Braneworld effective action and origin of inflation

A.O. Barvinsky$^{a,b}$

$^a$Theory Department, Lebedev Physics Institute, Leninsky Pr. 53, Moscow 117924, Russia
$^b$Physics Department, Ludwig Maximilians University, Theresienstr. 37, Munich, Germany

Abstract

We construct braneworld effective action in two brane Randall-Sundrum model and show that the radion mode plays the role of a scalar field localizing essentially nonlocal part of this action. Non-minimal curvature coupling of this field reflects the violation of AdS/CFT-correspondence for finite values of brane separation. Under small detuning of the brane tension from the Randall-Sundrum flat brane value, the radion mode can play the role of inflaton. Inflationary dynamics corresponds to branes moving apart in the field of repelling interbrane inflaton-radion potential and implies the existence acceleration stage caused by remnant cosmological constant at late (large brane separation) stages of evolution. We discuss the possibility of fixing initial conditions in this model within the concept of braneworld creation from the tunneling or no-boundary cosmological state, which formally replaces the conventional moduli stabilization mechanism.

$^\dagger$e-mail: barvin@td.lpi.ru

1. Introduction

Recently there has been a lot of interest in braneworld scenario [1], inspired, on one hand, by string theory and, on the other hand, strongly motivated by the attempts to resolve the hierarchy problem [2] culminating in 120 orders of magnitude gap between the Planck scale and the possible value of the present day cosmological constant [3]. A simple idea underlying a potential solution of the hierarchy problem is based on the possibility to generate exponentially big ratios of energy scales due to exponential nature of the warped compactification factor in braneworld models with multi-dimensional AdS bulk [4]. This idea was followed by the observation that in the observable long-distance approximation the usual 4-dimensional Einstein theory can be recovered on the 3-brane embedded into AdS 5-dimensional spacetime [5, 6], which fact turned out to be a manifestation of the widely
celebrated by string physicists AdS/CFT-correspondence [7, 8, 9, 10, 11, 12]. This made the whole approach extremely attractive and resulted in numerous attempts, apparently beginning with [13], to stabilize the moduli – the values of the warped compactification factor at the location of the visible (Planck) brane – in the range accounting for the hierarchy of scales.

Stabilization of moduli, in our opinion, has two important disadvantages. First, it usually assumes the presence of scalar fields in the bulk which in certain sense is against the spirit of brane-world scheme – its aesthetical purity implies the existence of only the gravitational field in the bulk. Secondly, stabilization makes braneworld static and, thus, does not leave a room for inflation and other stages of cosmological evolution. Question arises whether there exist other, more economical from the viewpoint of abundance of fields, schemes that could incorporate cosmological evolution in braneworld picture. The goal of this paper is an attempt of resolving this question within the two-brane Randall-Sundrum model in which the role of the inflaton is played by the radion mode – local field describing the brane separation.

This idea is not entirely new. Brane-separation model of inflaton was considered in [14] and revisited in [15]. Recently the model of ekpyrotic scenario [16] was put forward as an alternative to inflation. Inflation in brane-antibrane Universe was considered in [17]. These models are based on certain assumptions on the shape of the inter-brane potential and qualitatively describe the situation of approaching and then colliding branes. Their collision serves either as the end of inflation [14] (accompanied by tachyon mediated brane pairs annihilation and reheating in [17]), or as the big bang somehow solving the problems of standard scenario [16] (see the criticism of the latter in [18]).

Qualitatively our model is quite opposite to these suggestions and, effectively, much simpler because, instead of rather involved assumptions on the structure of string induced potentials, it relies on basic properties of Randall-Sundrum model. Moreover, this model brings together a number of issues the combination of which, even despite maybe their phenomenological inconsistency, sounds promising and deserving further studies. These issues include braneworld picture, AdS/CFT-correspondence and the nature of its violation implemented in non-minimal curvature coupling of radion, radion induced inflation corresponding to diverging branes rather than colliding ones in [14, 16]. They also include the issue of initial conditions for inflation actually replacing the concept of stabilization and, finally, incorporate the phenomenon of cosmological acceleration. The unifying framework for all these issues is the formalism of effective action induced on branes from the dynamics of gravity in the bulk.

One of the models that served as a motivation for this radion induced inflation was the model of the low-energy quantum origin of inflationary Universe suggested in [19, 20]. In this model the initial conditions for inflation were generated within the concept of tunneling cosmological wavefunction as a probability peak in the distribution of the inflaton field $P(\phi)$. This distribution is given by the exponentiated classical Euclidean action of the DeSitter instanton $I(\phi)$ corrected by the loop term which begins with the one-loop quantum effective action $\Gamma_{1\text{-loop}}(\phi)$ of all fields inhabiting the instanton background [20],
\[ P(\varphi) \simeq \exp[\mp I(\varphi) - \Gamma_{1\text{-loop}}(\varphi)] \] (\mp \text{ sign here is related to no-boundary [21] and tunneling [22] prescriptions respectively}). Applied to the model with strong non-minimal curvature coupling of the inflaton

\[
S[g, \varphi] = \int d^4x \sqrt{g} \left\{ \frac{m_p^2}{16\pi} R - \frac{1}{2} \xi \varphi^2 R + \frac{1}{2} \partial^2 \varphi - \frac{\lambda \varphi^4}{4} \right\}, \quad -\xi = |\xi| \gg 1,
\]

(1.1)

this distribution features a sharp probability peak at the value of inflaton \( \varphi_I \) which corresponds to the inflationary Hubble constant \( H(\varphi_I) \sim m_p \sqrt{\lambda/|\xi|} \) [19]. CMBR anisotropy in this model reads in terms of coupling constants \( \lambda \) and \( \xi \) as \( \Delta T/T \sim \sqrt{\lambda/|\xi|} \) [23, 24]. Therefore, the energy scale of inflation turns out to be much below the Planck scale – actually even below the GUT scale, \( H(\varphi_I) \sim 10^{-5} m_P \), so that one can trust semiclassical theory. Interestingly, the width of the probability peak is also given by the same ratio, \( \Delta H/H \sim \sqrt{\lambda/|\xi|} \), and the narrow probability peak can be interpreted as the source of initial conditions for inflation. The parameters of the matter sector of the model can be easily tuned to have sufficiently high e-folding number, \( N \geq 60 \), and this model as a whole can be pretty well compatible with observations.

Big value of \( |\xi| \) plays a crucial role in this model. The probability peak arises as an artifact of subtle balance between the tree-level and one-loop factors in \( P(\varphi) \). The one-loop effective action can be approximated by the anomalous scaling behaviour, \( \Gamma_{1\text{-loop}}(\varphi) \sim Z \ln H(\varphi) \), where \( H(\varphi) \) corresponds to the size – inverse radius – of the instanton, and \( Z \) is the anomalous dimensionality expressible in terms of conformal anomaly of quantum matter. In the slow roll approximation due to Higgs effects matter particles acquire masses that are much bigger than the curvature scale of the instanton. As a result the anomalous scaling \( Z \) is dominated by quartic powers of these masses and turns out to be quadratic in \( \xi \), \( Z \sim |\xi|^2 \). Thus, big \( |\xi| \) guarantees that the one-loop factor in \( P(\varphi) \) strongly suppresses high energy scales and generates at intermediate scales a sharp local peak of the above type.

One of the motivations for the present work was to consider the possibility of generating from the braneworld picture such a non-minimal inflaton model (1.1), thus clarifying the origin of inflaton, its non-minimal curvature coupling with big negative \( \xi \). As we shall see, the model similar to (1.1) can really be induced from the two-brane Randall-Sundrum model, the role of inflaton being played by the radion mode also non-minimally coupled to curvature. The sign and strength of this coupling, however, differs from that of (1.1) – the radion part of the effective action turns out to be conformal invariant. This does not, however, prevent from the inflation scenario. Moreover, this scenario is likely to include for late times acceleration stage of the Universe. Interestingly, the mechanism of generating initial conditions for this braneworld inflation via radion field can also be applied here, and it actually replaces the stabilization of moduli concept. This is because from the viewpoint of formalism this is the same stationarity of the effective action requirement as the one in stabilizing the moduli.

The paper is organized as follows. In Sect.2 we discuss the structure of the effective action induced on the brane from bulk gravitational dynamics. In particular, we clarify the
concept of radion as the field in terms of which one can localize essentially nonlocal metric action. Regarding the radion mode and its dynamical or gauge nature there exists a lot of controversial interpretations in current literature. What follows, we hope, sheds light on the gauge invariant role of the radion. In Sect. 3 we present the basics of two-brane Randal-Sundrum model and reformulate Garriga-Tanaka’s equations for gravitational perturbations in this model in the covariant form as an expansion in curvature. In Sect. 4 the structure of non-local coefficients of this expansion is considered in different limits of brane separation and interpreted in terms of AdS/CFT correspondence. The low-energy effective action generating these equations is then derived in the form containing a non-minimal coupling of the radion, which is responsible for strong violation of AdS/CFT-correspondence (and zero graviton mode delocalization) for small separation of branes. In Sect. 5 this action is cast to the form in which both the constant part of brane separation (modulus) and its spacetime dependent perturbation are absorbed into one field \( \varphi \) playing the role of inflaton. This inflaton is conformally coupled to Ricci curvature and acquires a nontrivial inflaton potential by slightly detuning the brane tension from the Randall-Sundrum value characteristic of exactly flat branes. We analyze the dynamical behaviour in the Einstein frame and show that two branes diverge because of the repelling force of the inflaton interbrane potential. Their propagation apart describes the inflationary dynamics on the positive tension brane. By applying the mechanism of braneworld creation from tunneling state we give the estimate for most probable initial conditions of this inflationary evolution. Concluding Sect. 6 contains summarizing comments.

2. Structure of bulk induced brane effective action

In the most general setting the braneworld effective action induced from the 5-dimensional bulk looks as follows. We start with the theory having the action

\[
S[G, g, \psi] = S_5[G] + \int_\Sigma d^4x \sqrt{g} [L_m(\psi, \nabla \psi) - \sigma],
\]

\[
S_5[G] = \int_{M^5} d^5x \sqrt{G} \left( R(G) - 2\Lambda_5 \right).
\]

For clarity of arguments we assume that only gravitational field with the metric \( G = G_{AB}(x, y) \), \( A = (\mu, 5), \mu = 0, 1, 2, 3 \), lives in the bulk spacetime with coordinates \( x^A = (x, y), x = x^\mu, x^5 = y \), while matter fields \( \psi \) with the Lagrangian \( L_m(\psi, \partial \psi) \) are confined to the brane \( \Sigma - 4 \)-dimensional timelike surface embedded in the bulk\(^1\). This setting can be generalized to the case of matter fields propagating in the bulk, but for reasons discussed in Introduction we avoid this in what follows. The brane has induced metric \( g = g_{\mu\nu}(x) \). The bulk and brane parts of the action are supplied with the 5-dimensional and 4-dimensional cosmological constants. Bulk cosmological constant \( \Lambda_5 \) is negative and, therefore, is capable

\(^1\)For brevity we do not include in the action (2.1) the surface Gibbons-Hawking term containing the traces of extrinsic curvatures associated with both sides of the brane [25].
of generating the AdS geometry, while the brane cosmological constant plays the role of 
brane tension $\sigma$ and, depending on the model, can be of either sign.

Full quantum effective action on the brane $S_{\text{eff}}[g, \psi]$ arises as the result of integration over 
bulk metric subject to the boundary condition on the brane – fixed induced metric which is 
the functional argument of $S_{\text{eff}}[g, \psi]$,

$$
\int DG \exp\{iS[G, g, \psi]\} \bigg|_{4G(\Sigma)=g} = \exp\{iS_{\text{eff}}[g, \psi]\}. 
$$

Note that, with this definition, the matter part of effective action coincides with that of the 
brane action in (2.1)

$$
S_{\text{eff}}[g, \psi] = W[g] + S_m[g, \psi], \quad (2.4)
$$

while all non-trivial dependence on $g$ arising from functional integration is contained in $W[g]$. 
Generically, the calculation of this quantity is available only by semiclassical expansion in 
the bulk in powers of $\bar{\hbar}$. In the tree-level approximation,

$$
W[g] = S_5[G(g)] - \int_\Sigma d^4x \sqrt{g} \sigma + O(\bar{\hbar}), \quad (2.6)
$$

$W[g]$ coincides with the 5-dimensional gravitational action on the solution of equations of 
motion in the bulk, $G = G(g)$, with given fixed induced metric on the brane

$$
\frac{\delta S_5[G]}{\delta G_{AB}} = 0, \quad (2.7)
$$

$$
^4G_{\mu\nu}(\Sigma) = g_{\mu\nu}. \quad (2.8)
$$

Given tree-level effective action, one can further apply the variational procedure, now 
with respect to the induced metric $g_{\mu\nu}$, to get the 4-dimensional equations of motion for the latter

$$
\frac{\delta S_{\text{eff}}}{\delta g_{\mu\nu}} = \frac{\delta W}{\delta g_{\mu\nu}} + \frac{1}{2} T^{\mu\nu} = 0, \quad (2.9)
$$

where $T^{\mu\nu} = (2/\sqrt{g}) \delta S_m/\delta g_{\mu\nu}$ is a stress tensor of matter fields on the brane. These equations 
are equivalent to the Israel junction conditions

$$
-\frac{1}{16\pi G_5} [K^{\mu\nu} - g^{\mu\nu} K] + \frac{1}{2} (T^{\mu\nu} - g^{\mu\nu} \sigma) = 0 \quad (2.10)
$$
in the conventional treatment of the full system of bulk-brane equations of motion (here 
$[K^{\mu\nu} - g^{\mu\nu} K]$ denotes the jump of the extrinsic curvature terms across the brane). In our 
treatment we split the procedure of solving this system into two stages. First we solve them 
in the bulk subject to Dirichlet boundary conditions on the brane and substitute the result 
into bulk action to get off-shell brane effective action. Stationarity of the latter with respect 
to the 4-dimensional metric comprises the remaining set of equations to be solved at the
second stage. Such splitting might be regarded as a redundant complication, but it allows one to formulate off-shell properties of effective braneworld theory and, in particular, analyze it from the viewpoint of the AdS/CFT-correspondence [9, 10, 11, 12], etc.

As we see, braneworld effective action induced from the bulk has as its arguments only the brane metric and brane matter fields. One does not at all meet in such a setting the variables describing the embedding of the brane into a bulk. From the perspective of 5-dimensional equations of motion this can be easily understood, because the dynamical equations for embedding variables are automatically enforced in virtue of the bulk 5-dimensional Einstein equations, Israel junction conditions (equivalent, as we have just mentioned, to the stationarity of the action with respect to the induced metric) and matter equations of motion on the brane\(^2\). Another way to look at this property is to borrow an old idea from the scope of canonical gravity – the fact that 3-geometry carries information about time [26]. In braneworld context, this idea implies that brane 4-geometry carries the information about the location of the brane in the bulk. Indeed, the function (2.6) can actually be viewed as the Hamilton-Jacobi function of the boundary geometry. The only distinction from the canonical theory is that the role of boundary is played by the timelike brane, rather than spacelike hypersurface, and the role of time is played by the fifth (spacelike) coordinate measuring the location of the brane (the conventional use of Hamilton-Jacobi equation in cosmology in long-wavelength limit [27] was recently extended to braneworld context in the number of works including [28, 29]).

Still, the radion mode is known as an essential ingredient of the calculational procedure [6, 30, 10, 12] in braneworld theory, and it is worth understanding its role from the viewpoint of effective action. Possible way to recover the radion mode is to consider the structure of the effective action \(W[g]\) introduced above. Already in the tree-level approximation this is a very complicated nonlocal functional of the metric. At maximum, we know it in the low-energy approximation when the curvature of the brane is small compared to the curvature of the AdS bulk induced by \(\Lambda_5\). The coefficients of the corresponding expansion in powers of the curvature are nonlocal. Their nonlocality, however, can be of two different types. One type is expressible in terms of the massive Green’s function of the operator \(\Box - M^2\) and, therefore, in the low-derivative limit it is reducible to the infinite sequence of quasi-local terms, like

\[ \int d^4x \sqrt{g} R \frac{1}{\Box - M^2} R \sim \int d^4x \sqrt{g} \frac{1}{M^2} \sum_n R \left( \frac{\Box}{M^2} \right)^n R. \]

(2.11)

In the low-energy limit all these terms are suppressed by inverse powers of the mass parameter given by the bulk curvature scale, \(M^2 \sim \Lambda_5\), \(\Box/M^2 \ll 1\), and therefore comprise small short-distance corrections. Another type of nonlocality, when there is no mass gap parameter like

\[ \int d^4x \sqrt{g} R \frac{1}{\Box} R, \quad \int d^4x \sqrt{g} R_{\mu\nu} \frac{1}{\Box} R^{\mu\nu}, \]

(2.12)

\(^2\)In view of the 5-dimensional diffeomorphism invariance of the full action its variational derivative with respect to the embedding variables can be linearly expressed in terms of the variational derivatives with respect to other fields, which explains this property.
is much stronger in the infrared regime and cannot be neglected. Counting the number of degrees of freedom in an essentially nonlocal field theory is rather tricky. However, sometimes the nonlocal action can be localized in terms of extra fields, which makes the local treatment of the theory manageable. For example, structurally the first of nonlocal actions above can be reformulated as an expression

\[ S[g, \varphi] \sim \int d^4x \sqrt{g} (\varphi R + \varphi \Box \varphi) \]  

(2.13)

containing extra scalar field \( \varphi \). The variational equation for \( \varphi, \delta \tilde{S}/\delta \varphi = 0 \), yields \( \varphi \sim (1/\Box) R \) as a solution, which when substituted into \( \tilde{S}[g, \varphi] \) renders the original nonlocal structure of (2.12). Thus we suggest that the scalar radion mode in the braneworld effective action can be recovered by a similar mechanism – localization of essentially nonlocal structures in curvature in terms of the radion field. The fact that on shell this field non-locally expresses in terms of curvature matches with the known calculations of Ref. [12], where the radion mode was given in terms of the conformal part of the metric perturbation. A similar mechanism in 2-dimensional context is the localization of the trace anomaly generated Polyakov action [32] in terms of the metric conformal factor. In the next two sections we show how such a localization takes place in the two-brane Randall-Sundrum scenario.

3. Two brane Randall-Sundrum model

Here we consider the two-brane Randall-Sundrum model with \( Z_2 \) orbifold identification of points on the compactification circle of the fifth coordinate [5]. The action of this model

\[ S_5[G, \psi] = \int_{M^5} d^5x \sqrt{G} \left[ \mathcal{R}(G) - 2\Lambda_5 \right] + \int_{\Sigma^+} d^4x \sqrt{g^+} \left[ L_m^+ - \sigma^+_\bot \right] + \int_{\Sigma^-} d^4x \sqrt{g^-} \left[ L_m^- - \sigma^- \right] \]  

(3.1)

contains the contribution of two branes with two brane tensions \( \sigma_{\bot} \) and matter Lagrangians \( L_m^\pm = L_m(\psi^\pm, \partial \psi^\pm) \). The branes \( \Sigma^\pm \) are located at antipodal points of the circle labelled by the values of \( y, y = y^\pm, y^+ = 0, |y_-| = d \). \( Z_2 \)-symmetry identifies the points on the circle \( y \) and \(-y\). When the brane tensions are opposite in signs and fine tuned in magnitude to the

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3The second of structures (2.12) seems to require for localization an extra symmetric tensor field. Interestingly, though, that within the curvature expansion the nonlocal Lagrangian \( R_{\mu\nu}(1/\Box)R^{\mu\nu} \) generates the variational derivative differing from that of \( R(1/\Box)R \) by the term proportional to the Einstein tensor [31], so that the difference between these two nonlocal Lagrangians can be simulated by the Einstein-Hilbert one.

4Nonlocal expression for \( \xi \) in terms of curvature and nonlocalities in the action require boundary conditions which depend on particular problem one is solving on the 4-dimensional braneworld. For scattering problem they are of Feynman chronological nature, for the Cauchy problem the non-localities imply retardation. In the no-boundary prescription of the cosmological state they can be derived by analytic continuation from the Euclidean section of the braneworld geometry [11, 12]. In what follows we shall not specify them explicitly.
values of the negative cosmological constant $\Lambda_5$ and the 5-dimensional gravitational constant $G_5$ according to the relations

$$\Lambda_5 = -\frac{6}{l^2}, \quad \sigma_+ = -\sigma_- = \frac{3}{4\pi G_5 l},$$  \hspace{1cm} (3.2)

then in the absence of matter on branes this model admits the solution with the AdS metric in the bulk ($l$ is its curvature radius),

$$ds^2 = dy^2 + e^{-2|y|/l} \eta_{\mu\nu} dx^\mu dx^\nu, \quad 0 = y_+ \leq |y| \leq y_- = d,$$  \hspace{1cm} (3.3)

and with flat induced metric $\eta_{\mu\nu}$ on both branes [5]. The metric on the negative tension brane is rescaled by the value of warped compactification factor $\exp(-2d/l)$ providing a possible solution for the hierarchy problem [4]. With fine tuning (3.2) this solution exists for arbitrary brane separation $d$ – two flat branes stay in equilibrium. Their flatness turns out to be the result of a compensation between the bulk cosmological constant and brane tensions. This situation includes, in particular, the limit of $d \to \infty$ corresponding to only one brane embedded into $Z_2$-identified AdS bulk.

Linearized gravity theory with small matter sources for metric perturbations $h_{AB}(x, y)$ on the background of this solution

$$ds^2 = dy^2 + e^{-2|y|/l} \eta_{\mu\nu} dx^\mu dx^\nu + h_{AB} dx^A dx^B,$$  \hspace{1cm} (3.4)

was considered in the series of papers starting with [5, 6, 10, 30]. First, the conclusion on the recovery of the 4-dimensional Einstein’s gravity theory on the brane in the low-energy limit was reached in [5] (graviton zero mode localization) for the case of a single brane. It was revised in [6], where it was shown that in the presence of the second brane one gets the Brans-Dicke type theory, rather than the Einstein gravity. The trace of stress tensor of matter becomes an additional source of the gravitational field due to a nontrivial contribution of the radion mode – the scalar field responsible for the location of brane(s) in the bulk. This field arises as follows.

By using 5-dimensional diffeomorphism invariance one can transform the solution of Einstein equations to the “Randall-Sundrum gauge”

$$h_{5\mu} = h_{55} = 0, \quad \partial^\mu h_{\mu\nu} = h^\mu_{\mu} = 0.$$  \hspace{1cm} (3.5)

In this gauge, however, the branes are no longer located at $y = y_\pm$. Their embedding equations

$$\Sigma_\pm : y = y_\pm + \xi^\pm(x)$$  \hspace{1cm} (3.6)

involve two scalar functions $\xi^\pm(x)$ [30], which in the linearized theory are of the same order of magnitude as metric perturbations, $\xi^\pm(x) \sim h_{\mu\nu}$. They satisfy 4-dimensional equations

\footnote{Strictly speaking this is not a gauge but, rather, a corollary of special coordinate conditions and 5A-components of Einstein equations in vacuum bulk.}
motion [6], \( \Box \xi^\pm (x) = \pm 8\pi G_5 T_\pm (x)/6 \), in terms of traces of matter stress tensors, \( T_\pm \equiv \eta^\mu_\nu T^\pm_{\mu\nu} \), and determine the deviation \( \Delta \) of the induced metrics on branes \( \bar{h}^\pm_{\mu\nu} \) from the metric components \( h_{\mu\nu}(x, y_\pm) \) in the Randall-Sundrum gauge

\[
\bar{h}^\pm_{\mu\nu}(x) = h_{\mu\nu}(x, y_\pm) + l \xi^\pm_{\mu\nu}(x) + 2 a^2 \eta_{\mu\nu} \xi^\pm_{\mu\nu}(x) + 2 a^2 \xi^\pm_{\mu\nu}(x).
\] (3.7)

Here \( a_\pm = a(y_\pm) = \exp(-y_\pm/l) \) are the values of warp factor on respective branes, and the 4-dimensional vector fields \( \xi^\pm_{\mu}(x) \) reflect the ambiguity in two independent 4-dimensional diffeomorphisms respectively on \( \Sigma^+ \) and \( \Sigma^- \).

Garriga and Tanaka [6] wrote down the equations of motion for \( \bar{h}^\pm_{\mu\nu}(x) \) in the non-covariant form under a certain choice of these 4-vector fields. These equations can be easily covariantized in terms of Ricci curvatures of brane metrics \( R^\pm_{\mu\nu} \). For a coupled system of metric perturbations of two branes these equations look rather involved and will be presented in the coming paper [31]. Here we consider a simplified case of empty negative tension brane \( T^-_{\mu\nu} = 0, \xi^- = 0 \).

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G(\Box) T_{\mu\nu} + 16\pi G_5 \eta(\Box) (\nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box) \frac{1}{2} T + O(R^2),
\] (3.9)

\[
\Box \xi = \frac{8\pi G_5}{6} T.
\] (3.10)

Here all the quantities are related to the positive tension brane \( \Sigma^+ \) and, therefore, we everywhere omit the + label. \( O(R^2) \) indicates that this is a linear order in the curvature\(^6\) and other basic quantities \( \xi \) and \( T_{\mu\nu} \) for which we use collective notation \( R = (R_{\mu\nu\alpha\beta}, \xi, T_{\mu\nu}) \). Finally, \( G(\Box) \) and \( G_5(\Box) \) are the (generally nonlocal) coefficients of the operator nature – functions of the 4-dimensional D’Alambertian. They follow from the Green’s function of the linearized 5-dimensional Einstein equations for metric perturbations in the bulk satisfying Neumann type (linearized Israel junction conditions) boundary conditions on branes. This Green’s function, as known since the work of Randall and Sundrum [5], includes the contribution of zero graviton mode and the tower of massive Kaluza-Klein modes. In the low energy limit, \( \Box \to 0 \), Kaluza-Klein modes give only short distance (higher-derivative) corrections, so that these operator coefficients

\[
G(\Box) = G_4 + O(\Box),
\] (3.11)

\[
G_\xi(\Box) = e^{-2d/l} G_4 + O(\Box),
\] (3.12)

\(^6\)Covariantization of Garriga-Tanaka’s equations of [6] in this order of perturbation theory in curvature boils down to recovering the Einstein tensor in the left hand side and acquiring the \( \nabla_\mu \nabla_\nu (1/\Box) T \) term in the right hand side.
express in terms of the effective 4-dimensional gravitational constant

\[ G_4 = \frac{G_5}{l} \frac{1}{1 - e^{-2d/l}}, \quad G_4 \rightarrow \frac{G_5}{l} \equiv m_P^2, d \rightarrow \infty. \]  

(3.13)

As we see, \( G_4 \) depends on brane separation \( d \) \([6, 30] \) and tends to the Randall-Sundrum value in the limit of a single brane, \( d \rightarrow \infty \). In the same limit the Brans-Dicke term containing the trace of \( T_{\mu\nu} \) vanishes, which corresponds to the recovery of Einstein theory in the long distance approximation.

4. AdS/CFT correspondence and non-minimal curvature coupling of radion

As is known, the recovery of low energy Einstein theory for a single brane case can be interpreted as a manifestation of the AdS/CFT-correspondence in context of braneworld scenario \([9, 10, 11, 12] \)\(^7\). It is instructive to observe this property by considering the structure of \( O(\Box) \) -corrections in (3.11)-(3.12). The Green’s function of 5-dimensional equations of motion was obtained by Garriga-Tanaka in the lowest order approximation in \( \Box \), leading to (3.11)-(3.12). Calculations in the subleading order of \( \Box \)-expansion \([31] \) show that one must distinguish several different energy scales.

First consider the limit of very large separation between the branes and the physical distance on the positive tension brane (measured by the magnitude of the quantity \( 1/\sqrt{\Box} \)) much bigger than the curvature radius of the AdS bulk. This corresponds to the following two inequalities

\[ l^2 \Box \ll 1, \quad l^2 e^{2d/l} \Box \gg 1. \]  

(4.1)

The second inequality actually means that, although the distance on the \( \Sigma_+ \) brane is much greater than \( l \), the physical distance on \( \Sigma_- \), \( e^{-d/l}/\sqrt{\Box} \), lies in the opposite range. In this domain the calculation of subleading terms give the result \([31]\]

\[ G(\Box) = G_4 \left\{ 1 + \frac{l^2 \Box}{4} \ln \left( l^2 \Box \right) + O \left[ (l^2 \Box)^2 \right] \right\}. \]  

(4.2)

It contains the logarithmic correction typical of polarization effects in local field theory. The coefficients of this logarithmic term can apparently be identified with that of a specific CFT treated within the \( 1/N \)-expansion – this has been done in many papers on AdS/CFT-correspondence (see, for example, \([7, 8, 11, 12] \) regarding the asymptotic dependence of \( G_5 \) and \( G_4 \) on the number of flavors \( N \) in the dual CFT theory). Validity of AdS/CFT correspondence in the domain (4.1) is obvious – this range implies that the negative tension brane is removed far away which, as mentioned above, is equivalent to a non-compact single

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\(^7\)See also \([29]\) for the holographic interpretation of bulk Weyl and curvature squared corrections to Einstein equations.
brane case with a continuous spectrum of Kaluza-Klein modes. In this case usual arguments of AdS/CFT-correspondence apply and we have the logarithmic terms of the above type.

Consider now another range of distances

\[ l^2 \Box \ll 1, \ l^2 e^{2d/l} \Box \ll 1. \]  

(4.3)

This region can be achieved by moving the branes towards one another, and it corresponds to the case when the physical distance on the negative tension brane, \( e^{-d/l}/\sqrt{\Box} \) becomes bigger than the AdS curvature radius. In this range the form factor \( G(\Box) \) does not have any log-type corrections [31] and reads

\[ G(\Box) = G_4 \left\{ 1 - \frac{l^2 \Box}{2} \frac{d/l}{1 - e^{-2d/l}} + O\left[(l^2 \Box)^2\right] \right\}. \]

(4.4)

This can obviously be interpreted as a violation of AdS/CFT correspondence in the two-brane case. Duality between the supergravity theory in the AdS bulk and CFT on its boundary holds only in case of a single boundary approaching the infinity of 5-dimensional AdS spacetime. Thus the AdS/CFT correspondence (equivalent in this context to the recovery of Einstein theory on the brane) holds only for large brane separation. For small separation it gets violated, and the Einstein theory gets deformed into the Brans-Dicke type model.

In what follows we get back to the leading order behaviour of nonlocal formfactors (3.11)-(3.12) and substitute them into the system of Garriga-Tanaka’s equations (3.9)-(3.10). Our goal will be to derive the action that generates these equations of motion. To do this note that by the very definition of \( g_{\mu\nu} \) as induced metric on the brane it can be directly coupled to the matter stress tensor but not to its trace \( \Box \). This means that the trace of \( T_{\mu\nu} \) has to be eradicated from the right hand side of (3.9) in favour of \( \xi \). The radion field \( \xi \) for similar reasons cannot be coupled to \( T \) as well. Therefore \( T \) should also be excluded from the right hand side of (3.10) – in terms of the Ricci scalar. As a result we have the system of equations

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_4 T_{\mu\nu} + \frac{2/\sqrt{\Box}}{e^{2d/l} - 1}(\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box)\xi + O(R^2, \Box^2), \]

(4.5)

\[ \Box \xi + \frac{l}{6} R = 0. \]

(4.6)

It is easy to guess the action that directly generates these equations by variational procedure with respect to \( g_{\mu\nu} \) and \( \xi \). It looks like

\[ S_{\text{eff}}[g, \xi, \psi] = \int d^4x \sqrt{g} \left\{ l \frac{1 - e^{-2d/l}}{16\pi G_5} R + \frac{e^{-2d/l}}{8\pi G_5} \left( R\xi + 3\xi \Box \xi \right) + L_m(\psi, \partial \psi) \right\}, \]

(4.7)

\(^8\text{There is a lot of notational confusion about this fact in current literature. When the argument of effective action is not the induced metric itself but its conformally rescaled version, then at the linearized level the trace of } T_{\mu\nu} \text{ can be coupled to perturbations of this artificial metric.}\)
where the 4-dimensional gravitational constant $G_4$ is expressed in terms of the fundamental 5-dimensional one, $G_5$, in order to have explicit dependence on the brane separation $d$. As we see, the radion mode $\xi$ is non-minimally coupled to metric via Ricci curvature, and there is no direct coupling of $\xi$ to matter. Quite similarly, matter is coupled to the brane metric via the matter Lagrangian which generates $T_{\mu\nu}$ by usual variational procedure. Thus, the low-energy effective action turns out to be a local functional in terms of metric and the radion mode. However, the exclusion of $\xi$ in terms of the curvature scalar, $\xi = -(l/6)(1/\Box)R$, leads, along the lines outlined in the previous section, to essentially nonlocal metric part (2.6) of the whole effective action

$$W[g] = \frac{l}{16\pi G_5} \int d^4x \sqrt{g} \left\{ (1 - e^{-2d/l})R - \frac{1}{6} e^{-2d/l}R\Box R + O(R^3) \right\}. \quad (4.8)$$

This confirms the mechanism suggested in Sect.2.

5. Radion mode as an inflaton field

Let us get back to the local action (4.7). It was obtained in the approximation quadratic in curvature and radion mode. However, its structure clearly indicates that it can be regarded as an expansion in $\xi$ of the following action

$$S_{\text{eff}}[g, \varphi] = \int d^4x \sqrt{g} \left\{ \frac{m_p^2}{16\pi} R + \frac{1}{2} \left( \varphi \Box \varphi - \frac{1}{6} R \varphi^2 \right) \right\} \quad (5.1)$$

with the new scalar field $\varphi$ exponentially related to the original small radion mode

$$\varphi(x) = \sqrt{\frac{3}{4\pi}} m_p \exp \left[ -\frac{d + \xi(x)}{l} \right], \quad m_p^2 = \frac{l}{G_5}. \quad (5.2)$$

The fact that the initial separation of branes $d$ (moduli) combines with the spacetime dependent perturbation of this separation $\xi(x)$ into the variable $d + \xi(x)$ is geometrically very natural, because this variable describes a movement and local bending of the brane relative to the (invisible) negative tension brane. In this parametrization the dependence of effective action on brane separation is entirely absorbed into the new field $\varphi(x)$. The limit of infinitely big separation between branes corresponds to vanishing $\varphi$, $\varphi(x) \to 0$, $d + \xi(x) \to \infty$, and a complete recovery of Einstein theory. For intermediate range of brane separation the field $\varphi$ becomes dynamically important. It is non-minimally coupled to curvature like in (1.1), but the coupling constant, $\xi = 1/6$, is positive and renders the $\varphi$-dependent part of the action to be conformally invariant$^9$. The kinetic term of $\varphi$ has a good positive sign, so that there is no instability associated with the positive tension brane$^{10}$.  

$^9$This form of the action was also obtained in Ref.[33] by Kaluza-Klein reduction, without revealing, however, its conformal nature. I am grateful to H.Tye for drawing my attention to this work.

$^{10}$Instability of two brane system is associated with the negative tension brane, on which the corresponding brane bending mode has a ghost nature [31, 34].
As a whole the action (5.1) is not conformal invariant because of the first – Einstein-Hilbert – term, which gives hope that the radion field $\varphi$ can play the role of inflaton. However, this field does not, thus far, have any potential that could have induced the slow roll scenario. A possible mechanism to get a radion potential is to make a small detuning [35] of the brane tension $\sigma = \sigma_+$ from the Randall-Sundrum value (3.2). Let us denote excessive part of the brane tension on the positive tension brane by $\sigma_e$, $\sigma = 3/4\pi G_5 l + \sigma_e$. This additional positive tension can be viewed as a result of the replacement of the original matter Lagrangian by the one containing small 4-dimensional cosmological term

$$L_m(\psi, \nabla \psi) \rightarrow L_m(\psi, \nabla \psi) - \sigma_e.$$ (5.3)

The inclusion of this term into the metric-radion part of the action gives rise to the action

$$S_{\text{eff}}[g, \varphi] = \int d^4x \sqrt{g} \left\{ \left( \frac{m_p^2}{16\pi} - \frac{1}{12}\varphi^2 \right) R + \frac{1}{2}\varphi \Box \varphi - \sigma_e \right\},$$ (5.4)

which still does not have a good inflaton potential as a growing function of $\varphi$. Note, however, that $\varphi$ is non-minimally coupled to curvature, which makes inflationary implications of this model less trivial [23, 24, 36, 37]. To analyse them we go over to the Einstein frame of the conformally related metric $\bar{g}_{\mu\nu}$ and new scalar field, $(g_{\mu\nu}, \varphi) \rightarrow (\bar{g}_{\mu\nu}, \phi)$ [38, 39],

$$g_{\mu\nu} = \cosh^2 \left( \sqrt{4\pi/3} \frac{\phi}{m_P} \right) \bar{g}_{\mu\nu},$$ (5.5)

$$\phi = \sqrt{3/4\pi} m_P \tanh \left( \sqrt{4\pi/3} \frac{\phi}{m_P} \right).$$ (5.6)

In this parametrization the range of the original field $\varphi$, $|\varphi| \leq \sqrt{3/4\pi m_P}$, in which the effective $\varphi$-dependent gravitational constant of the action (5.4)

$$\frac{1}{16\pi G_4(\varphi)} = \frac{m_p^2}{16\pi} - \frac{1}{12}\varphi^2$$ (5.7)

is positive, is mapped onto the infinite range of $|\phi| < \infty$. This guarantees that we are considering a stable phase of the theory free of ghosts.$^{11}$

The action in the Einstein frame (barred quantities are defined relative to the barred metric $\bar{g}_{\mu\nu}$)

$$\bar{S}_{\text{eff}}[\bar{g}, \phi] = \int d^4x \sqrt{\bar{g}} \left\{ \frac{m_p^2}{16\pi} \bar{R} + \frac{1}{2}\phi \Box \phi - \bar{V}(\phi) \right\},$$ (5.8)

$$\bar{V}(\phi) = \sigma_e \cosh^4 \left( \sqrt{4\pi/3} \frac{\phi}{m_P} \right).$$ (5.9)

$^{11}$Graviton modes have a ghost nature outside of this range which, however, corresponds to $d + \xi(x) < 0$, and thus can be ruled out on physical ground, because $d + \xi(x) = 0$ implies the limit when the branes locally touch one another at the point $x$. 13
has a minimally coupled field $\phi$ and the potential *monotonically growing* from its minimum value to infinity. Minimum of the potential corresponds to an infinite separation of branes, while the infinity of $\bar{V}(\phi)$ describes the limit of coincident branes with $\varphi = \sqrt{3/4\pi m_P}$, $d + \xi = 0$. This means that the branes are *repelling* in our setting: the dynamical roll of the radion field down the potential well leads to the diverging branes. The visible brane with slightly detuned tension is curved and in the slow-roll regime has quasi-DeSitter geometry embedded into an AdS 5-dimensional bulk. For large initial values of $\phi$ (small brane separation) this potential might be too steep to maintain the slow roll regime, but it can realize the power law inflation scenario, while for small $\phi$ ($\phi \ll \sqrt{3/4\pi m_P}$, big separation) both conditions of the slow roll approximation are satisfied perfectly well. Thus, the radion mode can really be a candidate for the inflaton field generating inflation.

Note that in our model no stabilization mechanism is involved for fixing the value of the radion moduli. Two branes initially located at some separation start diverging from one another and induce inflation on the positive tension brane. The parameters of this inflation certainly depend on the initial conditions. To determine them one can invoke the ideas of quantum cosmology – the creation of the braneworld described by the gravitational instanton either within the no-boundary or tunneling proposals for the cosmological wavefunction. In the tree-level approximation the brane-world creation was considered in [40]. It was shown that for large brane separation of concentric DeSitter branes the probability amplitude is dominated by the contribution of the 4-dimensional instanton action of the positive tension brane. In our model this is given in terms of the radion potential as

$$I(\phi) = -\frac{3m_P^4}{8V(\phi)}. \quad (5.10)$$

Modified by the contribution of one-loop effective action (in the approximation of the overall scaling behaviour with $H(\phi) \sim \cosh(\sqrt{4\pi/3\phi/m_P})$), the distribution function of the initial value of the radion field (cf. Introduction)

$$P(\phi) \sim \exp\left(\pm I(\phi) - Z \ln \cosh(\sqrt{4\pi/3\phi/m_P})\right) \quad (5.11)$$

can have a probability maximum for the tunneling case (sign $+$ in the exponential) corresponding to the effective Hubble constant $H \sim (\sigma_e/Z)^{1/4}$, provided the parameter $\sigma_e$ satisfies the upper bound $\sigma_e \leq 3m_P^4/2Z$. For very big values of the anomalous scaling parameter $Z$ of quantum matter inhabiting the brane these estimates can bring us to reasonable parameters of the inflationary scenario\(^{12}\). In fact, the extremum of the radion probability distribution which supplies us with initial conditions replaces the equations for stabilizing braneworld moduli (because in both cases the effective action is being analyzed for its extremum). However, in contrast with the stabilization scheme, we fix only initial conditions, the further evolution of the radion is not frozen and mimicks inflation.

\(^{12}\)For $SU(N)$ conformal field theory $Z$ – conformal anomaly integrated over the instanton 4-volume – is proportional to $N^2$ [11, 12] and therefore can be big within $1/N$-expansion.
Important fact is that the potential does not vanish at its minimum, so that inflation never ends. This might be bad from the viewpoint of classical inflation scenario. However, in view of the recent discovery of acceleration of the Universe [3], this property can be interpreted as the fact that the radion induced inflation scenario also includes the mechanism for the present day acceleration of the Universe.

6. Conclusions

Thus we have shown that the radion mode in braneworld dynamics arises as a scalar field localizing essentially nonlocal part of braneworld metric effective action. For a positive tension brane, this mode has good dynamical properties of its kinetic term and, in the case of a positive detuning of the brane tension on this brane in the two brane Randall-Sundrum model, it can generate inflationary scenario via non-minimal coupling to the curvature. Inflationary evolution arises as a manifestation of the fact that two branes move apart from one another due to the repelling force mediated by this non-minimal coupling. Thus, this scenario is quite opposite to the picture of ekpyrotic Universe [16] and brane separation models of [14, 15] and [17]. On the other hand, the suggested model does not incorporate any stabilization mechanisms for brane moduli. Rather, the formalism quite similar to that of stabilization leads to the determination of the initial conditions for radion dynamics via extremizing the (loop corrected) probability distribution of the radion field.

Full analyses of the inflationary scenario in this model goes beyond the scope of this paper. Phenomenologically it might be too naive to accommodate the observational bounds on inflation and acceleration of our Universe. One might, for example, expect serious difficulties with the preheating mechanism of transition from the inflationary stage to the radiation dominated one. Moreover, it can hardly explain the magnitude of the present day acceleration, even though this model contains a natural mechanism of this acceleration due to remnant cosmological constant that survives the limit of large brane separation (corresponding to present time). However, as a whole it looks very promising because it naturally brings together the whole set of intriguing issues – fundamental origin of inflaton as a mechanism of violation of AdS/CFT-correspondence, issue of initial conditions for inflation replacing the idea of stabilization mechanisms and cosmological acceleration. It is quite possible that the synthesis of this model with subtle mechanisms for generating bulk cosmological constants, brane tensions [41] and mechanisms of their detuning [35] can result in phenomenologically viable model approaching the solution of cosmological hierarchy problem.

Acknowledgements

The author benefitted from helpful discussions with C.Armendariz-Picon, L.Kofman, V.Rubakov, I.Sachs, S.Solodukhin and H.Tye and is also grateful to Andreas Rathke for a

\[13\] The author is grateful to Lev Kofman for drawing attention to this fact.
hard task of checking many calculations in this paper. He is grateful for hospitality of the Physics Department of LMU, Munich, where a major part of this work has been done. This work was supported by the Russian Foundation for Basic Research under the grant No 99-02-16122, the grant of support of leading scientific schools No 00-15-96699 and the Russian Research program “Cosmomicrophysics”.

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