MEASUREMENT OF THE BIAS PARAMETER FROM WEAK LENSING

Henk Hoekstra\textsuperscript{1,2,3}, Howard K.C. Yee\textsuperscript{2,3}, and Michael D. Gladders\textsuperscript{2,3}


\textbf{ABSTRACT}

We have measured the correlation between the lensing signal induced by (dark) matter and number counts of galaxies on scales ranging from 0.15 – 3.0 \(h^{-1}\) Mpc (which correspond to aperture radii of 1 – 15 arcminutes). This provides a direct probe of the scale dependence of the ratio of the classical bias parameter \(b\) and the galaxy-mass correlation coefficient \(r\). The results presented here are based on 16 deg\(^2\) of \(R_C\) band data taken with the CFHT as part of the Red-Sequence Cluster Survey. We used a sample of lens galaxies with 19.5 < \(R_C\) < 21, and a sample of source galaxies with 21.5 < \(R_C\) < 24. The results are consistent with a scale independent value of \(b/r\), which provides valuable constraints on models of galaxy formation on scales that can only be probed through weak lensing. For the currently favored cosmology (\(\Omega_m = 0.3, \Omega_A = 0.7\)) we find \(b/r = 1.05^{+0.12}_{-0.10}\), similar to what is found on larger scales (\(\sim 10h^{-1}\) Mpc) from local dynamical estimates. These results support the hypothesis that light traces mass on scales ranging from 0.15 out to \(\sim 10h^{-1}\) Mpc. The accuracy of the measurement will improve significantly in the coming years, enabling us to measure both \(b\) and \(r\) separately as a function of scale.

\textit{Subject headings:} cosmology: observations – dark matter – gravitational lensing

1. INTRODUCTION

The weak distortions of the images of distant galaxies by intervening matter (cosmic shear) provide an important tool to study the projected mass distribution in the universe. The variance of the lensing signal averaged in circular apertures can be used to constrain cosmological parameters (e.g., Blandford et al. 1991; Kaiser 1992; Schneider et al. 1998). Recently, various detections of this cosmic shear signal have been reported (e.g., Bacon et al. 2000; Kaiser et al. 2000; van Waerbeke et al. 2000; van Waerbeke et al. 2001; Wittman et al. 2000), demonstrating the feasibility and prospects of this technique.

Other studies have concentrated on the dark matter halo properties of galaxies (e.g., Brainerd et al. 1996; Hudson et al. 1998; Fischer et al. 2000; Wilson et al. 2000; Hoekstra 2000). Of these studies, Fischer et al. (2000) were the first to measure the lensing signal with high accuracy. They used the averaged tangential distortion around their sample of lens galaxies to study the galaxy-mass correlation function, which allowed them to investigate the ratio of the bias parameter \(b\) (e.g., Kaiser 1987) and the galaxy-mass correlation coefficient \(r\) (e.g., Pen 1998; Dekel & Lahav 1999).

Other observational constraints of biasing come from dynamical measurements (see Strauss & Willick 1995) which provide a measurement of the parameter \(\beta = \Omega_m/b\) on relatively large scales (\(\geq\) a few Mpc). Current measurements yield \(\beta \sim 0.4\) (see the compilation by Berlind et al. 2001; Peacock et al. 2001) which suggest for \(\Omega_m \sim 0.3\) that \(b/r \sim 1\).

The combination of high resolution cosmological N-body simulations and semi-analytic modelling of galaxy formation has been used to predict galaxy biasing as a function of scale (e.g., Benson et al. 2000; Kauffmann et al. 1999). These studies find that on scales \(\leq 4h^{-1}\) Mpc the galaxies are less clustered than the dark matter, although the results depend on the specific clustering properties for different types of galaxies (e.g., Kauffmann et al. 1999). A measurement of the galaxy biasing as a function of scale is important as it can be used to rule out models of galaxy formation, thus improving our understanding of this complex process.

Schneider (1998) proposed a method that provides the unique opportunity to measure \(b\) and \(r\) as a function of scale. This has been studied in more detail by Van Waerbeke (1998) who concluded that the results depend only slightly on the assumed power spectrum of density fluctuations.

In this letter we demonstrate the usefulness of weak gravitational lensing for studies of galaxy biasing. We use \(R_C\)-band imaging data from the Red-Sequence Cluster Survey (RCS) (e.g., Gladders & Yee 2000), which is a 100 deg\(^2\) galaxy cluster survey designed to provide a large sample of optically selected clusters of galaxies with redshifts 0.1 < \(z\) < 1.4.

2. OBSERVATIONS AND ANALYSIS

We use 16 deg\(^2\) of \(R_C\)-band imaging data from the RCS taken with the CFHT 12k mosaic CCD camera. The integration times are 900s per pointing with seeing ranging from 0\′′7 to 0\′′9. The pipeline data reduction is described in detail in Gladders & Yee (2001). The CFHT part of the RCS consists of 10 widely separated patches of \(\sim 2.1 \times 2.3\) degrees. Here we use 52 pointings of the CFH12k camera, typically 5 pointings from each patch. The data analysed so far do not cover complete patches, and therefore we limit the analysis to the individual pointings.
For the weak lensing analysis we use the scheme outlined in Hoekstra et al. (1998; 2000), which is based on the method described in Kaiser, Squires, & Broadhurst (1995) and Luppino & Kaiser (1997). A detailed discussion of the object analysis and its accuracy is described in Hoekstra et al. (2001), where we present our measurement of cosmic shear. The results indicate that the object analysis, and the necessary corrections for observational distortions work well, which allows us to obtain accurate measurements of the weak lensing signal.

For the analysis presented here, we select a sample of lenses and sources on the basis of their apparent $R_C$ magnitude. We use galaxies with $19.5 < R_C < 21$ as lenses, and galaxies with $21.5 < R_C < 24$ as sources which are used to measure the lensing signal. This selection yields a sample of 36226 lenses and $\sim 6 \times 10^5$ sources.

3. Aperture Mass and Number Counts

To study the galaxy biasing as a function of scale, we essentially measure the ratio of the cross-correlation between mass and galaxies, and the galaxy auto-correlation function. The method is described in detail in Schneider (1998) and Van Waerbeke (1998), and we discuss it only briefly here. The aperture mass is defined as (Schneider et al. 1998)

$$M_{ap}(\theta) = \int d^2 \phi \ U(\phi) \kappa(\phi).$$

(1)

If $U(\phi)$ is a compensated filter, i.e., $\int d\phi \ U(\phi) = 0$, with $U(\phi) = 0$ for $\phi > \theta$, the aperture mass statistic $M_{ap}$ can be directly expressed in terms of the (in the weak lensing regime) observable tangential shear $\gamma_T$

$$M_{ap}(\theta) = \int d^2 \phi \ Q(|\phi|) \gamma_T(\phi),$$

(2)

where

$$Q(\phi) = \frac{2}{\theta^2} \int_0^\theta d\phi \ U(\phi) - U(\theta).$$

(3)

We will take (Schneider 1998; Van Waerbeke et al. 2001)

$$U(\phi) = \frac{9}{\pi \theta_{ap}^2} \left[ 1 - \left( \frac{\phi}{\theta_{ap}} \right)^2 \right] \left[ \frac{1}{3} - \left( \frac{\phi}{\theta_{ap}} \right)^2 \right],$$

(4)

with the corresponding $Q(\phi)$

$$Q(\phi) = \frac{6}{\pi \theta_{ap}^2} \left( \frac{\phi}{\theta_{ap}} \right)^2 \left[ 1 - \left( \frac{\phi}{\theta_{ap}} \right)^2 \right] .$$

(5)

The real bias relation can be complicated as it depends on the process of galaxy formation (e.g., Dekel & Lahav 1999), but in the standard, deterministic, linear bias theory, the galaxy density contrast $\delta_g$ is related to the mass density contrast $\delta$ as (e.g., Kaiser 1987) $\delta_g = b \delta$, and the number density contrast of galaxies is given by

$$\Delta n_g(\theta) = \frac{N(\theta) - \bar{N}}{\bar{N}} = b \int dw p_f(w) \delta(f_K(w); w),$$

(6)

$1$ We found that the ratios presented in Figure 5 from Van Waerbeke (1998) are in fact proportional to $\sqrt{\Omega_m}$, which obviously does not alter the conclusion that $\mathcal{R}$ is constant with scale

where $\bar{N}$ is the average number density of lens galaxies, $w$ is the comoving distance, $f_K(w)$ is the comoving angular diameter distance, and where $p_f(w)dw$ corresponds to the redshift distribution of lens galaxies.

We define the filtered number counts $N(\theta_{ap})$ as (Schneider 1998)

$$N(\theta_{ap}) = \int d^2 \phi U(\phi) \Delta n_g(\phi).$$

(7)

For our choice of the filter function $U(\phi)$ we define the ‘filtered’ power spectrum $P_{\text{filter}}(w; \theta_{ap})$ as (e.g., Schneider et al. 1998)

$$P_{\text{filter}}(w; \theta_{ap}) = \int ds \ P_{3d} \left( \frac{s}{f_K(w)} \right)^2 \left[ \frac{12 J_4(s \theta_{ap})}{s^2 (s \theta_{ap})^2} \right] ,$$

(8)

where $P_{3d}$ is the time-evolving 3-D power spectrum, and $J_4(x)$ is the fourth order Bessel function of the first kind. As has been shown by Jain & Seljak (1997), it is important to use the non-linear power spectrum in the calculations, for which the results from Peacock & Dodds (1996) are used.

We can write the auto-correlation of $N$ as (Schneider 1998; Van Waerbeke 1998)

$$\langle N^2(\theta_c) \rangle = 2 \pi b^2 \int dw \frac{p_f^2(w)}{f_K(w)} P_{\text{filter}}(w; \theta_{ap}),$$

(9)

and the cross-correlation $\langle M_{ap}(\theta_c) N(\theta_{ap}) \rangle$ as

$$\langle M_{ap}(\theta_c) N(\theta_{ap}) \rangle = 3 \pi (\frac{b \Omega_m}{a(w) f_K(w)})^2$$

$$\int dw \frac{p_f(w) g(w)}{f_K(w)} P_{\text{filter}}(w; \theta_c),$$

(10)

where $\Omega_m$ is the density parameter, and $a$ is the cosmic expansion factor.

Schneider (1998) and Van Waerbeke (1998) used the simple, deterministic, linear biasing theory, which assumes a perfect correlation between the galaxy and mass density fields. However, as pointed out by Pen (1998), and Dekel & Lahav (1999), the biasing relation need not be deterministic, but might be stochastic. In this case the galaxy-mass cross-correlation coefficient $r$ is less than 1. Hence, we allow for stochastic biasing, and include the parameter $r$. The function $g(w)$ depends on the redshift distribution of the (background) sources $p_b(w)dw$ as

$$g(w) = \int_w^{w_H} dw' \ p_b(w') \frac{f_K(w' - w)}{f_K(w)} .$$

(11)

The predicted correlations $\langle N^2 \rangle$ and $\langle M_{ap} N \rangle$ depend on the choice of the power spectrum. Van Waerbeke (1998) showed that the ratio

$$\mathcal{R}(\theta_{ap}) = \frac{\langle M_{ap}(\theta_{ap}) N(\theta_{ap}) \rangle}{\langle N^2(\theta_{ap}) \rangle} ,$$

(12)

depends little on the choice of power spectrum, provided that $b$ and $r$ are independent of scale. In this case, the ratio only depends on $\Omega_m$ and $\Omega\Lambda$, and is constant with scale. Thus we can write...
\[ R = \Omega_m \frac{r}{b} \times f(\Omega_m, \Omega_A), \]  

where \( f(\Omega_m, \Omega_A) \) is a constant for a given cosmology. Thus the measurement of \( R \) as a function of scale provides a unique way to examine whether \( b/r \) depends on scale or not. In fact, weak gravitational lensing allows one to estimate \( b \) and \( r \) separately, when the ratio \( B = \langle M_{ap}^2 \rangle / \langle N^2 \rangle \propto (\Omega_m / b)^2 \) is also measured. Although this ratio is not constant with scale, \( B \) does not depend much on the assumed power spectrum (in particular on scales less than 10 arcmin.). Unfortunately our data are not sufficient to obtain a good measurement of \( \langle M_{ap}^2 \rangle \), but with more data we will be able to measure both \( r \) and \( b \) as a function of scale.

![Figure 1](image-url)

**Fig. 1.**—(a) The observed ratio of \( R \) as a function of aperture radius \( \theta_{ap} \). Note that the points are somewhat correlated. The errorbars are computed using the scatter in the measurements of the individual fields. The dashed line indicates the model predictions for an OCDM model, and the dotted line corresponds to an LCDM model. We find an average value of \( R \approx 0.021 \pm 0.002 \) (indicated by the hatched region), where we have used the full covariance matrix in order to account for the correlation between the points. (b) The measurement when the phase of the shear is increased by \( \pi/2 \), which should vanish if the signal in (a) is caused by lensing. The results are indeed consistent with no signal. The signal also vanishes when we correlate \( N(\theta_{ap}) \) of a given field with \( M_{ap}(\theta_{ap}) \) measured from the other pointings.

### 4. Redshift Distributions

In order to interpret the observed value of \( R \), we have to evaluate equations 9 and 10, which requires knowledge of the redshift distributions of the lens galaxies and the source galaxies.

For the sample of lens galaxies we use the redshift distribution found from the CNOC2 Field Galaxy Redshift Survey (e.g., Lin et al. 1999; Yee et al. 2000; Carlberg et al. 2000). The CNOC2 survey provides a well determined (spectroscopically) redshift distribution for field galaxies down to \( R_C = 21.5 \), which is ideal, given our limits of 19.5 < \( R_C < 21 \). The adopted redshift distribution gives a median redshift \( z = 0.35 \) for the lens galaxies.

For the source galaxies the situation is more complicated. These galaxies are generally too faint for spectroscopic surveys, although recently Cohen et al. (2000) measured spectroscopic redshifts around the Hubble Deep Field North down to \( R_C \approx 24 \). Cohen et al. (2000) find that the spectroscopic redshifts agree well with the photometric redshifts derived from multi color photometry. Because of likely field-to-field variations in the redshift distribution, we prefer to use the photometric redshift distributions derived from both Hubble Deep Fields (Fernández-Soto et al. 1999; Chen et al. 1998). Photometric redshift distributions generally work well, as has been demonstrated by Hoekstra et al. (2000). This redshift distribution yields a median redshift of \( z = 0.53 \) for the source galaxies.

We computed the value of \( R \) for a range of cosmological parameters, and find that, for the adopted redshift distributions, \( R \) can be approximated with fractional accuracy of 2% using

\[ R = \frac{r \Omega_m}{100b} \left[ (5.8 - 1.6\Omega_m^{0.63}) + (4.6 - 2.6\Omega_m^{0.63})\Omega_m^{1.23} \right]. \]

(14)

### 5. Measurement of the Bias Parameter

To measure \( \langle M_{ap}N \rangle \) and \( \langle N^2 \rangle \) from the data we use the estimators for \( M_{ap} \) and \( N \) introduced by Schneider (1998)

\[ \tilde{M}_{ap} = \pi \theta_{ap}^2 \frac{\sum_{i=1}^{N_f} Q(\theta_i)w_i \gamma r_i}{\sum_{i=1}^{N_f} w_i}, \quad \text{and} \quad \tilde{N} = \frac{1}{N} \sum_{i=1}^{N_f} U(\theta_i), \]

(15)

where \( N_f \) and \( N_i \) are respectively the number of lens and source galaxies found in the aperture of radius \( \theta_{ap} \). The weights \( w_i \) correspond to the inverse square of the uncertainty in the shape measurement (see Hoekstra et al. 2000 for a detailed discussion).

The observed value of \( R \) as a function of aperture size is presented in Figure 1a. We note that the points are somewhat correlated. A significant signal is detected at all scales. The results are consistent with a value of \( R \) that is constant with scale, which implies that \( b/r \) is constant as well. This is an important result, as the smallest scales we are probing are comparable to the sizes of galaxy halos. We obtain an average value of \( R \approx 0.021 \pm 0.002 \), where we have used the covariance matrix to account for the correlation between the points at different scales.

To examine possible systematic effects, we also computed \( \langle M_{ap}N \rangle \) when the galaxies are rotated by 45 degrees. This signal should vanish in the case of lensing. The results presented in Figure 1b are consistent with no signal, indicating that the corrections for the systematic distortions have worked well (more details will be provided in Hoekstra et al. 2001). As another check, we correlated \( N(\theta_{ap}) \) for each field with \( M_{ap}(\theta_{ap}) \) of the other pointings, and find that the signal also vanishes in this case.
Fig. 2.—The value of $b/r$ as a function of angular scale, under the assumption $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. Note that the points are slightly correlated. The errorbars (which indicate the 68\% confidence limits) are computed using the scatter in the measurements of the individual fields. The upper axis indicates the effective physical scale probed by the compensated filter $U(\phi)$ at the median redshift of the lenses ($z = 0.35$). The results are consistent with a value of $b/r$ that is independent of scale. For this cosmology we find $b/r = 1.05^{+0.12}_{-0.10}$ (indicated by the hatched region), whereas for an open model ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.0$) we obtain $b/r = 0.73^{+0.08}_{-0.07}$. The error on the average has been computed using the full covariance matrix, in order to account for the correlation between the points at various scales.

In Figure 2 we present the resulting value of $b/r$ as a function of aperture radius for the currently favored cosmology ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$). In this case we find $b/r = 1.05^{+0.12}_{-0.10}$. For an open model ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.0$) we obtain $b/r = 0.73^{+0.08}_{-0.07}$. For comparison, we have also indicated the effective physical scale ($\sim$ FWHM of the filter function) probed by the compensated filter $U(\phi)$ at the median redshift of the lenses ($z = 0.35$).

A direct comparison with dynamical studies is difficult because different galaxy types cluster differently and because of the different scales probed in our study. However, our results are in fair agreement with the results from dynamical studies, in the sense that we find $b/r \sim 1$ (e.g., Berlind et al. 2001; Peacock et al. 2001). Therefore from scales ranging from $0.15 h_{50}^{-1}$ Mpc out to $\sim 10 h_{50}^{-1}$ Mpc, i.e. from the scales of galaxy halos out to the linear regime, the measurements are consistent with a value $b/r \sim 1$, suggesting that the light distribution traces the dark matter distribution quite well.

6. PROSPECTS

For the first time we have measured the parameter $b/r$ as a function of scale using weak lensing using 16 deg$^2$ of data from the Red-Sequence Cluster Survey. With the analysis of the full survey, the errorbars are expected to decrease by a factor $\sim 2$, thus improving the constraints on possible variation of $b/r$ with scale. Also we will be able to probe larger scales, as we have limited the analysis to the individual pointings, rather than the full patches which are $\sim 2.1 \times 2.3$ degrees. Other cosmic shear surveys will place additional constraints, eventually allowing us to measure $r$ and $b$ separately as a function of scale.

The lens galaxies were selected on the basis of their apparent magnitude, but with planned multi-color photometry, it is also possible to measure the biasing properties as a function of galaxy type or luminosity (using photometric redshifts). Eventually, using bigger surveys, it might even be possible to study the evolution of galaxy biasing as a function of redshift.

REFERENCES