly constrain the acceptable parameter space.](image-url)
for all the data sets combined is shown in Fig. 2d, we find $\Omega_Q = 0.64 \pm 0.07$. These results are in good agreement with the analysis of Netterfield et al. (2001).

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5. Conclusion

The location of the sound horizon is very sensitive to the dark energy contribution. Due to the strong degeneracy in the shape of the CMB spectrum, a certain class of Quintessence models can be better constrained using only the acoustic peaks. We have applied a likelihood analysis to constrain the shape of the quintessence potential, based on both the supernovae type Ia data and the positions of the CMB peaks. Assuming a flat space-time the combined analysis gives the best fit for $\Omega_Q = 0.64 \pm 0.07$. We have found that, even considering large uncertainties in the CMB measurements, the third peak limits the equation of state parameter at 2$\sigma$ in the range $-1 \leq w_Q^0 \leq -0.96$ for $\Omega_Q$ with this prior value. This result has an important implication for minimally coupled quintessence models. Actually they must behave similarly to a cosmological constant, therefore inverse power law and a class of SUGRA inspired models are disfavoured. In fact, an equation of state parameter $w_Q^0 \sim -1$ implies the quintessence field is undergoing small damped oscillations around a minimum or evolving in a very flat region of the potential. For these reasons models like the double exponential potential (Barreiro et al. 2000) or the single modified exponential potential (Skordis & Albrecht 2000) pass this constraint, even though they are not included in our analysis. Another important caveat is that this study does not take into account quintessence scenarios where the contribution of the dark en-
nergy density in radiation or early in matter dominated eras is not negligible. In such a case we would have to take into account physical effects not only on distance measurements, but also on the structure formation process itself. These models and the non-minimally coupled ones therefore could yet be distinguished from a pure $\Lambda CDM$ model. We still require a more complete analysis to understand the nature of the dark energy, but this paper points out that it is possible to constrain certain classes of models far more than was previously realised.

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REFERENCES

