It may be possible to construct a laser interferometer gravitational wave antenna in space with $h_{\text{ren}} \sim 10^{-27}$ at $f \sim 0.1$ Hz in this century. We show possible specification of this antenna which we call DECIGO. Using this antenna we show that 1) $\sim 10^5$ chirp signals of coalescing binary neutron stars per year may be detected with $S/N \sim 10^4$. 2) The time variation of the scale factor of our universe may be determined for ten years observation of binary neutron stars so that we can directly measure the acceleration of the universe. 3) the stochastic gravitational waves of order $\Omega_{\text{GW}} \gtrsim 10^{-20}$ predicted by the inflationary universe paradigm may be detected by correlation analysis for which effects of the recent cosmic acceleration would become highly important.

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I. INTRODUCTION

There are at least four methods to detect gravitational waves. They are ;1) Resonant type antenna covering $\sim$kHz band; 2-a) Laser interferometers on the ground covering 10Hz$\sim$ kHz band; 2-b) Laser interferometers in space like LISA [1] covering $10^{-4} \sim 10^{-2}$Hz band; 3) Residuals of pulsar timing covering $\sim 10^{-8}$Hz band; 4) Doppler tracking of the spacecraft covering $10^{-4} \sim 10^{-2}$Hz band. It is quite interesting to note that little has been argued on possible detectors in $10^{-2} \sim 10$Hz band. In this Letter we argue in §2 possible specification of such a detector which we call DECIGO(DECi hertz Interferometer Gravitational wave Observatory). In §3 we argue that the direct measurement of the acceleration of the universe is possible using DECIGO. §4 will be devoted to discussions.

II. SPECIFICATION OF DECIGO

The sensitivity of a space antenna with an arm length of $1/10$ of LISA and yet the same assumption of the technology level, such as a laser power of 1 W, the optics of 30 cm, etc. will be $4 \times 10^{-21}$Hz$^{-1/2}$ around 0.1 Hz in terms of strain, a factor of 10 better than the planned LISA sensitivity around 0.1 Hz. The sensitivity could be improved by a factor of 1000 for the next generation of a space antenna with more sophisticated technologies such as implementation of higher-power lasers and larger optics in order to increase the effective laser power available on the detectors, and thus to reduce the shot noise. The ultimate sensitivity of a space antenna in the far future could be, however, $3 \times 10^{-27}$ around 0.1 Hz in terms of strain, assuming the quantum limit sensitivity for a 100 kg mass and an arm length of $1/10$ of LISA (See also brief discussion [2] for possibility of sensitivity $\sim 10^{-30}$Hz$^{-1/2}$ at frequency range $10^{-4} \sim 1$Hz). We name this detector DECIGO. This requires an enormous amount of effective laser power, and also requires that the other noise sources, such as gravity gradient noise, thermal noise, practical noise, etc. should be all suppressed below the quantum noise. Here we assume that such an antenna may be available by the end of this century. Note here that when the pioneering efforts to detect the gravitational waves started in the last century using resonant type detectors as well as laser interferometers, few people expected the present achievement in resonant type detectors such as IGEC(bar) [3] and in laser interferometers such as TAMA300 [4], LIGO [5], GEO600 [6], and VIRGO [7]. Therefore all the experimentalists and the theorists on gravitational waves should not be restricted to the present levels of the detectors. Our point of view in this Letter is believing the proverb “Necessity is the mother of the invention” so that we argue why a detector like DECIGO is necessary to measure some important parameters in cosmology.

The sensitivity of DECIGO, which is optimized at 0.1 Hz, is assumed to be limited only by radiation pressure noise below 0.1 Hz and shot noise above 0.1 Hz. The contributions of the two noise sources are equal to each other at 0.1 Hz, giving the quantum limit sensitivity at this frequency. The radiation pressure noise has a frequency dependence of $\propto f^{-2}$ (in units of Hz$^{-1/2}$) because of the inertia of the mass, while the shot noise has a dependence of approximately $\propto f^{-1}$ (in units of Hz$^{-1/2}$) because of the signal canceling effect due to the long arm length.
In figure 1 we show sensitivity of various detectors and characteristic amplitude $h_c$ for a chirping NS-NS binary at $z = 1$. The required sensitivity ($S/N=1$) for detecting stochastic gravitational wave background by 10 years correlation analysis is also shown.

### III. DIRECT MEASUREMENT OF THE ACCELERATION OF THE UNIVERSE

Recent distance measurements for high-redshift supernovae suggest that the expansion of our universe is accelerating [8] which means that the equation of the state of the universe is dominated by “dark energy” with $\rho + 3p < 0$. SuperNova / Acceleration Probe (SNAP, http://lbl.gov) project will observe $\sim 2000$ Type Ia supernovae per year up to the redshift $z \sim 1.7$ so that we may get the accurate luminosity distance $d_L(z)$ in near future. Gravitational wave would be also a powerful tool to determine $d_L(z)$ [9].

From accurate $d_L(z)$ one may think that it is possible to determine the energy density $\rho(z)$ and the pressure $p(z)$ as functions of the redshift [10]. However as shown by Weinberg [11] and Nakamura & Chiba [12], $\rho(z)$ and $p(z)$ can not be determined uniquely from $d_L(z)$ but they depend on one free parameter $\Omega_k$ (the spatial curvature).

Recent measurement of the first peak of the anisotropy of CMB is consistent with a flat universe ($\Omega_k = 0$) for primordially scale-invariant spectrum predicted by slow-roll inflation [13] under the assumption of $\Lambda$ cosmology. However it is important to determine the curvature of the universe irrespective of the theoretical assumption on the equation of the state and the primordial spectra also. In other words an independent determination of $\Omega_k$ is indispensable since $\Omega_k$ is by far the important parameter. As discussed in [12], the direct measurement of the cosmic acceleration [14] can be used for this purpose. Here we point out that the gravitational waves from the coalescing binary neutron stars at $z \sim 1$ observed by DECIGO may be used to determine $\Omega_k$.

#### A. Cosmic Acceleration

We consider the propagation of gravitational wave in our isotropic and homogeneous universe. The metric is given by $ds^2 = -dt^2 + a(t)^2(dx^2 + r(x)^2(1 + \sin^2 \theta)dz^2))$, where $a(t)$ is the scale factor and $a(t)r(x)$ represents the angular distance. The relation between the observed time of the gravitational waves $t_o$ at $x = 0$ and the emitted time $t_e$ at the fixed comoving coordinate $x$ is given by $\int_{t_e}^{t_o} \frac{dt}{a(t)} = \Delta \equiv const$. Then we have $\frac{dt_o}{dt_e} = a_o/a_e = (1 + z)$ and

$$\frac{d^2t_o}{dt_e^2} = (1 + z)a_e^{-1}(\partial t_o - \partial t_e) \equiv g_{cos}(z) = (1 + z)(1 + z)H_0 - H(z)), \quad (1)$$

where $H(z)$ is the Hubble parameter at the redshift $z$ and $H_0$ is the present Hubble parameter. For an emitter at the cosmological distance $z \gtrsim 1$ we have $g_{cos}(z) \sim O(t_0^{-1})$ where $t_0$ is the age of universe $t_0 \sim 3 \times 10^{17}$ sec. From above equations we have $\Delta t_o = \Delta t_e (1 + z) + \frac{g_{cos(z)}}{2} \Delta t_e^2 + \cdots$, where $\Delta t_o$ and $\Delta t_e$ are the arrival time at the observer and the time at the emitter, respectively. When we observe the gravitational waves from the cosmological distance, we have $\Delta t_o = \Delta T + X(z)\Delta T^2 + \cdots$, with $X(z) \equiv g_{cos(z)}(1 + z)^2 = \frac{(H_0 - H(z)/(1 + z))}{2}$, where $\Delta T = (1 + z)\Delta t_e$ is the arrival time neglecting the cosmic acceleration/ deceleration (the second term). Now for $\Delta T \sim 10^9$ sec, the time lag of the arrival time due to the cosmic acceleration/ deceleration amounts to the order of second $\sim 10^{13}/(3 \times 10^{17}) \sim 1$ [sec]. From Eq. (1), if $X(z)$ is positive, then $\partial t_o - \partial t_e > \partial t_o - \partial t_e$. This clearly means that our universe is accelerating. Therefore the value of this time lag is the direct evidence for the acceleration/ deceleration of the universe.

As shown in [12], if the accurate value of $X(z)$ at a single point $z_e$ is available it is possible to determine $\Omega_k$ as

$$\Omega_k = \frac{1 - (dr(z_e)/dz_e)^2(1 + z_e)^2(H_0 - 2X(z_e))}{r(z_e)^2H_0^2}, \quad (2)$$

where we have assumed that the quantity $r(z) \equiv d_L(z)/(1 + z)$ is obtained accurately, for example, by SNAP. Even if the accurate values of $X(z)$ are not available for any points, we may apply the maximal likelihood method to determine $\Omega_k$. Using the value of $\Omega_k$ thus determined, we can obtain the equation of state of our universe without any theoretical assumption on the matter content of our universe [12]. Note here the expected value of $X(z)$ for the flat $\Lambda$ cosmology is obtained as $X(z)/H_0 = 0.5(1 - \sqrt{\Omega_m(1 + z)^3 + 1 - \Omega_m/(1 + z))}$. At $z = 1$ we have $X(z)/H_0 = 0.06$ for $\Omega_m = 0.3$ and $X(z)/H_0 = -0.21$ for $\Omega_m = 1.0$ (Einstein de-Sitter universe).
Let us study an inspiraling compact binary system that evolves secularly by radiating gravitational wave [15]. For simplicity we study a circular orbit and evaluate the gravitational wave amplitude and the energy loss rate by Newtonian quadrupole formula [16]. We basically follow analysis of Cutler & Flanagan [15] but properly take into account of effects of accelerating motion. The Fourier transform \( \tilde{h}(f) = \int_{-\infty}^{\infty} e^{2\pi i ft} h(t) dt \) for the wave \( h(t) \) is evaluated using the stationary phase approximation as \( \tilde{h}(f) = K d_L(z)^{-1} M_c^{5/6} f^{-7/6} \exp[i \Phi(f)] \), where \( K \) is determined by the angular position and the orientation of the binary relative to the detector, and \( M_c \) is the chirp mass of the system. Keeping the first order term of the coefficient \( X(z) \), the phase \( \Phi(f) \) of the gravitational wave becomes

\[
\Phi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{4}(8\pi M_{cz} f)^{-5/3} - \frac{25}{32768} X(z) f^{-13/3} M_{cz}^{-10/3} \pi^{-13/3},
\]

where \( t_c \) and \( \phi_c \) are integral constants and \( M_{cz} = M_c(1 + z) \) is the redshifted chirp mass.

If we include the post-Newtonian (PN) effects up to \( 1.5 \)-order, the term \( 3/4(8\pi M_{cz} f)^{-5/3} \) in eq.(3) should be modified as\( \frac{2}{5}(8\pi M_{cz} f)^{-5/3} \left[ 1 + \frac{25}{2} \left( \frac{z}{1+z} + \frac{11}{16} \right) x + \left( 4/\beta - 16 \pi \right) x^3/2 + \cdots \right] \), where \( x = (\pi M_t f(1 + z))^{2/3} = o(a^2/c^2) \) is the PN expansion parameter with \( M_t \) being the total mass of the binaries. The term proportional to \( \beta \) in \( 1.5 \)-order (\( \propto x^{3/2} \)) is caused by the spin effect [15,17]. In general PN contribution depends on the frequency \( f \) as \( o(f^{-5-2N}/3) \) and is largely different from the dependence \( f^{-13/3} \) caused by the cosmic acceleration. This difference means that the correlation between PN and the effect of the cosmic acceleration in Fisher matrix is weak, which is very preferable for the actual signal analysis. In the next subsection we evaluate the signal to noise ratio \( (S/N) \) and parameter estimation errors using standard formulas for the Matched filtering analysis [15].

C. the estimation error

For the circular orbit of the binary neutron stars of mass \( M_1 \) and \( M_2 \) with the separation \( a \) at the redshift \( z \), the frequency of the gravitational waves \( f \) is given by \( f = 0.1Hz(1+z)^{-1}(M_l/2.8M_\odot)^{1/2}(a/15500km)^{-3/2} \). The coalescing time \( t_c \), the number of cycles \( N_{cycle} \), and the characteristic amplitude of the gravitational waves \( h_c \) are given by

\[
t_c = 7(1+z)(M_1/1.4M_\odot)^{-1}(M_2/1.4M_\odot)^{-1}(M_l/2.8M_\odot)^{-1}(a/15500km)^4yr
\]

\[
N_{cycle} = 1.66 \times 10^7 (M_1/1.4M_\odot)^{-1}(M_2/1.4M_\odot)^{-1}(M_l/2.8M_\odot)^{-1/2}(a/15500km)^{5/2}
\]

\[
h_c = 1.45 \times 10^{-23}(1+z)^{5/6}(M_c/1.2M_\odot)^{5/6}(f/0.1Hz)^{-1/6}(d_L/10Gpc)^{-1}
\]

Let us evaluate how accurately we can fit the parameter \( X(z) \). We take six parameters \( \lambda_i = \{ A, M_{cz}, \mu_z, t_c, \phi_c, M_{cz}^{-10/3} X(z) \} \) in the matched filtering analysis up to 1PN-order for the phase \( \Phi(f) \) and Newtonian order for the amplitude [15]. Here \( A \) is the amplitude of signal \( K d_L(z)^{-1} M_c^{5/6} \) in the previous subsection and \( \mu_z \) is the redshifted reduced mass \( \mu_z = (1+z)M_1M_2/M_c \). As the chirp mass \( M_{cz} \) can be determined quite accurately, we simply put \( \Delta X(z) = \Delta \left\{ M_{cz}^{-10/3} X(z) \right\} / M_{cz}^{-10/3} \). For simplicity we fix the redshift of sources at \( z = 1 \) and calculate \( S/N \) and the error \( \Delta X \) for equal mass binaries with various integration time \( \Delta t \) before coalescence. We use the effective factor \( 1/\sqrt{5} \) for reduction of antenna sensitivity due to its rotation [16]. For the present analysis we neglect the binary confusion noise since double White Dwarf binaries do not exist at frequency \( f \gtrsim 0.1Hz \) [18].

As shown in figure 2 we can detect NS-NS binaries at \( z = 1 \) with \( S/N \approx 20000 \) and \( \Delta X/t_0^{-1} \approx 7.0 \times 10^{-3} \) for integration time \( \Delta t = 16yr \) \( (N_{cycle} \sim 10^7 \) orbital cycles) and \( \Delta X/t_0^{-1} \approx 1.26 \) for \( \Delta t = 1yr \). With this detector it would be possible to determine \( X(z) \) and obtain the information of the cosmic acceleration quite accurately. With \( \Delta T = 16yr \) we have the estimation error for the redshifted masses as \( \Delta M_{cz}/M_{cz} = 1.5 \times 10^{-13}, \Delta \mu_z/\mu_z = 4.2 \times 10^{-8} \) and for the wave amplitude \( \Delta A/A = (S/N)^{-1} = 5 \times 10^{-5} \). Although the more detailed study is needed to estimate the error of the binary inclination angle, it is expected that the luminosity distance \( d_L \) can be determined accurately so that the redshift \( z \) can be determined using the inverse function \( z = d_L^{-1}(distance) \) of the accurate luminosity distance from e.g. SNAP. As a result we can know two (not redshifted) masses \( M_1 \) and \( M_2 \) for \( \sim 10^5 \) binaries per year up to \( z = 1 \) [19]. This number will be large enough to establish the mass function of “Neutron Stars” which would bring us important implications for the equation of the state of the high density matter and the explosion mechanisms of TypeII supernovae.

As the \( S/N \) and the estimation error scale as \( S/N \propto h_{rms}^{-1} \) and \( \Delta X \propto h_{rms} \), we can attain \( \Delta X/t_0^{-1} \approx 7.0 \) for the integration time \( T = 16yr \) using a less sensitive detector with \( h_{rms} \sim 10^{-24} \) (1000 times worse). Even though the
D. Acceleration in the Very Early Universe

In the inflationary phase there was an extremely rapid acceleration of the universe. In this phase the gravitational waves were generated by quantum fluctuation (see [20] for a recent review). With CMB quadrupole anisotropies measured by COBE [21], the slow-roll inflation model predicts a constraint on the stochastic background $\Omega_{GW} \lesssim 10^{-15} - 10^{-16}$ at $f \sim 0.1\text{Hz}$ [22]. Ungarelli and Vecchio [23] discussed that the strain sensitivity $h_{\text{rms}} \sim 10^{-24}$ is the required level at $f \sim 0.1\text{Hz}$ for detecting the stochastic background of the order of $\Omega_{GW} \sim 10^{-16}$ by correlating two detectors for decades (see also figure 1). It is important to note that the band $f > 0.1\text{ Hz}$ is free from stochastic backgrounds generated by White Dwarf binaries. However the radiation from neutron stars binaries is present in this band and it is indispensable to remove their contributions accurately from data stream, where effects of the cosmic acceleration would be highly important. Thus measurement of the present-day cosmic acceleration is closely related to detection of the primordial gravitational wave background that is one of the most interesting targets in cosmology. If DECIGO with $h_{\text{rms}} \sim 2 \times 10^{-27}$ at $f \sim 0.1\text{Hz}$ is available we can detect the primordial gravitational waves background even if the energy density is extremely low $\Omega_{GW} \sim 10^{-20}$ by correlating two detectors for a decade. We may say that we can either confirm or deny the existence of the primordial gravitational waves background by DECIGO.

IV. DISCUSSIONS

The determination of the angular position of the source is crucial for matching the phase as the position and the orientation of the detector change in the observational period. The phase modulation at the orbital radius 1AU corresponds to $2\text{AU}/c \sim 1000\text{[sec]}$. Thus, in order to match the phase within the accuracy of 0.1[sec] we need to determine the angular position with precision $\sim 0.1/1000 \text{[rad]} \sim 20''$. In the matched filtering analysis we can simultaneously fit parameters of the angular position as well as the relative acceleration between the source and the barycenter of the solar system. The former is related to the annual signal modulation and is different from the secular effects by the latter. However due to their correlation in the Fisher matrix, the measured acceleration would be somewhat degraded if we cannot determine the angular position by other observational methods. First, using the gravitational wave alone, we can, in advance, specify the coalescence time and the angular position of the source within some error box. If coalescence of NS-NS binaries would release the optical signal (e.g. Gamma Ray Bursts as proposed by [24]) we may measure the angular position accurately by pointing telescopes toward the error box at the expected coalescence time from the chirp signal. Therefore we have not tried to fit the angular position of the source in the matched filtering method [25]. We have also assumed that we can determine the redshift of the source by using optical information of host galaxies or the measured luminosity distance $d_L$ that can be obtained by gravitational wave alone [9] with the inverse function $z = d_L^{-1}(\text{distance})$ of the accurate luminosity distance.

Let us discuss the effects of the local motion $g_{\text{local}}$ of the emitter at the cosmological distance on the second derivative $d^2t_o/dt^2$ (see e.g. Ref. [26]). We can easily confirm that the effects of the large-scale structure of the time scale $\sim 10^3t_0(600\text{km s}^{-1}/V_{\text{flow}})^2$ with $V_{\text{flow}}$ of the typical bulk velocity of galaxies is much smaller than that of the cosmic acceleration. Next we estimate the internal acceleration within the galaxy based on the observational result of NS-NS binary PSR 1913+16. As shown in Table 1 of [26], the dominant contribution of its acceleration $\ddot{x}$ comes from the global Galactic potential field and has time scale $c/\ddot{x} \sim 10t_0(R_e/10\text{kpc})(V_{\text{rot}}/200\text{km s}^{-1})$ that can be comparable to the cosmic signal $g_{\text{cos}}$ where $R_e$ is the effective radius of the acceleration and $V_{\text{rot}}$ is galactic rotation velocity. However the contamination of local effect $g_{\text{local}}$ can be reduced by taking the statistical average of many binaries as $\langle g_{\text{cos}} + g_{\text{local}} \rangle = \langle g_{\text{cos}} \rangle$. If we can specify the position of binaries in host galaxies by the follow-up optical observation, the prediction of $g_{\text{local}}$ might be also possible using structure models of the observed host galaxies.

In conclusion we would like to encourage the further design study of DECIGO and the theoretical study of the sources of gravitational waves for DECIGO. Even if we may not see the construction of DECIGO in our life since the highly advanced technology is needed, we are sure that our children or grandchildren will decide and go DECIGO.
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FIG. 1. Sensitivity (effectively S/N=1) for various detectors (LISA, DECIGO, LIGOII and a detector $10^3$ times less sensitive than DECIGO) in the form of $h_{\text{rms}}$ (solid lines). The dashed line represents evolution of the characteristic amplitude $h_c$ for NS-NS binary at $z=1$ (filled triangles; wave frequencies at 1yr and 10 yr before coalescence). The dotted lines represent the required sensitivity for detecting stochastic background with $\Omega_{GW}=10^{-16}$ and $\Omega_{GW}=10^{-20}$ by ten years correlation analysis (S/N=1).

FIG. 2. S/N and the estimation error $\Delta X$ for NS-NS ($1.4M_\odot$) or BH-BH ($10M_\odot$) binaries at $z=1$. We evaluate them for DECIGO (sensitivity: $h_{\text{rms}} \sim 10^{-27}$ at $f \sim 0.1\text{Hz}$).