Matrix Elements of Twist-2 Operators in Quenched Chiral Perturbation Theory

Jiunn-Wei Chen\textsuperscript{a} and Martin J. Savage\textsuperscript{b}

\textsuperscript{a} Department of Physics, University of Maryland, College Park, MD 20742-4111.
\textsuperscript{b} Department of Physics, University of Washington, Seattle, WA 98195.

Abstract

We compute the leading non-analytic quark mass dependence of the matrix elements of isovector twist-2 operators between octet baryon states in quenched QCD using quenched chiral perturbation theory. There are contributions of the form $m_q \log m_q$, in analogy with QCD, but there are also contributions of the form $\log m_q$ from hairpin interactions. The nucleon does not receive such hairpin contributions.
I. INTRODUCTION

The parton distribution functions (PDF’s) of the nucleon are fundamental quantities associated with the strong interactions. Extensive experimental investigations have been undertaken during the past three decades to measure these distributions via deep-inelastic scattering (DIS) of leptons from protons and light nuclei. Due to the intrinsically non-perturbative nature of the strong interactions in the low-momentum region, theoretical efforts to understand these distributions have had only limited success. With the ever increasing power of computers and significant developments in algorithms used to numerically simulate QCD on the lattice, it is hoped that properties of the PDF’s can be determined from first principle \[1,2\] at some point in the not so distant future (for a review see Ref. \[3\]). It is the forward matrix elements of twist-2 operators that are computed numerically and these matrix elements are directly related to moments of the PDF’s. Of course, only matrix elements computed with unquenched QCD with the physical values of the quark masses, $m_q$, are to be directly compared with experimental data, but at this point in time such computations are not possible. All present computations are performed with lattice quark masses, $m_{q}^{\text{latt}}$, that give a pion mass of $m_{\pi}^{\text{latt}} \sim 500$ MeV, and most simulations are quenched. Despite the fact that quenched computations require significantly less computer time, they cannot, unfortunately, be connected to QCD in any way. While solid progress is being made toward unquenched calculations \[2\] of the moments of the PDF’s, it is likely that partially-quenched \[4\] computations will first provide a reliable connection between lattice computations and nature by allowing for calculations with smaller $m_q$ and thereby minimizing the impact of the $m_q$-extrapolation. However, it is of interest, from a theoretical standpoint, to know the moments of the PDF’s in quenched QCD (QQCD). To extrapolate from $m_{q}^{\text{latt}}$ down to $m_q$, the $m_{q}$-dependence of the matrix elements is required \[5\], and recently chiral perturbation theory ($\chi$PT) has been used to determine the leading $m_q$-dependence in an expansion about the chiral limit in QCD \[6,7\] and large-$N_c$ QCD \[8\]. $\chi$PT also allows for a connection to be made with existing convolution models of PDF’s, and in addition, shows how to make them consistent with QCD \[9\]. In this work we determine the leading $m_q$-dependence of the matrix elements of isovector twist-2 operators about the chiral limit in QQCD using quenched chiral perturbation theory (Q$\chi$PT) \[10–14\].

II. Q$\chi$PT

The lagrange density of QQCD is

\[
\mathcal{L} = \sum_{a,b,u,d,s} \bar{q}^a \left[ i \slashed{D} - m_q \right]^b_a q_b + \sum_{\tilde{a},\tilde{b}=\tilde{u},\tilde{d},\tilde{s}} \bar{\tilde{q}}^{\tilde{a}} \left[ i \slashed{D} - m_{\tilde{q}} \right]^\tilde{b}_{\tilde{a}} \tilde{q}_{\tilde{b}} \\
= \sum_{j,k=u,d,s,\tilde{u},\tilde{d},\tilde{s}} \overline{Q}^j \left[ i \slashed{D} - m_Q \right]^k_j Q_k ,
\]

where $q_a$ are the three light-quarks, $u$, $d$, and $s$, and $\tilde{q}$ are three light bosonic quarks $\tilde{u}$, $\tilde{d}$, and $\tilde{s}$. The super-quark field, $Q_j$, is a six-component column vector with the three light-quarks, $u$, $d$, and $s$, in the upper three entries and the three ghost-light-quarks, $\tilde{u}$, $\tilde{d}$, and $\tilde{s}$, in the lower three entries. The graded equal-time commutation relations for two fields is
\[ Q_i^\alpha(x)Q_j^{\beta\dagger}(y) - (-)^{\eta_i\eta_j}Q_j^{\beta\dagger}(y)Q_i^\alpha(x) = \delta^{\alpha\beta}\delta_{ij}\delta^3(x - y) \] ,

where \( \alpha, \beta \) are spin-indices and \( i, j \) are flavor indices. The objects \( \eta_k \) correspond to the parity of the component of \( Q_k \), with \( \eta_k = +1 \) for \( k = 1, 2, 3 \) and \( \eta_k = 0 \) for \( k = 4, 5, 6 \). The diagonal super mass-matrix, \( m_Q \), has entries \( m_Q = \text{diag}(m_u, m_d, m_s, m_u, m_d, m_s) \), i.e. \( m_\bar{u} = m_u \), \( m_\bar{d} = m_d \) and \( m_\bar{s} = m_s \), so that the contribution to the determinant in the path integral from the \( \bar{q} \)'s exactly cancel.

In the absence of quark masses, the lagrange density in eq. (1) has a graded symmetry \( U(3|3)_L \otimes U(3|3)_R \), where the left- and right-handed quark fields transform as \( Q_L \rightarrow U_LQ_L \) and \( Q_R \rightarrow U_RQ_R \) respectively. However, the functional integral associated with this Lagrange density does not converge unless the transformations on the left- and right-handed fields are related, \( \text{sdet}(U_L) = \text{sdet}(U_R) \), where \( \text{sdet() denotes a superdeterminant} \) [4,11,12], leaving the theory to have a symmetry \([SU(3|3)_L \otimes SU(3|3)_R] \times U(1)_V \), where the “\( \times \)” denotes a semi-direct product as opposed to a direct product, “\( \otimes \)”. It is assumed that this symmetry is spontaneously broken \([SU(3|3)_L \otimes SU(3|3)_R] \times U(1)_V \rightarrow SU(3|3)_V \times U(1)_V \) so that an identification with QCD can be made.

The pseudo-Goldstone bosons of QQCD form a \( 6 \times 6 \) matrix, \( \Phi \), that can be written in block form

\[ \Phi = \begin{pmatrix} \pi & \chi^\dagger \\ \chi & \bar{\pi} \end{pmatrix} \] ,

where \( \pi \) is the \( 3 \times 3 \) matrix of pseudo-Goldstone bosons including the \( \eta' \) with quantum numbers of \( \bar{q}q \) pairs, \( \bar{\pi} \) is a \( 3 \times 3 \) matrix of pseudo-Goldstone bosons including the \( \bar{\eta}' \) with quantum numbers of \( \bar{q}q \) pairs, and \( \chi \) is a \( 3 \times 3 \) matrix of pseudo-Goldstone fermions with quantum numbers of \( \bar{q}q \) pairs,

\[
\begin{align*}
\pi &= \begin{pmatrix} \eta_u & \pi^+ & K^+ \\ \pi^- & \eta_d & K^0 \\ K^- & \bar{K}^0 & \eta_s \end{pmatrix}, \\
\bar{\pi} &= \begin{pmatrix} \bar{\eta}_u & \bar{\pi}^+ & \bar{K}^+ \\ \bar{\pi}^- & \bar{\eta}_d & \bar{K}^0 \\ \bar{K}^- & \bar{K}^0 & \bar{\eta}_s \end{pmatrix}, \\
\chi &= \begin{pmatrix} \chi_{\eta_u} & \chi_{\pi^+} & \chi_{K^+} \\ \chi_{\pi^-} & \chi_{\eta_d} & \chi_{K^0} \\ \chi_{K^-} & \chi_{\bar{K}^0} & \chi_{\eta_s} \end{pmatrix}.
\end{align*}
\]

As the object

\[ \Phi_0 = \frac{1}{\sqrt{6}} \text{str}(\Phi) = \frac{1}{\sqrt{2}}(\eta' - \bar{\eta}') \] ,

is invariant under \([SU(3|3)_L \otimes SU(3|3)_R] \times U(1)_V \) the most general lagrange density that describes low-momentum dynamics will contain arbitrary functions of \( \Phi_0 \) [11,12]. At lowest order in the chiral expansion, the Lagrange density that describes the dynamics of the pseudo-Goldstone bosons is, using the notation of Ref. [13],

\[ L = \frac{f^2}{8} \text{str} \left[ \partial^\mu \Sigma^i \partial_\mu \Sigma^i \right] + \lambda \text{str} \left[ m_Q \Sigma + m_Q^\dagger \Sigma^i \right] + \alpha_\Phi \partial^\mu \Phi_0 \partial_\mu \Phi_0 - M_0^2 \Phi_0^2 \] ,

where the parameter \( \lambda \) is chosen to reproduce the meson masses, and \( \Sigma \) is the exponential of the \( \Phi \) field,

\[ \Sigma = \exp \left( \frac{2i}{f} \Phi \right) \] .

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With this normalization, \( f \sim 132 \text{ MeV} \) in QCD. In addition, it is understood that the operators with coefficients \( \alpha \Phi \) and \( M_0^2 \), the hairpin interactions, are inserted perturbatively. Expanding out the Lagrange density in eq. (6) to quadratic order in the meson fields, one finds relations between the meson masses in the isospin limit,

\[
m_{\eta_0}^2 = 2m_K^2 - m_{\pi}^2, \quad m_{\eta_8}^2 = m_{\eta_d}^2 = m_{\pi}^2.
\]

The Lagrange density in eq. (6) has been used to compute several observables in the meson sector, such as \( f_K, f_{\pi} \) [15] and the meson masses, to one-loop in perturbation theory [11,12,16] (for a review see Ref. [17]).

The inclusion of the lowest-lying baryons, the octet of spin-\( \frac{1}{2} \) baryons and the decuplet of spin-\( \frac{3}{2} \) baryon resonances, is detailed in Ref. [13]. An interpolating field that has non-zero overlap with the baryon octet (when the \( ijk \) indices are restricted to 1, 2, 3) is [13]

\[
B_{ij} = (-)^{1+\eta_i \eta_j} B_{ikj}, \quad B_{ij} + (-)^{1+\eta_i \eta_j} B_{ijk} + (-)^{1+\eta_i \eta_j + \eta_i \eta_k} B_{kji} = 0.
\]

The object \( B_{ijk} \) describes a 70-dimensional representation of \( SU(3|3)_V \) [13]. It is convenient to decompose the irreducible representations of \( SU(3|3)_V \) into irreducible representations \( SU(3)_q \otimes SU(3)_{\bar{q}} \otimes U(1) \) [14]. The subscript denotes where the “\( SU(3) \)” acts, either on the \( q \)’s or on the \( \bar{q} \)’s. The ground floor of the 70-dimensional representation contains baryons that are comprised of three quarks, \( qqq \), and transforms as an \( (8,1) \) of \( SU(3)_q \otimes SU(3)_{\bar{q}} \).

The octet baryons are embedded as

\[
B_{abc} = \frac{1}{\sqrt{6}} \left( \epsilon_{abc} B^d_c + \epsilon_{acd} B^d_b \right), \quad (11)
\]

where the indices are restricted to take the values \( a, b, c = 1, 2, 3 \) only. The octet baryon matrix is

\[
B = \begin{pmatrix}
\frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\
\Sigma^- & \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & n \\
\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda
\end{pmatrix}.
\]

The first floor of the 70-dimensional representation contains baryons that are composed of two quarks and one ghost-quark, \( \bar{q}qq \), and transforms as an \( (6,3) \oplus (\bar{3},\bar{3}) \) of \( SU(3)_q \otimes SU(3)_{\bar{q}} \).

The tensor representation \( \tilde{s}_{ab} \) of the \( (6,3) \) multiplet, where \( \tilde{a} = 1, 2, 3 \) runs over the \( \bar{q} \) indices and \( a, b = 1, 2, 3 \) run over the \( q \) indices, has baryon assignment

\[
\tilde{s}_{11} = \Sigma^+_{\tilde{a}}, \quad \tilde{s}_{12} = \tilde{a}s_{21} = \frac{1}{\sqrt{2}} \Sigma^0_{\tilde{a}}, \quad \tilde{s}_{22} = \Sigma^{-1}_{\tilde{a}}
\]

\[
\tilde{s}_{13} = \tilde{a}s_{31} = \frac{1}{\sqrt{2}} (6)_{\tilde{a}}^{+1}, \quad \tilde{s}_{23} = \tilde{a}s_{32} = \frac{1}{\sqrt{2}} (6)_{\tilde{a}}^{-1}, \quad \tilde{s}_{33} = \Omega^0_{\tilde{a}}.
\]
The right superscript denotes the third component of q-isospin, while the left subscript denotes the \( \bar{q} \) flavor. The tensor representation \( \bar{a}t^a \) of the \((3,3)\) multiplet, where \( \bar{a} = 1, 2, 3 \) runs over the \( \bar{q} \) indices and \( a = 1, 2, 3 \) run over the \( q \) indices, has baryon assignment

\[
\bar{a}t^1 = (3) \tilde{z}^{1 \bar{a}}_a , \quad \bar{a}t^2 = (3) \tilde{z}^{1 \bar{a}}_a , \quad \bar{a}t^3 = \Lambda^0 \bar{a} . \tag{14}
\]

The \( \bar{a}s_{ab} \) and the \( \bar{a}t^a \) are uniquely embedded into \( B_{ijk} \) (up to field redefinitions), constrained by the relations in eq. (10):

\[
B_{ijk} = \sqrt{\frac{2}{3}} i_{-3}s_{jk} \quad \text{for} \quad i = 4, 5, 6 \quad \text{and} \quad j, k = 1, 2, 3
\]
\[
B_{ijk} = \frac{1}{2} j_{-3} t^\sigma \varepsilon_{\sigma ik} + \frac{1}{\sqrt{6}} j_{-3}s_{ik} \quad \text{for} \quad j = 4, 5, 6 \quad \text{and} \quad i, k, \sigma = 1, 2, 3
\]
\[
B_{ijk} = -\frac{1}{2} k_{-3} t^\sigma \varepsilon_{\sigma ij} - \frac{1}{\sqrt{6}} k_{-3}s_{ij} \quad \text{for} \quad k = 4, 5, 6 \quad \text{and} \quad i, j, \sigma = 1, 2, 3 . \tag{15}
\]

As we are only interested in one-loop contributions to observables with \(qqq\)-baryons in the asymptotic states, we do not explicitly show the second and third floors of the \(70\).

An interpolating field that contains the spin-\(\frac{3}{2}\) decuplet as the ground floor is [13]

\[
T^a_{ijk} \sim \left[ Q_{i}^{a,b}Q_{j}^{b,c}Q_{k}^{c,a} + Q_{i}^{b,c}Q_{j}^{c,a}Q_{k}^{a,b} + Q_{i}^{c,a}Q_{j}^{a,b}Q_{k}^{b,c} \right] \varepsilon_{abc} (C_{\gamma \mu})_{\beta \gamma} , \tag{16}
\]

where the indices \(i, j, k\) run from 1 to 6. Neglecting spin indices, one finds that under the interchange of flavor indices [13]

\[
T_{ijk} = (-1)^{1+\eta_i \eta_j} T_{jk i} = (-1)^{1+\eta_i \eta_k} T_{ik j} . \tag{17}
\]

\( T_{ijk} \) describes a \(38\) dimensional representation of \(SU(3)_q \otimes SU(3)_{\bar{q}}\), which has a ground floor transforming as \((10, 1)\) under \(SU(3)_q \otimes SU(3)_{\bar{q}}\) with

\[
T_{abc} = T_{abc} , \tag{18}
\]

where the indices are restricted to take the values \(a, b, c = 1, 2, 3\), and where \(T_{abc}\) is the totally symmetric tensor containing the decuplet of baryon resonances,

\[
T_{111} = \Delta^{++} , \quad T_{112} = \frac{1}{\sqrt{3}} \Delta^+ , \quad T_{122} = \frac{1}{\sqrt{3}} \Delta^- , \quad T_{222} = \Delta^0 , \quad T_{223} = \frac{1}{\sqrt{3}} \Sigma^+, \quad T_{223} = \frac{1}{\sqrt{3}} \Sigma^0 , \quad T_{333} = \Omega^- . \tag{19}
\]

The first floor transforms as a \((6,3)\) under \(SU(3)_q \otimes SU(3)_{\bar{q}}\) and has a tensor representation, \(\bar{a}x_{ij}\), with baryon assignment

\[
\bar{a}x_{11} = \Sigma^{*+1}_a , \quad \bar{a}x_{12} = \bar{a}x_{21} = \frac{1}{\sqrt{2}} \Sigma^* a , \quad \bar{a}x_{22} = \Sigma^{-1}_a , \quad \bar{a}x_{13} = \Sigma^{*+1}_a , \quad \bar{a}x_{23} = \bar{a}x_{32} = \frac{1}{\sqrt{2}} \Sigma^* a , \quad \bar{a}x_{33} = \Omega^* a . \tag{20}
\]

\[:5]
The embedding of $x_{ij}$ into $T_{ijk}$ is unique (up to field redefinitions), constrained by the symmetry properties in eq. (17):

$$T_{ijk} = + \frac{1}{\sqrt{3}} i_{-3} x_{jk} \quad \text{for} \quad i = 4, 5, 6 \quad \text{and} \quad j, k = 1, 2, 3$$

$$T_{ijk} = - \frac{1}{\sqrt{3}} j_{-3} x_{ik} \quad \text{for} \quad j = 4, 5, 6 \quad \text{and} \quad i, k = 1, 2, 3$$

$$T_{ijk} = + \frac{1}{\sqrt{3}} k_{-3} x_{ij} \quad \text{for} \quad k = 4, 5, 6 \quad \text{and} \quad i, j = 1, 2, 3 \quad .$$

The free Lagrange density for the $B_{ijk}$ and $T_{ijk}$ fields is [13,14], at leading order in the heavy baryon expansion [18–22],

$$\mathcal{L} = i \left( B_v \cdot D B \right) + 2 \alpha_M \left( B B M_+ \right) + 2 \beta_M \left( B M_+ B \right) + 2 \sigma_M \left( B B \right) \text{str} (M_+)$$

$$- i \left( T'^\nu \cdot D T_\mu \right) + \Delta \left( T'^\nu T_\mu \right) + 2 \gamma_M \left( T'^\nu M_+ T_\mu \right) - 2 \sigma_M \left( T'^\nu T_\mu \right) \text{str} (M_+) \quad ,$$

where $M_+ = \frac{1}{2} \left( \xi^\dagger m_Q \xi^\dagger + \xi m_Q \xi \right), \Delta$ is the decuplet-octet mass splitting, $v_\mu$ is the baryon four-velocity, and $\xi = \sqrt{\Sigma}$. The brackets, ( ... ) denote contraction of lorentz and flavor indices as defined in Refs. [13,14].

The Lagrange density describing the interactions of the baryons with the pseudo-Goldstone bosons is [13]

$$\mathcal{L} = 2 \alpha \left( B S^\mu B A_\mu \right) + 2 \beta \left( B S^\mu A_\mu B \right) + 2 \gamma \left( B S^\mu B \right) \text{str} (A_\mu)$$

$$+ 2 \mathcal{H} \left( T'^\nu S^\mu A_\mu T_\nu \right) + \sqrt{\frac{3}{2}} \mathcal{C} \left[ \left( T'^\nu A_\nu B \right) + \left( B A_\nu T'^\nu \right) \right] + 2 \gamma' \left( T'^\nu S^\mu T_\nu \right) \text{str} (A_\mu) \quad ,$$

where $S^\mu$ is the covariant spin-vector [18–20]. Restricting oneself to the qqq sector, it is straightforward to show that in QCD

$$\alpha = \frac{2}{3} D + 2 F \quad , \quad \beta = - \frac{5}{3} D + F \quad ,$$

where $D$ and $F$ are constants that multiply the $SU(3)_q$ invariants that are commonly used in QCD, but it should be stressed that the $F$ and $D$ in QQCD will not have the numerical values of those of QCD. In QQCD there is an additional coupling that must be considered, $\gamma$, a hairpin interaction [13], that is usually not considered in the $\chi$PT description of low-energy QCD. In our calculation of the matrix elements of twist-2 operators we will replace $\alpha$ and $\beta$ with $F$ and $D$, but we will keep $\gamma$ explicit. In the above discussion, vector and axial-vector meson fields have been introduced in direct analogy with QCD. The covariant derivative acting on either the $B$ or $T$ fields has the form

$$(D^\mu B)_{ijk} = \partial^\mu B_{ijk} + (V^\mu)_i^j B_{jk} + (-)^{\eta_j + \eta_m} (V^\mu)_j^m B_{imk} + (-)^{\eta_i + \eta_j} (V^\mu)_k^m B_{ijn} \quad ,$$

where the vector and axial-vector meson fields are

$$V^\mu = \frac{1}{2} \left( \xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi \right) \quad , \quad A^\mu = \frac{i}{2} \left( \xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi \right) \quad .$$
III. ISOVECTOR TWIST-2 OPERATORS

In QCD, the nonsinglet twist-2 operators have the form,

$$O^{(n),a}_{\mu_1\mu_2...\mu_n} = \frac{1}{n!} \gamma^a \lambda^\alpha \gamma_{\mu_1} \left( i \overleftrightarrow{D}_{\mu_2} \right) ... \left( i \overleftrightarrow{D}_{\mu_n} \right) q - \text{traces}, \quad (27)$$

where the \{\ldots\} denotes symmetrization on all Lorentz indices, and where \(\lambda^a\) are Gell-Mann matrices acting in flavor-space. The \(O^{(n),a}_{\mu_1\mu_2...\mu_n}\) transform as \((8, 1) \oplus (1, 8)\) under \(SU(3)_L \otimes SU(3)_R\) chiral transformations \([6,7]\). Of particular interest to us are the isovector operators where \(\lambda^3 = \text{diag}(1, -1, 0)\). In QCD the nonsinglet twist-2 operators have the form

$$QO^{(n),a}_{\mu_1\mu_2...\mu_n} = \frac{1}{n!} Q \gamma^a \lambda^\alpha \gamma_{\mu_1} \left( i \overleftrightarrow{D}_{\mu_2} \right) ... \left( i \overleftrightarrow{D}_{\mu_n} \right) Q - \text{traces}, \quad (28)$$

where the \(\overrightarrow{\lambda}\) are super Gell-Mann matrices, and we are interested in the isovector matrix elements with \(\overrightarrow{\lambda}^0 = \text{diag}(1, -1, 0, 1, -1, 0)\). The operators \(QO^{(n),3}_{\mu_1\mu_2...\mu_n}\) transform as \((8, 1, 1, 1) \oplus (1, 8, 1, 1) \oplus (1, 1, 8, 1) \oplus (1, 1, 1, 8)\) under \(SU(3)_{Lq} \otimes SU(3)_{Lq} \otimes SU(3)_{Rq} \otimes SU(3)_{Rq}\) graded chiral transformations.

A. Matrix Elements in the Pion

At leading order in the chiral expansion, matrix elements of the isovector operator \(QO^{(n),3}_{\mu_1\mu_2...\mu_n}\) between meson states are reproduced by operators of the form \([6]\)

$$QO^{(n),3}_{\mu_1\mu_2...\mu_n} \rightarrow a^{(n)} (i)^n \frac{f^2}{4} \left( \frac{1}{\Lambda_\chi} \right)^{n-1} \text{str} \left[ \Sigma^i \overrightarrow{\lambda}^3 \overrightarrow{\partial}_{\mu_1} \overrightarrow{\partial}_{\mu_2} ... \overrightarrow{\partial}_{\mu_n} \Sigma + \Sigma \overrightarrow{\lambda}^3 \overrightarrow{\partial}_{\mu_1} \overrightarrow{\partial}_{\mu_2} ... \overrightarrow{\partial}_{\mu_n} \Sigma^i \right] - \text{traces}, \quad (29)$$

where \(\Lambda_\chi = 4\pi f\) is the scale of chiral symmetry breaking. Operators involving more derivatives on the meson fields or more insertions of \(m_q\) make contributions to the matrix elements that are higher order in the chiral expansion \([6]\). In QCD, the one-loop diagrams shown in fig. 1 with only the octet mesons in the loop give rise to the leading non-analytic terms in \(m_q\), of the form \(m_q \log m_q\). Such diagrams have the potential to give analogous contributions to the matrix elements in QQCD. However, explicit computation proves that such non-analytic contributions to the matrix element of \(QO^{(n),3}_{\mu_1\mu_2...\mu_n}\) resulting from the diagrams shown in fig. 1 vanish, due to a cancellation between the “\(\pi\)”-loops and “\(\chi\)”-loops. Therefore, at one-loop order there are no nonanalytic contributions and the matrix element of \(QO^{(n),3}_{\mu_1\mu_2...\mu_n}\) in the pion is

$$\langle QO^{(n),3}_{\mu_1\mu_2...\mu_n} \rangle_\pi = i \ 4 \ a^{(n)} \left( \frac{1}{\Lambda_\chi} \right)^{n-1} \varepsilon^{\alpha\beta\gamma} q_{\mu_1} ... q_{\mu_n} - \text{traces} + ... \quad , \quad (30)$$

for \(n\)-odd, between an initial pion with isospin index \(\alpha\) and momentum \(q_\mu\), and a final pion with isospin index \(\beta\). The matrix elements vanish for \(n\)-even and the ellipses denote terms that are analytic in \(m_q\), or are higher order in the chiral expansion.
FIG. 1. One-loop diagrams that contribute to the matrix elements of $Q^{(n),3}_{\mu_1 \mu_2 \ldots \mu_n}$ in the pion. The crossed circle denotes an insertion of an operator from eq. (29), arising directly from the twist-2 operator. The smaller solid circle denotes an insertion of a leading order strong-interaction vertex from eq. (6). Diagrams (a) and (b) are vertex corrections while diagram (c) denotes wavefunction renormalization.

B. Matrix Elements in the Octet Baryons

At leading order in the chiral expansion, the matrix elements of $Q^{(n),3}_{\mu_1 \mu_2 \ldots \mu_n}$ are described by

$$Q^{(n),3}_{\mu_1 \mu_2 \ldots \mu_n} \rightarrow \alpha^{(n)} v_{\mu_1} v_{\mu_2} \ldots v_{\mu_n} \left( \mathcal{B} \mathcal{B} \overline{x}_{\xi+}^3 \right) + \beta^{(n)} v_{\mu_1} v_{\mu_2} \ldots v_{\mu_n} \left( \mathcal{B} \overline{x}_{\xi+}^3 \mathcal{B} \right)$$

$$+ \gamma^{(n)} v_{\mu_1} v_{\mu_2} \ldots v_{\mu_n} \left( \mathcal{T}^\alpha \overline{x}_{\xi+}^3 \mathcal{T}_\alpha \right) + \sigma^{(n)} \frac{1}{n!} v_{\{\mu_1} v_{\mu_2} \ldots v_{\mu_{n-2}} \left( \mathcal{T}_{\mu_{n-1}} \overline{x}_{\xi+}^3 \mathcal{T}_{\mu_n} \right) ,$$

where the $\{\ldots\}$ brackets denote complete symmetrization of the enclosed indices, $v_{\mu}$ is the four-velocity of the heavy baryon, and

$$\overline{x}_{\xi+}^3 = \frac{1}{2} \left( \xi \overline{\xi}^\dagger + \xi^\dagger \overline{\xi} \right) ,$$

in complete analogy with the work of Refs. [6,7]. The index contractions denoted by $\{\ldots\}$ are the same as those in eq. (23) defined in Refs. [13,14]. In general, the coefficients $\alpha^{(n)}, \beta^{(n)}, \gamma^{(n)}$ and $\sigma^{(n)}$ are not constrained by symmetries and must be determined from elsewhere. However for $n = 1$ they are fixed by the isospin charge of the baryons to be

$$\alpha^{(1)} = +2 \quad , \quad \beta^{(1)} = +1 \quad , \quad \gamma^{(1)} = -3 \quad , \quad \sigma^{(1)} = 0 .$$

An expression analogous to eq. (31) exists in QCD, and can be obtained from eq. (31) by restricting the flavor-indices in each sum to take values $i = 1, 2, 3$ only. However, one must bear in mind that, in general, the values for $\alpha^{(n)}, \beta^{(n)}, \gamma^{(n)}$ and $\sigma^{(n)}$ in QQCD will differ from those in QCD for $n \neq 1$. At tree-level the matrix elements in an octet baryon "$i$" is

$$\langle Q^{(n),3}_{\mu_1 \mu_2 \ldots \mu_n}\rangle_{\text{tree}}^{\text{tree}} = v_{\mu_1} v_{\mu_2} \ldots v_{\mu_n} \mathcal{M}_{\text{tree}}^{\text{tree}} , \quad \langle Q^{(n),3}_{\mu_1 \mu_2 \ldots \mu_n}\rangle_{\text{tree}}^{\text{tree}} = v_{\mu_1} v_{\mu_2} \ldots v_{\mu_n} Q \mathcal{M}_{\text{tree}}^{\text{tree}} ,$$

8
where the Clebsch-Gordan coefficients, $\mathcal{M}_i^{\text{tree}}$ and $Q\mathcal{M}_i^{\text{tree}}$, are given in Table I. Parametrically, but not numerically, the matrix elements in QCD and QQCD are the same at tree-level. We consider matrix elements in only the proton, $\Sigma^+$ and $\Xi^0$, as matrix elements in the other members of the octet are either trivially related to these, or vanish by isospin symmetry.

In QCD, contributions that are non-analytic in $m_q$ first appear at next-to-leading order (NLO) and have the form $m_q \log m_q$ [5]. However, in QQCD there are additional contributions of the form $m_q \log m_q$ to baryons other than the nucleon from hairpin interactions. We will consider the two distinct contributions separately, and write the matrix element at one-loop level as

$$
\langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{tree}} = \langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{tree}} + \langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{non-HP}}
$$

$$
Q \langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{tree}} = Q \langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{tree}} + Q \langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{non-HP}} + Q \langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{HP}}.
$$

The superscripts on the one-loop contributions indicate whether or not a hairpin interaction (HP) appears in the diagram. Of course, there are no HP contributions in QCD.

### 1. Non-Hairpin Contributions

The one-loop diagrams in figs. (2) give contributions that are non-analytic in $m_q$ of the form $m_q \log m_q$. The diagrams in figs. (3) also give terms of the form $m_q \log m_q$ in the limit that the decuplet-octet mass splitting, $\Delta$, vanishes, but in general give a more complicated expression. We write the non-hairpin one-loop contribution to the forward matrix element in terms of the contributions from the meson mass eigenstates,

$$
\langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{non-HP}} = v_{\mu_1} v_{\mu_2} ... v_{\mu_n} \frac{1 - \delta^{i1}}{8\pi^2 f^2} \left[ \sum_j \mathcal{M}_j^{(j)} H_j + \sum_j N_j^{(j)} J_j \right]
$$

$$
Q \langle O_{\mu_1\mu_2...\mu_n}^{(n)} \rangle_i^{\text{non-HP}} = v_{\mu_1} v_{\mu_2} ... v_{\mu_n} \frac{1 - \delta^{i1}}{8\pi^2 f^2} \left[ \sum_j Q\mathcal{M}_j^{(j)} H_j + \sum_j QN_j^{(j)} J_j \right].
$$

As we are assuming isospin symmetry in our calculation, the sum over mesons for QCD corresponds to $j = \pi, K, \eta$, while for QQCD the sum corresponds to $j = \pi, K, \eta_s$. The coefficients resulting from mesons with the mass of the pion, $\mathcal{M}(\pi)$ and $Q\mathcal{M}(\pi)$ [ $\mathcal{N}(\pi)$ and $Q\mathcal{N}(\pi)$ ], from the diagrams in fig. (2) [ fig. (3) ] are tabulated in Table II [ Table V ], while the coefficients resulting from mesons with the mass of the kaon, $\mathcal{M}(K)$ and $Q\mathcal{M}(K)$ [ $\mathcal{N}(K)$ and $Q\mathcal{N}(K)$ ], are tabulated in Table III [ Table VI ]. The pionic contribution to the

<table>
<thead>
<tr>
<th>Baryon</th>
<th>$\mathcal{M}^{\text{tree}}$, $Q\mathcal{M}^{\text{tree}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$\frac{1}{3}(2\alpha(n) - \beta(n))$</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$\frac{1}{5}(5\alpha(n) + 2\beta(n))$</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>$\frac{1}{6}(\alpha(n) + 4\beta(n))$</td>
</tr>
</tbody>
</table>

**TABLE I.** The tree-level matrix elements of the isovector twist-2 operators between octet baryon states. The constants $\alpha(n)$ and $\beta(n)$ are defined in eq. (31).
FIG. 2. The meson loop diagrams that give leading non-analytic contributions of the form 
m_q \log m_q \text{ to the matrix element of } Q^{O_{\mu_1\mu_2...\mu_n}} \text{ between octet baryon states. The crossed circle denotes an insertion of an operator from eq. (29) or eq. (31), arising directly from the twist-2 operator. The smaller solid circle denotes an insertion of the strong two-pion-nucleon interaction from the nucleon kinetic energy term in eq. (22), while the square denotes an insertion of the axial-vector interaction } \propto F, D, \gamma. \text{ The label “π” denotes an octet meson, and “χ” denotes an octet fermionic meson. Diagrams (a)-(d) are vertex corrections while diagram (e) denotes wavefunction renormalization.}

FIG. 3. One-loop diagrams with $T_{ijk}$ intermediate states that contribute to the matrix elements of $Q^{O_{\mu_1\mu_2...\mu_n}} \text{ between octet baryon states. The thick solid line inside the loop denotes a } T_{ijk} \text{ propagator, while the dashed line denotes a meson propagator. The crossed circle denotes an insertion of an operator from eq. (29) or eq. (31), arising directly from the twist-2 operator, and the square denotes an insertion of the strong interaction vertex } \propto C. \text{ Diagrams (a) and (b) are vertex corrections while diagram (c) denotes wavefunction renormalization.}
\( \Sigma^0 \) - \( [1 + 3(D - F)^2] \mathcal{M}^{\text{tree}}_{\Sigma^0} \) + \( \frac{3}{2} \alpha(n) D(D + F) \) + \( \frac{1}{2} \left( 3 \alpha(n) + 2 \beta(n) \right) D^2 \)

**TABLE IV.** Contributions to the matrix elements of \( \mathcal{O}^{(n)}_{\mu_1 \mu_2 \ldots \mu_n} \) proportional to \( H_\eta \) in QCD and the matrix elements of \( Q\mathcal{O}^{(n)}_{\mu_1 \mu_2 \ldots \mu_n} \) proportional to \( H_\eta \) in QQCD in the octet baryons.

\begin{align*}
B & \quad \mathcal{M}^{(\pi)} (\text{QCD}) & \quad Q\mathcal{M}^{(\pi)} (\text{QQCD}) \\
p & \quad -C^2 \left\{ \frac{2 \mathcal{M}^{\text{tree}}_p + \frac{15}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) }{ } \right\} & \quad -C^2 \left\{ \frac{\mathcal{M}^{\text{tree}}_p + \frac{1}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) }{ } \right\} \\
\Sigma^+ & \quad -C^2 \left\{ \frac{3}{5} \mathcal{M}^{\text{tree}}_{\Sigma^+} + \frac{1}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right\} & \quad -C^2 \left\{ \frac{\mathcal{M}^{\text{tree}}_{\Sigma^+} + \frac{1}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) }{ } \right\} \\
\Xi^0 & \quad -C^2 \left\{ \frac{1}{3} \mathcal{M}^{\text{tree}}_{\Xi^0} - \frac{1}{18} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right\} & \quad 0
\end{align*}

**TABLE V.** Contributions to the matrix elements of \( \mathcal{O}^{(n)}_{\mu_1 \mu_2 \ldots \mu_n} \) and \( Q\mathcal{O}^{(n)}_{\mu_1 \mu_2 \ldots \mu_n} \) in the octet baryons that are proportional to \( J_\pi \).
Hairpin interactions in QQCD. Explicit computation of the diagrams in fig. 4 yields

\[
\Sigma^0 - C^2 \left[ \frac{2}{3} M_{\Sigma^0}^{\text{tree}} + \frac{2}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right]
\]

\[
\Xi^0 - C^2 \left[ \frac{3}{2} M_{\Xi^0}^{\text{tree}} + \frac{2}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right]
\]

\[
\Sigma - C^2 \left[ \frac{2}{3} M_{\Sigma}^{\text{tree}} + \frac{11}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right]
\]

\[
\Xi - C^2 \left[ \frac{5}{6} M_{\Xi}^{\text{tree}} + \frac{13}{18} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right]
\]

**Table VI.** Contributions to the matrix elements of \( O_{\mu_1 \mu_2 \ldots \mu_n}^{(n)} \) and \( Q O_{\mu_1 \mu_2 \ldots \mu_n}^{(n)} \) in the octet baryons that are proportional to \( J_K \).

<table>
<thead>
<tr>
<th>B</th>
<th>( N^{(K)} ) (QCD)</th>
<th>( Q N^{(K)} ) (QQCD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>(-C^2 \left[ \frac{2}{3} M_{p}^{\text{tree}} + \frac{2}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right] )</td>
<td>0</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>(-C^2 \left[ \frac{2}{3} M_{\Sigma^+}^{\text{tree}} + \frac{11}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right] )</td>
<td>(-C^2 \left[ \frac{5}{6} M_{\Sigma^+}^{\text{tree}} + \frac{13}{18} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right] )</td>
</tr>
<tr>
<td>( \Xi^0 )</td>
<td>(-C^2 \left[ \frac{3}{2} M_{\Xi^0}^{\text{tree}} + \frac{2}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right] )</td>
<td>(-C^2 \left[ \frac{5}{6} M_{\Xi^0}^{\text{tree}} + \frac{18}{9} \left( \gamma(n) - \frac{\sigma(n)}{3} \right) \right] )</td>
</tr>
</tbody>
</table>

**Table VII.** Contributions to the matrix elements of \( O_{\mu_1 \mu_2 \ldots \mu_n}^{(n)} \) proportional to \( J_\eta \) in QCD and the matrix elements of \( Q O_{\mu_1 \mu_2 \ldots \mu_n}^{(n)} \) proportional to \( J_{\eta_1} \) in QQCD in the octet baryons.

\[
J_j = \left( m_j^2 - 2\Delta^2 \right) \log \left( \frac{m_j^2}{\mu^2} \right) + 2\Delta \sqrt{\Delta^2 - m_j^2} \log \left( \frac{\Delta - \sqrt{\Delta^2 - m_j^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m_j^2 + i\epsilon}} \right)
\]

\[
H_j = m_j^2 \log \left( \frac{m_j^2}{\mu^2} \right), \quad (37)
\]

where \( \Delta \) is the decuplet-octet mass splitting and \( m_j \) is the mass of the meson, and we have retained only those terms that are non-analytic in \( m_j \) or are required for the correct decoupling in the \( \Delta \rightarrow \infty \) limit. It is clear that for \( \Delta = 0 \), \( J_j = H_j \).

### 2. Hairpin Contributions

There are contributions to some matrix elements from one-loop diagrams involving the hairpin interactions in QQCD. Explicit computation of the diagrams in fig. 4 yields

\[
\langle Q O_{\mu_1 \mu_2 \ldots \mu_n}^{(n)} \rangle_{\text{HP}} = -v_{\mu_1} v_{\mu_2} \ldots v_{\mu_n} \left[ \frac{2C^2}{9} \right] \left[ M_0^2 \left( T_{ss}^{\Delta \Delta} + T_{uu}^{\Delta \Delta} - 2T_{us}^{\Delta \Delta} \right) - \alpha_\phi \left( m_{\eta_\phi}^2 \left( T_{ss}^{\Delta \Delta} - T_{us}^{\Delta \Delta} \right) + m_{\eta_u}^2 \left( T_{us}^{\Delta \Delta} - T_{us}^{\Delta \Delta} \right) \right) \right] , \quad (38)
\]

where \( \alpha_\phi \) and \( M_0 \) are defined in eq. (6) and \( C \) is the axial coupling between the decuplet and octet. The integrals \( T_{ab}^{\Delta \Delta} \) are defined to be [14] \( T_{qq'}^{\Delta \Delta} = \tilde{T}(m_{qq'}, m_{q'q'}, \Delta, \Delta, \mu) \), where

\[
\tilde{T}(m_1, m_2, \Delta_1, \Delta_2, \mu) = \frac{[Y(m_1, \Delta_1, \mu) + Y(m_2, \Delta_2, \mu) - Y(m_1, \Delta_2, \mu) - Y(m_2, \Delta_1, \mu)]}{[\Delta_1 - \Delta_2][m_1^2 - m_2^2]} , \quad (39)
\]

with

\[
Y(m, \Delta, \mu) = \left[ m^2 - \frac{2}{3} \Delta^2 \right] \Delta \log \left( \frac{m^2}{\mu^2} \right) + \frac{2}{3} \left[ \Delta^2 - m^2 \right] \log \left( \frac{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}} \right) . \quad (40)
\]
FIG. 4. One-loop hairpin diagrams with $T_{ijk}$ intermediate states that contribute to the matrix elements of $Q^{[n],3}_{\mu_1 \mu_2 ... \mu_n}$ between octet baryon states. The thick solid line inside the loop denotes a $T_{ijk}$ propagator, while the dashed line denotes an $\eta = \eta_{u,d,s}$ or $\tilde{\eta} = \tilde{\eta}_{u,d,s}$ propagator. The crossed circle denotes an insertion of an operator from eq. (31), arising directly from the twist-2 operator, and the square denotes an insertion of the strong interaction vertex $\propto C$. The cross on the meson propagator denotes an insertion of a hairpin interaction with coefficient $M_0^2$ or $\alpha_\Phi$. Diagram (a) is a vertex correction while diagram (b) denotes wavefunction renormalization.

The Clebsch-Gordan coefficients, $h_i$, appearing in eq. (38) are given in Table VIII. It is important to note that there is no contribution from one-loop hairpin diagrams with $B_{ijk}$ (containing the octet) intermediate states, since the vertex diagrams are exactly canceled by wavefunction renormalization. Further, the hairpin diagrams do not contribute to the matrix elements in the nucleon in the limit of exact isospin symmetry, as the possible intermediate states in $T_{ijk}$ are inaccessible.

<table>
<thead>
<tr>
<th>B</th>
<th>$h_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>0</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$\frac{1}{6} \left( 5\alpha^{(n)} + 2\beta^{(n)} + 4\gamma^{(n)} - \frac{4}{3}\sigma^{(n)} \right)$</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>$\frac{1}{6} \left( \alpha^{(n)} + 4\beta^{(n)} + 2\gamma^{(n)} - \frac{2}{3}\sigma^{(n)} \right)$</td>
</tr>
</tbody>
</table>

TABLE VIII. Contributions to the matrix elements of the isovector twist-2 operators $Q^{[n],3}_{\mu_1 \mu_2 ... \mu_n}$ from one-loop hairpin diagrams involving $T_{ijk}$ intermediate states.

C. Matrix Elements in the Nucleon

Our motivation for undertaking this work is to construct a framework in which experimental data can be compared with the predictions of lattice QCD. As data exists only for the nucleon, it is natural to ask why we bothered with the other members of the baryon octet. As the present values of $m_{latt}^q$ are large, and in fact $m_{latt}^s \sim m_{latt}^u, d$, we consider three light quarks as opposed to just two. Thus the quantities $\alpha^{(n)}, \beta^{(n)}, \gamma^{(n)}$ and $\sigma^{(n)}$ must all be determined, in addition to the strong interaction couplings $F, D$ and $C$ in order to perform the quark mass extrapolation at one-loop order. For a one-loop analysis as we have presented here, the coefficients $\gamma^{(n)}$ and $\sigma^{(n)}$ can be determined at tree-level from calculations of matrix elements in the baryon decuplet, and it is consistent to use these values in the one-loop expression for the octet baryons. Calculation of the proton, $\Sigma^+$ and $\Xi^0$ matrix elements will allow for a determination of $\alpha^{(n)}$ and $\beta^{(n)}$, and hence the $m_q$-dependence of the nucleon matrix element. While the hairpin interactions, $M_0^2$ and $\alpha_\Phi$, do not contribute
directly to the matrix elements in the nucleon, they do contribute to the matrix elements in the other octet baryons, and so will indirectly influence the calculation of the matrix elements in the nucleon.

To get a feel for the $m_q$-dependence of the matrix elements in the proton, we pick some values for the parameters describing the proton matrix element and vary the pion mass. For illustrative purposes, we choose $\gamma^{(n)} = \sigma^{(n)} = 0$, and $\alpha^{(n)} = +2$ and $\beta^{(n)} = +1$ for $n \neq 1$ (this does not eliminate contributions from the $T_{ijk}$ intermediate states, as they contribute in wavefunction renormalization). Also, we use the tree-level QCD values for the axial coupling constants $F = 0.5$, $D = 0.8$ [23] and $C = 1.8$ [24], and the measured value of the decuplet-octet mass splitting, $\Delta$.

![Graph showing the variation of the matrix element of the isovector twist-2 operator in the proton for a specific choice of couplings](image)

**FIG. 5.** The variation of the matrix element of the isovector twist-2 operator in the proton for a specific choice of couplings, $\gamma^{(n)} = \sigma^{(n)} = 0$, $\alpha^{(n)} = +2$, $\beta^{(n)} = +1$ and the tree-level QCD values for the axial couplings, as a function of the pion mass. The kaon mass is fixed to its experimental value, as is the decuplet-octet mass splitting, $\Delta$. The dashed curve corresponds to QCD while the solid curve corresponds to QQCD.

**IV. CONCLUSIONS**

It is important to know the parton distribution functions of the nucleon in QQCD. In order to obtain these predictions from the lattice computations of the foreseeable future, extrapolations in $m_q$ from the lattice values down to the physical values will still be required. We have taken the first step toward establishing the theoretical $m_q$-dependence by computing the leading nonanalytic $m_q$-dependence of the forward matrix element of the isovector twist-2 operators in the octet baryons in QQCD using $Q\chi$PT. It is certainly true that higher order calculations are required, not only to establish the convergence of the expansion about the chiral limit, but also to allow an extrapolation to the relatively large values of the lattice quark masses. Even in QCD, the convergence of $SU(3)$ chiral perturbation theory varies from observable to observable due to the large value of the kaon mass. Having said that, we look forward to a complete analysis of the quenched lattice data, including an extraction of the constants that describe these matrix elements and eventual predictions at the physical value of $m_\pi$. 
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