MUON $g - 2$ AND ELECTRIC DIPOLE MOMENTS IN SUGRA MODELS

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The SUSY contribution to the muon magnetic moment anomaly, $a_{\mu}^{\text{SUGRA}}$, and the electron electric dipole moment, $d_e$, is discussed within the framework of a modified mSUGRA model where the magnitudes of the soft breaking masses are universal, but arbitrary phases are allowed. It is shown analytically how the cancellation mechanism can allow for large phases (i.e. $\theta_B \lesssim 0.4$) and still suppress the value of $d_e$ below its current experimental bound. The dependence of $a_{\mu}^{\text{SUGRA}}$ on the CP violating phases are analytically examined, and seen to decrease it but by at most a factor of about two. This reduction would then decrease the upper bound on $m_{1/2}$ due to the lower bound of Brookhaven data, and hence lower the SUSY mass spectrum, making it more accessible to accelerators. At the electroweak scale, the phases have to be specified to within a few percent to satisfy the experimental bound on $d_e$, but at the GUT scale, fine tuning below 1% is required for lower values of $m_{1/2}$. This fine tuning problem will become more serious if the bound on $d_e$ is decreased.

1 Introduction

In supersymmetry, the interaction of charginos ($\tilde{\chi}_i^\pm$, $i = 1,2$), neutralinos ($\tilde{\chi}_j^0$, $j = 1,2,3,4$) and sleptons ($\tilde{\ell}_k$, $\tilde{\nu}_k$, $k = 1,2$; $\tilde{\nu}_e$, $\tilde{\nu}_\mu$) with the leptons ($l_m = e, \mu, \tau$) gives rise to electromagnetic vertices $l_m - l_n - \gamma$. Thus the basic diagrams for the diagonal muon interaction shown in Fig. 1 gives rise to anomalous contributions to the muon magnetic moment $a_{\mu} = (g - 2)/2$, and could possibly produce an electric dipole moment, $d_{\mu}$. Similar diagrams with $\mu \to e$ and $\tilde{\nu}_\mu, \tilde{\mu} \to \tilde{\nu}_e, \tilde{\ell}$ can give rise to $a_e$ and $d_e$, while the off diagonal diagram could allow for the decay $\mu \to e + \gamma$. The different possibilities, however, involve different physics. Corrections to the anomalous magnetic moments of the leptons are always present in supersymmetry, and the recent results of the Brookhaven E821 experiment \cite{1} showing a 2.6 $\sigma$ deviation from the Standard Model prediction,

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 43(16) \times 10^{-10}$$

(1)

shows the possibility of a SUSY contribution. Indeed, in the initial calculations \cite{2,3} based on supergravity grand unification models (mSUGRA \cite{4}), it was predicted \cite{3} that a deviation should show up when the experimental sensitivity

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reached that of the Brookhaven experiment. However, for electric dipole moments to be non-zero requires in addition the presence of CP violating phases. The current experimental bounds on $d_e$ is $^5$

$$d_e < 4.3 \times 10^{-27}$$  \hspace{1cm} (2)

and this bound is expected to be reduced by a factor of 2-3 in the near future $^6$.

![Diagrams](image)

Figure 1. Diagrams contributing to $a_{\mu}$ and $d_{\mu}$ involving intermediate chargino-sneutrino states and intermediate neutralino-smuon states.

In this paper, we examine the magnetic moment and electric dipole moment phenomena within the framework of gravity mediated SUGRA models, and assume that the Brookhaven anomaly is real $^7$ and due to SUSY. We will assume a 2 $\sigma$ range for $a_{\mu}^{\text{SUGRA}}$

$$11 \times 10^{-10} < a_{\mu}^{\text{SUGRA}} < 75 \times 10^{-10}$$  \hspace{1cm} (3)

If we assume universal soft breaking in the first two generation, then $m_{\tilde{\mu}_k} = m_{\tilde{\nu}_k}$, and $m_{\tilde{\nu}_e} = m_{\tilde{\nu}_e}$. Then both $a_{\mu}$ and $d_e$ can be obtained from the same complex amplitude $A$:

$$a_{\mu}^{\text{SUGRA}} = -\frac{\alpha}{4\pi \sin^2 \theta_W} m_{\mu}^2 \text{Re}[A]$$  \hspace{1cm} (4)

$$d_e = -\frac{\alpha}{8\pi \sin^2 \theta_W} m_e \text{Im}[A]$$  \hspace{1cm} (5)

There has been a great deal of recent analysis of $a_{\mu}$ within the framework of mSUGRA models $^8,^9$ showing that indeed for current theory constrained by all accelerator and non-accelerator bounds, $a_{\mu}^{\text{SUGRA}}$ can be expected to lie within the range of Eq. (3). It is thus at first sight puzzling that $d_e$ is so small, since it arises from the same amplitude $A$ that gives rise to a large $a_{\mu}^{\text{SUGRA}}$. Two possible explanations for this are the following: (1) The CP violating phases appearing in $A$ are anomalously small, and in fact it is possible to build reasonably natural models where this can happen $^{10}$. (2) The CP violating phases are indeed $O(1)$ (as the CKM phase is), but there are cancelations.
between the two diagrams of Fig. 1 suppressing the value of $d_e$. This possibility appears more preferable, and there has been considerable analysis within that framework and we will consider it here. However, this leads immediately to the following question: if cancellations occur in $d_e$, why don’t corresponding cancellations not occur also in $a_{SUGRA}^\mu$? In the following we will answer this question and show analytically how one may have large phases such that $d_e$ is suppressed to nearly zero, but $a_{SUGRA}^\mu$ is reduced by less than a factor of 2 (so that agreement with the Brookhaven data is maintained).

The remaining question then is whether fine tuning of the phases is needed to suppress $d_e$ to the level of Eq. (2) when the phases are large. We will see that significant fine tuning has started to occur at the GUT scale, and this problem will become more serious if the upper bound on $d_e$ is reduced further.

2 SUGRA Models

We consider here a generalization of the usual mSUGRA model allowing for CP violating phases. At the GUT scale $M_G$, the theory depends upon the following parameters: $m_0$, the universal scalar soft breaking mass; $m_i = |m_{1/2}| e^{i \phi_i}, i = 1, 2, 3$ the three gaugino masses; $A_0 = |A_0| e^{i \alpha_0}$, the cubic soft breaking mass; $B_0 = |B_0| e^{i \beta_0}$, the quadratic soft breaking mass; and $\mu_0 = |\mu_0| e^{i \theta_0}$, the Higgs mixing mass. One is always free to set one of the gaugino phases to zero and we chose $\phi_2 = 0$. Radiative breaking of $SU(2) \times U(1)$ at the electroweak scale determines $\theta_\mu$ according to (with the convenient choice that the Higgs VEVs be real) $\theta_\mu = -\theta_B + f_1(-\theta_B + \alpha_l, -\theta_B + \alpha_q)$ where $f_1$ is a loop correction, $\alpha_{(l,q)}$ are the (lepton, quark) phases of $A$ at the electroweak scale, and $\theta_B$ is the $B$ phase at the electroweak scale. In addition $|\mu|^2$ and $|B|$ are determined by the usual formulae. Thus the theory is defined by four real parameters, $m_0, |m_{1/2}|, |A_0|$, and $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$, and four new CP violating phases: $\theta_{B_0}, \phi_1, \phi_3, \alpha_0$. However, since we are here examining only the electron EDM, our results depend only weakly on $\phi_3$ and $\alpha_0$. Thus the two important phases are $\theta_{B_0}$ and $\phi_1$.

In carrying out the calculations discussed below, it is important to impose all the known accelerator and non-accelerator bounds on the SUSY parameter space (including coannihilation effects in $\Omega_{\chi^0} h^2$), and these bounds need to be calculated accurately. For a discussion of these see Ref. 13. Finally we mention that we scan the parameter space over the range $m_0, |m_{1/2}| < 1$ TeV, $|A_0|/m_{1/2} < 4$, and $2 < \tan \beta < 50$. 

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3 Suppression of $d_e$

It was realized from the beginning that $a_\mu$ is an increasing function of $\tan \beta$ \cite{ref1,ref2}. The Brookhaven data favors $\tan \beta > 5 - 7$ \cite{ref5}, and larger $\tan \beta$ is consistent with the data. In order to see analytically the effect of the CP violating phases then, we consider the leading part of the complex amplitude for large $\tan \beta$ calculated in Ref. 13. Using Eq. (4), one can write $a_\mu^{SUGRA}$ in the form

$$a_\mu^{SUGRA} = A \cos \theta_\mu + B \cos(\theta_\mu + \phi_1) + b \cos \theta_\mu + c$$  \hspace{1cm} (6)

$A$ comes from the chargino diagram, while $B$, $b$ and $c$ arise from the neutralino diagram. In general, the chargino contributions are largest and the parameter $a = B/(A+b)$ is $\approx 0.10 - 0.45$. ($b$ and $c$ are generally small). Similarly, Eq.(5) for $d_e$ has the general form

$$\frac{d_e}{e} = -\frac{m_e}{2m_\mu} (A + b) [\sin \theta_\mu + a \sin(\theta_\mu + \phi_1)]$$  \hspace{1cm} (7)

One sees that a priori, the amplitudes for $a_\mu^{SUGRA}$ and $d_e$ are of large size, being scaled by the largest amplitudes $A + b$. However, one can suppress $d_e$, and in fact even obtain $d_e = 0$ if the phases obey $\sin \theta_\mu + a \sin(\theta_\mu + \phi_1) = 0$, or alternately, using Eq. (6) (neglecting the small loop corrections), if \cite{ref13}

$$\tan \beta(\theta_B) = \frac{a \sin \phi_1}{1 + a \cos \phi_1}$$  \hspace{1cm} (8)

Since $a$ is not small, one sees that $\theta_B$ need not be small to accommodate even $d_e = 0$. This is essentially the origin of the cancellation effect \cite{ref11} for large SUSY CP violating phases.

One may now insert Eq. (8) into Eq. (6) to see the effect the phases have on $a_\mu^{SUGRA}$:

$$a_\mu^{SUGRA}(\theta_B, \phi_1) = a_\mu^{SUGRA}(0,0)\frac{\cos \theta_B}{|\cos \theta_B|} Q(\phi_1)$$  \hspace{1cm} (9)

where

$$Q(\phi_1) = \frac{[1 + 2a \cos \phi_1 + a^2]^{1/2}}{(1 + a)}; \hspace{1cm} 0.5 \lesssim Q \leq 1$$  \hspace{1cm} (10)

Thus the effect of the non-zero phases which reduce $d_e$ to zero is to reduce the magnitude of $a_\mu^{SUGRA}$ by at most a factor of two. Further, since experimentally $a_\mu^{SUGRA} > 0$, one requires $\cos \theta_B > 0$ and so

$$\theta_B > 0 \hspace{0.5cm} \text{for} \hspace{0.5cm} 0 < \phi_1 < \pi; \hspace{0.5cm} \theta_B < 0 \hspace{0.5cm} \text{for} \hspace{0.5cm} \pi < \phi_1 < 2\pi$$  \hspace{1cm} (11)
and so as $\phi_1$ varies over the entire range, one has $|\theta_B| \lesssim 0.4$.

In summary then, even if $d_e$ were zero, the cancellation mechanism can accommodate large phases ($|\theta_B| \lesssim 0.4$), and give acceptable predictions for the muon magnetic moment anomaly. The question, however, is: given the current bounds Eq.(2) on $d_e$, can one have large CP violating phases without unreasonable fine tuning of $\theta_B$ or $\phi_1$? We turn to this question next.

4 Large Phases and Fine Tuning

Since the experimental upper bound on $d_e$ is so small, one might expect that when the SUSY CP violating phases are large, fine tuning might be required to obtain the necessary amount of suppression. In order to quantify this, we define for any phase $\phi$ the quantity

$$R(\phi) = \frac{(\phi_1 - \phi_2)}{(\phi_1 + \phi_2)/2}$$

(12)

where $\phi_1$ and $\phi_2$ are the largest and smallest values of $\phi$ that satisfy the bound of Eq. (2). Thus $R(\phi)$ is a measure of how tightly constrained a phase must be to satisfy the experimental bounds. How much fine tuning one can tolerate is, of course, a matter of individual taste. However, we will take here as a benchmark the requirement that for any phase, $R(\phi) > 0.01$.

For a grand unified model, presumably parameters at the GUT scale are the more fundamental ones, and they are to be determined by some higher theory. Thus fine tuning at $M_G$ can represent a serious problem. The sensitive parameter in this case is $\theta_{B_0}$ (the $B$ phase at $M_G$). To understand analytically what may be occurring, we consider the RGE for the low and intermediate $\tan \beta$ region where an analytic expression exists. One finds

$$B = B_0 - \frac{1}{2}(1 - D_0)A_0 - \Phi_i |m_{1/2}| e^{i\phi_i}$$

(13)

where $D_0 = 1 - m_t^2/(200 \sin \beta)^2 \lesssim 0.25$ and $\Phi_i = O(1)$. Taking the imaginary part one finds

$$|B| \sin \theta_B = |B_0| \sin \theta_{B_0} - \frac{1}{2}(1 - D_0)|A_0| \sin \alpha_0 - \Phi_i m_{1/2} \sin \phi_i$$

(14)

For fixed $\alpha_0$ and $\phi_i$, one can relate the range of $\theta_B$, $\Delta \theta_B$, allowed by radiative electroweak breaking in terms of the range of $\theta_{B_0}$, $\Delta \theta_{B_0}$:

$$\Delta (\theta_{B_0}) \cong \frac{|B|}{|B_0|} \Delta (\theta_B)$$

(15)
However, radiative breaking at the electroweak scale shows that $|B|$ gets small as $\tan\beta$ grows i.e. $|B| = (1/2) \sin 2\beta (m_3^2/|\mu|)$. Hence we expect that

$$\Delta \theta_{B_0} \ll \Delta \theta_B$$

(16)

and fine tuning may occur at the GUT scale. An example of what happens is shown in Fig.3 where $R(\theta_{B_0})$ is plotted as a function of $m_{1/2}$ for $\tan\beta = 40$, $A_0 = 0$ for $\phi_1 = 1.6, 1.2, 0.9, 2.3,$ and $2.6$. One sees that for a wide range of $\phi_1$, $R(\theta_{B_0})$ falls below 0.01, and if one were to impose the condition that $R > 0.01$, it would eliminate a significant portion of the low $m_{1/2}$ parameter space. In the near future, one may expect the bounds on $d_e$ to be reduced by a factor of 2 to 3. This would exacerbate the fine tuning problem at the GUT scale.

![Figure 2. $R(\theta_{B_0})$ as a function of $m_{1/2}$ for $\tan\beta = 40$, $A_0 = 0$ for (from bottom to top) $\phi_1 = 1.6, 1.2, 0.9, 2.3$ and $2.6$.]

5 Conclusion

If the SUSY CP violating phases are small, i.e. $O(10^{-2})$, then the bounds on $d_e$ can be satisfied, and the effects of $d_e$ disconnect from $g_\mu - 2$. The Brookhaven E821 experiment, plus other experimental constraints, then leads to the following results for mSUGRA models

The light Higgs, $h$, mass bound combined with the $b \to s + \gamma$ constraint gives rise to a lower bound on $m_{1/2}$ of $m_{1/2} > 300 - 400$ GeV. The lower bound on the $a_\mu$ anomaly then produces an upper bound on $m_{1/2}$ of $m_{1/2} < 585(845)$ GeV at the 90% (95%)
C.L. for $\tan \beta \leq 50$. For mSUGRA, $a_{\mu}^{\text{SU}G\text{RA}}$ is bounded from above with $a_{\mu}^{\text{SU}G\text{RA}} \lesssim 50 \times 10^{-10}$. One can then predict the mSUGRA discovery reaches for accelerators. Thus at the 90% C.L. one finds that the Tevatron should see the light Higgs only, while a 500 GeV NLC would be able to see only $h$, the light stop squark and perhaps the light selectron. (One would need a higher energy LC to see more of the SUSY spectrum.) The LHC, of course, could see the entire SUSY spectrum. In addition, the lower bounds of dark matter detection rates would be raised, since the upper bound on $m_{1/2}$ has been lowered, and $\mu > 0$.

If the CP violating phases are large, they effect both $d_e$ and $a_\mu$. The experimental bound on $d_e$ can then still be satisfied if the cancelation mechanism occurs. One finds then the following: The cancelations can be understood analytically [Eq. (11)] and are seen to lead to $\theta_B$ as large as $\sim 0.4$ with large gaugino phase $\phi_1$. The value of $a_{\mu}^{\text{SU}G\text{RA}}$ is reduced by at most a factor of about two, and so agreement with the Brookhaven $a_\mu$ data can still be satisfied. (The reduction of $a_{\mu}^{\text{SU}G\text{RA}}$ will, however, lower the upper bound on $m_{1/2}$ and thus lower the SUSY mass spectrum, increasing the reach of accelerators.) The experimental bounds on $d_e$ can generally be satisfied with a fine tuning of phases of a few percent at the electroweak scale. However at $M_G$, fine tuning < 1% is needed for $\theta_{B_0}$ in the lower $m_{1/2}$ region. This fine tuning will become more serious if the experimental bound on $d_e$ is lowered further.

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**References**

6. Private communication from David DeMille.