A brief overview is given of recent developments in the analyses of large phases and CP violation in supersymmetric unified models. The problem of experimental electric dipole moment constraints and large phases is discussed. Implications of large phases on supersymmetric phenomena are reviewed. The possibility of generating a muon electric dipole moment much larger than implied by the scaling relation \( d_\mu / d_e \simeq m_\mu / m_e \) from lepton flavor nonuniversality and within reach of the recently proposed Brookhaven experiment for a sensitive probe of \( d_\mu \) is also discussed.

1 Introduction

In this paper we will give an overview of the subject of large phases and CP violation in supersymmetric (SUSY) unified models. Specifically we will discuss the EDM problem in SUSY arising from the current experimental EDMs constraints\(^1\), their satisfaction with large phases and the effect of large phases on SUSY phenomena. We will also discuss the possibility of generating a muon edm (EDM) significantly larger than that dictated by a linear scaling in the lepton mass. The muon EDM is of considerable current interest in view of a recent proposal for a sensitive measurement of it at Brookhaven\(^2\). We begin our discussion regarding the situation in the electro-weak sector of standard model. Here there is only one CP phase which arises in the Kobayashi-Maskawa mass matrix and this phase contributes to the lepton EDMs only at the multiloop level and consequently the lepton EDMs in the Standard Model are extremely
small and beyond the reach of current experiment and also beyond the reach of any conceivable experiment in the near future. It is known that baryogenesis requires a new source of CP violation beyond what is in the standard model. Thus new CP violating phases must exist in nature beyond what is in the SM. Such new phases would also contribute to the lepton EDMs and consequently the lepton EDMs provide a very clean window for discovering new physics. The QCD sector sector of the standard model is more complex as it brings in another phase arising from a topological term in the effective QCD Lagrangian, i.e., \( \theta_G \frac{\pi}{G} \). The effective parameter which controls CP violation is 
\[ \bar{\theta} = \theta_G + \text{arg}(\det M_u M_d) + ... \] which gives a neutron EDM \( d_n \simeq 1.2 \times 10^{-16} \text{ ecm} \). The current limit \( d_n < 6.5 \times 10^{-26} \text{ ecm} \) implies \( \bar{\theta} < 6 \times 10^{-10} \). The desired smallness of \( \theta \) is the well known problem of QCD which has been discussed quite extensively in the literature. The same problem, of course, also persists in supersymmetric theories (For a recent discussion of this problem see Ref.\( ^4 \)). However, even beyond the \( \theta \) problem in QCD there is a CP problem unique to SUSY. We discuss this in Sec.2

2 CP PHASES IN SUSY

Models based on soft breaking of supersymmetry contain an abundance of CP violating phases. Thus, for example, mSUGRA with CP violation depends on the parameters \( m_0, m_0, A_0, \tan \beta, \theta_\mu, \alpha_{A_0} \) where \( m_0 \) is the universal scalar mass, \( m_0 \) is the universal gaugino mass, \( A_0 = |A_0| \text{exp}(i \alpha_{A_0}) \) is the universal trilinear coupling, \( \tan \beta \) is the ratio of the two Higgs VEVs in MSSM, and \( \theta_\mu = \text{Arg}(\mu) \) where \( \mu \) is the Higgs mixing parameter \( \mu \) (we use the sign convention of Ref.\( ^5 \)). Thus there are two phases, i.e., \( \alpha_{A_0} \) and \( \theta_\mu \) that enter in mSUGRA\( ^6 \). The non-universal supergravity models and MSSM involve many more phases and the edms of quarks and leptons will depend on these. Thus in MSSM the electron EDM depends on three independent phases \( \xi_i + \theta_1 (i = 1, 2) \) and \( \alpha_{A_i} + \theta_\mu \) where \( \xi_i \) are the phases of the gauginos masses \( \tilde{m}_i \), i.e., \( \tilde{m}_i = |\tilde{m}_i| \text{exp}(i \xi_i) \) \( (i=1,2) \) corresponding to the U(1) and SU(2) gauginos. The quark EDMs depends on 9 phases \( \xi_i + \theta_1 (i = 1, 2, 3) \); \( \alpha_k + \theta_\mu (k = u, d, c, s, t, b) \). The electron and the neutron edms together depend on ten independent phases\( ^7 \).

In a broad class of SUSY, string and brane models we expect the CP phases of O(1) as there is no a priori reason for it to be otherwise. Phases of this size lead to an EDM of the electron and of the neutron which are significantly larger than their experimental lower limits. Possible solutions to this problem consist of choosing small phases\( ^8 \), assuming a heavy SUSY spectrum with masses O(several) TeV\( ^9 \), embedding the models in a left-right symmetric framework which suppresses the dangerous phases\( ^10 \), and the more recently
proposed mechanism of internal cancellations\textsuperscript{6,7,11}. There is also the possibility that the phases arise only in the third generation and hence their contributions to the EDM of the first generation quarks and leptons are suppressed. The dominant contributions to the lepton EDMs arise from the one loop chargino ($\chi^\pm$) and one loop neutralino ($\chi^0$) exchanges. For the case of the neutron EDM one has contributions from one loop chargino, neutralino and gluino ($\tilde{g}$) exchanges, and in addition contributions from the two loop stop-top and sbottom-bottom exchanges. In certain parts of the parameter space two loop contributions from CP odd Higgs exchange may also be important\textsuperscript{12}. The operators that contribute are the electric dipole operator $-\frac{i}{2}d_f\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu}$, the chromoelectric electric dipole operator $-\frac{i}{2}\tilde{d}_C\bar{\psi}\gamma_5T^a\psi G_{\mu\nu}A^a$, and the purely gluonic dimension six operator $-\frac{1}{6}\tilde{d}_G\bar{\psi}\gamma_5G_{\mu\nu}G_{\beta\gamma}G_{\delta\lambda}\sigma\epsilon^{\mu\nu\lambda\sigma}$. In extracting the effects of the chromoelectric and the purely gluonic operators one uses the so called naive dimensional analysis of Georgi-Manohar\textsuperscript{13}, i.e., $d_C = \frac{\xi}{4\pi}\tilde{d}_C\eta_C$, $d_G = \frac{\eta}{4\pi}\tilde{d}_G\eta_G$, where $\eta_C \approx \eta_G \approx 3.4$, $M = 1.19$ GeV is the chiral symmetry breaking scale. The neutron EDM $d_n$ is estimated using SU(6) quark model $d_n = (\frac{4}{3}d_d - \frac{1}{3}d_u)$. Another constraint recently imposed in some analyses is the experimental constraint of the EDM of atoms. For example, the EDM of the mercury atom is extremely accurately known\textsuperscript{14}, i.e., $d_{Hg} < 9 \times 10^{-28}$ ecm. However, a theoretical analysis of an atomic EDM depends on the Schiff moment and involves nuclear physics effects which are poorly understood. A more accurate understanding of the Schiff moment in terms of the parameters of the microscopic CP violating SUGRA or MSSM Lagrangian is needed to have confidence in such an analysis. If the phases are large they will affect low energy phenomena. Thus inclusion of CP phases will affect sparticle masses, decay branching ratios and cross-sections\textsuperscript{15}, neutralino relic density and detection rates in dark matter detectors\textsuperscript{16}, g-2\textsuperscript{17}, higgs system\textsuperscript{18,19,20,21,22,23}, trileptonic signal\textsuperscript{24,25}, $b\bar{b}$ system\textsuperscript{26}, baryogenesis\textsuperscript{27}, proton decay\textsuperscript{28}, and hadron collider phenomenology\textsuperscript{29} and $e^+e^-$ collider phenomenology\textsuperscript{30}. The possibility that soft SUSY phases may be the origin of all CP violation has also been considered\textsuperscript{31}. In the following we discuss the CP effects on the neutral Higgs system, and on g-2. We will also discuss the possibility of generating a muon EDM which is significantly larger than what is predicted by fermion mass scaling.

3 CP effects in neutral Higgs system

Soon after the possibility of large CP phases became feasible\textsuperscript{6,7,11} it was pointed out that CP violation through loops would generate mixing between the CP even and the CP odd sectors\textsuperscript{18}. The CP even -CP odd mixing was exhibited
using the stop exchange\textsuperscript{18,19,20}. More recently it was pointed out that for large tan $\beta$ effects of chargino exchange would be significant and may become as large or even larger than the stop exchange\textsuperscript{21}. We illustrate here the main elements of this analysis. In the presence of large CP violating phases the spontaneous symmetry breaking including one loop effects generates an induced phase so that

$$H_1 = \left( \begin{array}{cc} H_{11} & \phi_1 + i\psi_1 \\ \phi_1 + i\psi_1 & H_{11} \end{array} \right)$$

$$H_2 = \left( \begin{array}{cc} H_{12} & \phi_2 + i\psi_2 \\ \phi_2 + i\psi_2 & H_{12} \end{array} \right)$$

In the basis $\{\phi_1, \phi_2, \psi_{1D}, \psi_{2D}\}$ where $\psi_{1D} = \sin \beta \psi_1 + \cos \beta \psi_2$, and $\psi_{2D} = -\cos \beta \psi_1 + \sin \beta \psi_2$, $\psi_{2D}$ decouples and the remaining $3 \times 3$ mass\textsuperscript{2} matrix $M_{Higgs}^2$ is given by

$$M_{Higgs}^2 = \begin{pmatrix} M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 + \Delta_{11} & -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & \Delta_{13} \\ -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & M_Z^2 s_\beta^2 + M_A^2 c_\beta^2 + \Delta_{22} & \Delta_{23} \\ \Delta_{13} & \Delta_{23} & (M_A^2 + \Delta_{33}) \end{pmatrix}$$

In Ref.\textsuperscript{18,19,20} stop corrections to $m_A^2$ and to $\Delta_{ij}$ (i,j=1,2,3) were computed and it was shown that all of the Q scale dependence can be absorbed in $m_A^2$ and that $\Delta_{ij}$ are scale independent. One then finds that the diagonalization of the mass\textsuperscript{2} matrix of Eq.(3) leads to mixing in the mass diagonal eigenstates between the CP even and the CP odd components. In Ref.\textsuperscript{21} this analysis was extended to include the W-chargino($\chi^+$)-charged Higgs ($H^+$) exchange. It was shown that a composite treatment of $W - \chi^+ - H^+$ exchange allows one to absorb all the Q dependence in $m_A^2$ and the $\Delta_{ij}$ once again have no explicit Q dependence\textsuperscript{21}. With inclusion of the chargino exchange contribution $m_A^2$ now reads\textsuperscript{21}

$$m_A^2 = (\sin \beta \cos \beta)^{-1} \left( - m_2^2 \cos \theta_H + \frac{g_1^2}{16\pi^2} \tilde{m}_2^2 |\mu| \cos \gamma_2 f_1(m_2^2, m_{\chi_1^+}^2, m_{\chi_2^+}^2) \right) + \ldots$$

where $f_1(x, y) = -2 + \log(xy/Q^4) + ((y + x)/(y - x)) \log(y/x)$ and contains the explicit Q dependence and ...represent the contributions from the stops, sbottoms etc. The $W - W - H^+$ exchange contribution to the lightest higgs boson mass is typically negative and lies in the range of 1-2 GeV and one needs to include this effect in the precision analyses. $W - W - H^+$ also contributes to the CP even-CP odd Higgs mixing. While as in previous analyses the lightest higgs typically remains a CP even state, there is a significant mixing between the heavy CP even neutral Higgs boson $H^0$ and the CP odd Higgs
boson $A^0$. The relative strength of the chargino exchange contribution vs the stop exchange contribution depends on $\tan \beta$ and for $\tan \beta \geq 30$ the chargino contributions can dominate the stop contribution. If large CP phases exist, then CP even -CP odd Higgs mixing could be seen at $e^+e^-$ colliders and would provide a clear signal for the existence of such phases. Further, it was shown in Ref. that if CP even-CP odd Higgs mixing is seen experimentally then it is only the cancellation mechanism that can explain such a mixing consistent with EDM constraints.

4 CP Effects on g-2

One of the phenomena affected by SUSY CP phases is the supersymmetric contribution to $g_{\mu} - 2$. It was shown in Ref. that the supersymmetric contribution to $a_\mu = (g_{\mu} - 2)/2$ is strongly dependent on the phases $\theta_\mu$ and $\xi_2$ and also dependent, though somewhat less strongly, on the phases $\xi_1$ and on $a_{A_0}$. One may ask what the implication of this strong dependence is for CP phases in light of the recent Brookhaven data which finds a discrepancy between experiment and the Standard Model prediction such that $a_{\mu}^{exp} - a_{\mu}^{SM} = 43(16) \times 10^{-10}$. This question was investigated in Ref. and it is found that the BNL data constrains the CP phases very strongly. Thus one finds that as much as 60-90% of the parameter space in the $\theta_\mu - \xi_2$ plane is eliminated by the BNL constraint. Further, it is possible to construct models with large CP phases which satisfy the EDM constraints as well as the Brookhaven constraint on $g_{\mu}$. Five models of this type are exhibited in Table 1 where the phases are large, EDM constraints on the electron and on the neutron EDM are satisfied, and $a_{\mu}^{(SUSY)}$ lies in the range given by the BNL experiment. One also finds that all of the sparticle spectrum corresponding to Table 1 is consistent with naturalness constraints (see, e.g., Ref.) and would be accessible at hadron colliders and some of the spectra may also be accessible at linear colliders.

<table>
<thead>
<tr>
<th>(case)</th>
<th>$\xi_2$, $\theta_\mu$, $\xi_3$</th>
<th>$d_e$, $d_n$ (eem)</th>
<th>$a_{\mu}^{(SUSY)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$-0.63, 3.3, 37$</td>
<td>$-4.2 \times 10^{-27}$, $-5.3 \times 10^{-26}$</td>
<td>$47.0 \times 10^{-10}$</td>
</tr>
<tr>
<td>(b)</td>
<td>$-0.85, 4, 37$</td>
<td>$4.2 \times 10^{-27}$, $4.8 \times 10^{-26}$</td>
<td>$10.8 \times 10^{-10}$</td>
</tr>
<tr>
<td>(c)</td>
<td>$-.8, 2, 1.3$</td>
<td>$4.0 \times 10^{-27}$, $5.4 \times 10^{-26}$</td>
<td>$12.2 \times 10^{-10}$</td>
</tr>
<tr>
<td>(d)</td>
<td>$-.32, 3, -.28$</td>
<td>$-1.2 \times 10^{-27}$, $3.3 \times 10^{-26}$</td>
<td>$20.1 \times 10^{-10}$</td>
</tr>
<tr>
<td>(e)</td>
<td>$-.5, 49, -.5$</td>
<td>$1.8 \times 10^{-27}$, $-6.6 \times 10^{-27}$</td>
<td>$12.7 \times 10^{-10}$</td>
</tr>
</tbody>
</table>
5 Large Muon EDM

There is a recent Brookhaven proposal\textsuperscript{2} to probe $d_\mu$ with a sensitivity of $d_\mu \sim O(10^{-24}) \text{ecm}$. In most theoretical models the charge lepton edms scale, e.g., $\frac{d_\mu}{d_e} \simeq \frac{m_\mu}{m_e}$. Since experimentally $d_e < 4.3 \times 10^{-27} \text{ecm}$ the scaling relation if valid implies that $d_\mu \leq 10^{-24} \text{ecm}$ which, however, falls below the sensitivity of the proposed BNL experiment. Thus a large muon edms can be gotten only by the breakdown of scaling. Some models where this comes about consist of the two higgs doublet model\textsuperscript{36}, left-right symmetric models\textsuperscript{37,38}, and models with flavor non-universalsities in the slepton sector\textsuperscript{39}(see also Ref.\textsuperscript{40}). We discuss here the last possibility, i.e., models with slepton flavor nonuniversality. To illustrate in some detail how the scaling relation gets violated in this case, we consider the charge lepton edm arising from the exchange of charginos and neutralinos which is given by\textsuperscript{7}

$$d_l = \frac{\epsilon \alpha_{EM}}{4\pi \sin^2 \theta_W} \frac{\kappa_l}{m_{\tilde{\nu}_l}^2} \sum_{i=1}^{2} \tilde{m}_{\chi_i} I_m(U_{i2}^* V_{11}) A(\frac{\tilde{m}_{\chi_i}^2}{m_{\tilde{\nu}_l}^2})$$

$$+ \frac{\epsilon \alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{4} I_m(\eta_{i,k}^l) \frac{\tilde{m}_{\chi_i}^4}{M_{\tilde{l}_k}^2} B(\frac{\tilde{m}_{\chi_i}^2}{M_{\tilde{l}_k}^2})$$

where $\kappa_l = m_l/(\sqrt{2}m_W \cos \beta)$ and $I_m(\eta_{i,k}^l)$ is given by

$$I_m(\eta_{i,k}^l) = m_l(C_{j,k} + A_l d_{j,k} + ..)$$

We see now that if $A_l$ is universal, i.e., $A_e = A_\mu$, one has scaling for the EDMs, $\frac{d_\mu}{d_e} \simeq \frac{m_\mu}{m_e}$. However, in the presence of non-universality $A_\mu \neq A_e$ the scaling relation breaks down. In this case slepton flavor nonuniversality can upset the cancellation mechanism in the muon EDM even when such a cancellation occurs for the EDM of the electron. In this situation the cancellation mechanism produces an EDM of the electron consistent with the current experimental limit while the lack of cancellation in the muon channel produces an muon EDM much larger than what scaling predicts and in the range accessible to the proposed Brookhaven experiment.

Acknowledgments

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