Magnetic backgrounds and tachyonic instabilities in closed string theory

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Abstract

We consider closed superstrings in Melvin-type magnetic backgrounds. A 2-parameter class of such NS-NS backgrounds are exactly solvable as weakly coupled string models with spectrum containing tachyonic modes. Magnetic field allows one to interpolate between free superstring theories with periodic and antiperiodic boundary conditions for the fermions around some compact direction, and, in particular, between type 0 and type II string theories. Using “9–11” flip, this interpolation can be extended to M-theory and may be used to study the issue of tachyon condensation in type 0 string theory. We review related duality proposals, and, in particular, suggest a description of type 0 theory in terms of M-theory in a curved magnetic flux background in which the type 0 tachyon appears to correspond to a state in $d = 11$ supergravity fluctuation spectrum.


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Magnetic backgrounds play an important role in field theory and open string theory providing simple solvable models with nontrivial physics content. Similar (approximately) constant magnetic backgrounds in gravitational theories like closed string theory are necessarily curved with an example provided by the Melvin-type flux tube solutions (see, e.g., [1]). The role of the magnetic field(s) is played by the vector(s) originating from the metric and/or antisymmetric 2-tensor upon Kaluza-Klein reduction on a circle.

One important special case is the Kaluza-Klein Melvin background which has curved magnetic universe interpretation in 9 dimensions, but, remarkably, is represented by a flat (“twisted”) 10-dimensional space [2]. The non-trivial 3-dimensional part of this space is a twisted product of a K-K circle and a 2-plane: going around the circle must be accompanied by a rotation by angle $2\pi b$ in the plane. This is a continuous version of the Rohm model [3] where the 2-plane was replaced by a 2-torus so that the magnetic (“twist”) parameter was allowed to take only discreet values.

The flatness of the 10-d space allows one to solve the corresponding (super)string theory exactly [4, 5] determining the “Landau-type” spectrum of states which (for large enough magnetic field) contains tachyons in the winding sector. Other NS-NS Melvin-type models (which are no longer flat in 10 dimensions) are also solvable since they may be formally related to the K-K Melvin model by a generalized $O(d, d)$ T-duality transformation [6].

One can also construct similar magnetic backgrounds with R-R magnetic field by applying U-duality transformation, or equivalently, by lifting, e.g., the K-K Melvin space to 11 dimensions and then reducing down to 10 along a different dimension [7, 8]. Such R-R string models seem, however, hard to solve exactly.

One characteristic feature of these magnetic closed string models is that the magnetic field parameter introduces an effective phase for space-time fermions; in particular, switching on magnetic field allows one to interpolate between periodic and antiperiodic boundary conditions for the fermions along a spatial circle [4, 7]. Antiperiodic fermions appear in the context of superstrings at finite temperature [9] and in closely related type 0 string theory viewed as $(-1)^F$ orbifold of type II superstring theory [10, 11].

Using the orbifold interpretation of type 0 string theory and “9-11” flip it was suggested in [12] to interpret type 0 string theory as a similar non-supersymmetric orbifold compactification of 11-d M-theory. It was conjectured [12] that the tachyon of type 0A string in flat space gets $m^2 > 0$ at strong coupling where type 0A string becomes dual to 11-d M-theory on large circle with antiperiodic fermions.

In an interesting paper [13], this proposal was combined with the observation that K-K Melvin magnetic background in type II string theory (and its direct lift to 11 dimensions) may be used to interpolate [4, 7] between periodic and antiperiodic boundary conditions for space-time fermions. Since reducing the flat twisted 11-d space along a different – “mixed circle” – direction gives [2, 7] the R-R 7-brane background, it was suggested that type 0 string in this 10-d R-R Melvin background should be dual to type II theory in the same background but with shifted magnetic field. This implies that type 0 string tachyon should disappear at strong magnetic R-R field [13, 14].

In our recent work [8], which will be reviewed below, we extended the discussion in [7, 13]
to the case of more general class of Melvin-type backgrounds with two independent magnetic
parameters $b$ and $\tilde{b}$ (parametrizing the vector and axial vector 9-d fields originating from $G_9$
and $B_9$ components). This class includes the K-K Melvin ($b \neq 0, \tilde{b} = 0$) and the dilatonic
Melvin ($b = \tilde{b}$) solutions as special cases and is covariant under T-duality. We lifted these
type IIA magnetic solutions to 11 dimensions, getting a $\tilde{b} \neq 0$ generalization of the flat [2]
$d = 11$ background discussed in [7, 13]. Dimensional reduction along different directions (or
U-dualities directly in $d = 10$) led to a number of $d = 10$ supergravity backgrounds with
R-R magnetic fluxes, generalizing to $\tilde{b} \neq 0$ the R-R magnetic flux 7-brane [7, 13] of type IIA
theory.

We proposed the analog of the type II – type 0 duality in [13] now based on a curved
d = 11 background which is a lift of the T-dual $\tilde{b} \neq 0, b = 0$ Melvin background. Remarkably,
this background has manifest 9–11 symmetry, i.e. its two different 10-d reductions are
formally the same NS-NS Melvin spaces.\footnote{Though one corresponds to a weakly-coupled and another to a strongly coupled string theory as the values of the radii $R_9$ and $R_{11}$ are interchanged.} While in the original K-K Melvin set-up the
tachyon originates from a stringy winding mode (i.e. type 0 tachyon may be interpreted as
responding to a wound membrane state) in this T-dual setting the tachyon appears to
be a momentum mode, i.e. the type 0 tachyon appears to correspond to a state in $d = 11$
supergravity multiplet.

One motivation for the study of these magnetic backgrounds and their instabilities is to
get better understanding of closed string tachyons and their stabilization, i.e. of dynamical
interpolations between stable and unstable theories. In particular, one would like to
establish precise relations between non-supersymmetric backgrounds in type II superstring
theory and unstable backgrounds in type 0 theory. Another is to use interpolating magnetic
backgrounds to connect D-brane solutions in the two theories and related gauge theories.
Understanding the fate of closed string tachyon in such non-supersymmetric situations is
crucial, in particular, for the program of constructing string theory duals of non-supersymmetric
gauge theories [15, 16, 17, 18] (see also [19, 20, 21] for some recent related discussions).

2 NS-NS Melvin background in $d = 10$ superstring

Consider the following NS-NS bosonic background in type II string theory:

$$ds^2 = -dt^2 + dx_9^2 + dx_9^2 + dr^2 + \frac{r^2}{1 + \tilde{b}^2 r^2}[d\varphi + (b + \tilde{b})dx_9][d\varphi + (b - \tilde{b})dx_9] ,$$  \hspace{1cm} (1)

$$B_2 = \frac{\tilde{b} r^2}{1 + \tilde{b}^2 r^2} d\varphi \wedge dx_9 , \hspace{1cm} e^{2(\phi - \phi_0)} = \frac{1}{1 + \tilde{b}^2 r^2} .$$  \hspace{1cm} (2)

This is an exact solution of the string theory – the corresponding sigma model is conformal
to all orders in $\alpha'$ [4, 5]. $x_9$ is a periodic coordinate of radius $R = R_9$ and $\varphi$ is the angular
coordinate with period $2\pi$. This model is covariant under T-duality in $x_9$ direction: the
$(R, \tilde{b}, b)$ model is T-dual ($x_9 \rightarrow \tilde{x}_9$) to $(\tilde{R}, b, \tilde{b})$ model with $\tilde{R} \equiv \frac{\alpha'}{R}$. 

\footnote{Though one corresponds to a weakly-coupled and another to a strongly coupled string theory as the values of the radii $R_9$ and $R_{11}$ are interchanged.}
The constants $b$ and $\tilde{b}$ are the magnetic field parameters. The magnetic Melvin-type flux tube interpretation becomes apparent upon dimensional reduction in the $x_9$-direction. The resulting solution of $d = 9$ supergravity

$$
\begin{align*}
    ds_9^2 &= -dt^2 + dx_s^2 + dr^2 + f^{-1}f^{-1}r^2 d\varphi^2, \\
    A_\varphi &= bn^2f^{-1}, \\
    B_\varphi &= -\tilde{b}n^2f^{-1}, \\
    e^{2(\phi - \phi_0)} &= \tilde{f}^{-1}, \\
    e^{2\sigma} &= ff^{-1}, \\
    f &\equiv 1 + b^2r^2, \\
    \tilde{f} &\equiv 1 + \tilde{b}^2r^2,
\end{align*}
$$

(3)

(4)

(5)

describes a magnetic flux tube universe with the magnetic vector $A$ (coming from the metric) and the axial-vector $B$ (coming from the 2-form) and the non-constant dilaton $\phi$ and K-K scalar $\sigma$.

There are two special important cases: (i) $\tilde{b} = 0, b \neq 0$ and (ii) $b = 0, \tilde{b} \neq 0$. The corresponding string backgrounds are T-dual to each other. The first (K-K Melvin model) is represented by the flat 10-d space (but curved magnetic 9-d background)

$$
    ds^2 = -dt^2 + dx_s^2 + dx_9^2 + dr^2 + r^2(d\varphi + b dx_9)^2 .
$$

(6)

The second has non-trivial metric, 3-form and dilaton:

$$
\begin{align*}
    ds_{10}^2 &= -dt^2 + dx_s^2 + dr^2 + \tilde{f}^{-1}(dx_9^2 + r^2 d\varphi^2) . \\
    H_3 &= dB_2 = 2\tilde{b}\tilde{f}^{-2}r dr \wedge d\varphi \wedge dx_9 , \\
    e^{2(\phi - \phi_0)} &= \tilde{f}^{-1}.
\end{align*}
$$

(7)

(8)

They define equivalent conformal field theories, with the two different geometries being “seen”, as in other known T-dual situations, by point-like momentum and winding string modes respectively. ²

For generic values of $b$ and $\tilde{b}$ all of the type II supersymmetries are broken; supersymmetries are restored at the special values $bR = 2n_1$ and $\tilde{b}\tilde{R} = 2n_2$ ($n_i = 0, \pm 1, \ldots$), where the conformal model describes the standard flat-space type II superstring theory.

As follows from (6), a shift of $x_9$ by period of the circle $2\pi R$ implies rotation in the plane by angle $2\pi bR$. Thus in the special case of $bR = 2k + 1$ ($k = 0, \pm 1, \ldots$) the metric becomes topologically trivial. However, the superstring theory in this background is still non-trivial (not equivalent to that of standard flat space) since space-time fermions change sign under $2\pi$ rotation in the plane. That means that the $bR = 1$ case (all models with $bR = 2k + 1$ are equivalent) describes superstrings with antiperiodic boundary condition in $x_9$, just as in the twisted 3-torus model of [3] or in the finite temperature case [9]. Type IIA string in the space (6) with $bR = 1$ is equivalent, for $R \rightarrow 0$, to type 0A string in $R^{1,8} \times S^1_{R\rightarrow0}$ or type 0B string in $R^{1,8} \times S^1_{R\rightarrow\infty}$. The tachyon of type 0 theory originates from a particular winding mode present in the $bR = 1$ type II model spectrum.

The string model corresponding to the above 2-parameter background can be solved in terms of free fields and its spectrum is found to be

$$
\frac{1}{2}\alpha' M^2 \equiv \frac{1}{2}\alpha'(E^2 - p_s^2) = \hat{N}_R + \hat{N}_L + \frac{1}{2}\tilde{R}R^{-1}(m - bR\hat{J})^2
$$

²While the curved 2-parameter background (1) may look much more complicated than the flat one (6), it is the natural T-duality (O(2,2) duality) “completion” of (6). A possible analogy is to think of (6) as a counterpart of a plane wave background; then (7) corresponds to the fundamental string background, and (1) – to a superposition of a fundamental string and a wave.
where $\tilde{N}_R - \tilde{N}_L = mw$, and $\gamma \equiv bRw + \tilde{b}\tilde{R}m - bR\tilde{b}\tilde{R}J$. Here $p_s$ are continuous momenta in the 6 free directions, the integers $m$ and $w$ represent quantized momentum and winding numbers in the compact $x_9$ direction, $\tilde{N}_R$ and $\tilde{N}_L$ are the number of states operators, which have the standard free string theory form in terms of normal-ordered bilinears of bosonic and fermionic oscillators, and $\tilde{J} \equiv \tilde{J}_L + \tilde{J}_R$ and $\tilde{J}_{LR}$ are the angular momentum operators in the 2-plane with the eigenvalues $\tilde{J}_{LR} = \pm (l_{LR} + \frac{1}{2}) + S_{LR}$, $\tilde{J} = l_L - l_R + S_L + S_R$. The orbital momenta $l_{LR} = 0, 1, 2, \ldots$ (which replace the continuous linear momenta $p_1, p_2$ in the $(r, \varphi)$ 2-plane for non-zero values of $\gamma$) are the analogues of the orbital quantum number $l$ and the radial quantum number $k$ in the Landau problem, and $S_{R,L}$ are the spin components.

The spectrum is thus similar to the one for charge particles in constant magnetic field orthogonal to the plane of motion (with $Q_{L,R} = mR^{-1} \pm w\tilde{R}^{-1}$ playing the role of charges of string states in 9-dimensional description). In contrast to the open string case, here not only the masses and spins but also charges of string states can take arbitrarily large values. The terms in $M^2$ that are linear in angular momenta reflect gyromagnetic interaction, while the terms quadratic in $J$'s represent the effect of gravitational back reaction of the magnetic field.

Since $\tilde{J}$ can take both integer (in NS-NS, R-R sectors) and half-integer (in NS-R, R-NS sectors) values, the symmetry of the bosonic part of the spectrum $b \to b + k_1R^{-1}$, $\tilde{b} \to \tilde{b} + k_2\tilde{R}^{-1}$ is not a symmetry of its fermionic part, i.e. the full superstring spectrum is invariant under the shifts with even integer $k_i = 2n_i$ only. In the case of $bR = 2n_1$, $\tilde{b}\tilde{R} = 2n_2$ (i.e. $\gamma =$even integer, $\tilde{\gamma} = 0$) the spectrum is thus equivalent to that of the standard free superstring compactified on a circle. In the two cases (a) $bR = \text{odd}$, $\tilde{b}\tilde{R} = \text{even}$ or (b) $bR = \text{even}$, $\tilde{b}\tilde{R} = \text{odd}$ the spectrum is the same as that of the free superstring compactified on a circle with antiperiodic boundary conditions for space-time fermions.

The T-duality symmetry in the compact Kaluza-Klein direction $x_9$ is manifest in the spectrum: $M^2$ is invariant under $R \leftrightarrow \tilde{R} \equiv \alpha'R^{-1}$, $b \leftrightarrow \tilde{b}$, $m \leftrightarrow w$. Thus the two T-dual models $b = 0$ one and $\tilde{b} = 0$ one have equivalent spectra.

The only states which may become tachyonic are bosonic states that lie on the first Regge trajectory with maximal value of $S_R$, minimal value of $S_L$, and zero orbital momentum, i.e. $\tilde{J}_R = S_R - \frac{1}{2} = \tilde{N}_R + \frac{1}{2}$, $\tilde{J}_L = S_L + \frac{1}{2} = -\tilde{N}_L - \frac{1}{2}$, so that $\tilde{J}_R - \tilde{J}_L = \tilde{N}_R + \tilde{N}_L + 1$. For general $b$ and $\tilde{b}$, there are instabilities (associated with states with high spin and charge) for arbitrarily small values of the magnetic field parameters. The only exception is the $\tilde{b} = 0$ model and equivalent T-dual $b = 0$ model. For $\tilde{b} = 0$ model the type II superstring has no tachyons if the value of $b$ is smaller than some finite critical value $b_{\text{cr}}$. The spectrum of the $\tilde{b} = 0$ model ($\gamma = bRw$) has the first (lowest mass) potentially tachyonic state that appears as $b$ is increased from zero for $\tilde{J} = 0$, $\tilde{J}_R - \tilde{J}_L = 1$: it has $\tilde{N}_R = \tilde{N}_L = 0$, $m = 0$, $w = 1$, $l_R = l_L = 0$, $S_R = -S_L = 1$, and thus

$$\alpha'M^2 = \frac{R^2}{\alpha'} - 2bR \equiv 2R(b_{\text{cr}} - b), \quad b_{\text{cr}} = \frac{R}{2\alpha'},$$

i.e. we get tachyon in the winding sector when $b > b_{\text{cr}}$. In the T-dual model with $b = 0$, $\tilde{b} \neq 0$, which has the equivalent spectrum, the first tachyon appears in momentum sector for
\[ \tilde{b} > \tilde{b}_{cr} = \tilde{R} \frac{2}{2\alpha} = \frac{1}{2\tilde{R}}. \]

3 Related \( d = 11 \) backgrounds and U-dual \( d = 10 \) R-R flux branes

Lifting the type IIA background (1) to \( d = 11 \) one gets the following “6-brane” (or “8-brane” with “defects” along \( x_9, x_{11} \)) solution of \( d = 11 \) supergravity [8]

\[ ds_{11}^2 = \tilde{f}^{1/3}(-dt^2 + dx_s^2) + \tilde{f}^{-2/3}[dx_{11}^2 + \tilde{f}dr^2 + r^2f^{-1}d\varphi^2 + f(dx_9 + br^2f^{-1}d\varphi)^2], \quad (10) \]

\[ C_3 = \tilde{b}r^2\tilde{f}^{-1}dx_9 \wedge dx_{11} \wedge d\varphi, \quad f = 1 + b^2r^2, \quad \tilde{f} = 1 + \tilde{b}^2r^2. \]

This background is regular, with curvature being constant near the core \( r = 0 \) and going to zero at large \( r \).

The two special cases are \( \tilde{b} = 0 \) when the \( d = 11 \) space becomes flat (this is the direct \( d = 11 \) lift of (6))

\[ ds_{11}^2 = -dt^2 + dx_s^2 + dx_9^2 + dr^2 + r^2(d\varphi + b dx_9)^2 + dx_{11}^2, \quad C_3 = 0, \quad (11) \]

and \( b = 0 \) when the metric and \( C_3 \) are still non-trivial

\[ ds_{11}^2 = \tilde{f}^{1/3}(-dt^2 + dx_s^2) + \tilde{f}^{-2/3}(\tilde{f}dr^2 + r^2d\varphi^2 + dx_9^2 + dx_{11}^2). \quad (12) \]

\[ C_3 = \tilde{b}r^2\tilde{f}^{-1}dx_9 \wedge dx_{11} \wedge d\varphi. \]

Remarkably, this metric and \( |C_3| \) turn out to be symmetric under the “9-11” flip.

The reduction of (10) along \( x_{11} \) gives back (1), while the reduction along \( x_9 \) leads to the following “mixed” (NS-NS/R-R) type IIA background related to (1) by a U-duality (\( T_{x_9 ST x_9} \) duality transformation)

\[ ds_{10}^2 = f^{1/2}(-dt^2 + dx_s^2 + \tilde{f}^{-1}dx_9^2 + dr^2 + f^{-1}\tilde{f}^{-1}r^2d\varphi^2), \quad (13) \]

\[ A = br^2\tilde{f}^{-1}d\varphi, \quad B_2 = \tilde{b}r^2\tilde{f}^{-1}dx_9, \quad e^{2(\tilde{\phi} - \phi_0)} = f^{3/2}\tilde{f}^{-1}. \]

Here we renamed \( x_{11} \rightarrow x_9 \). The two parameters \( b \) and \( \tilde{b} \) now control the strengths of the magnetic 1-form R-R field \( A \) and of the NS-NS 2-form field \( B_2 \). This solution may be interpreted as a R-R magnetic flux 7-brane with a “defect” along \( x_9 \).

In the special case of \( b = 0 \) we get the R-R flux 7-brane background [7] which is the reduction of the flat \( d = 11 \) background (11) along the other - \( x_9 \) - direction (and thus is U-dual of the flat K-K Melvin space (6) which is the trivial reduction along \( x_{11} \))

\[ ds_{10}^2 = f^{1/2}(-dt^2 + dx_s^2 + dx_9^2 + dr^2 + r^2f^{-1}d\varphi^2), \quad (14) \]

\[ A_\varphi = br^2f^{-1}, \quad e^{2(\tilde{\phi} - \phi_0)} = f^{3/2}. \]

\(^3\)Note that while (1) had magnetic interpretation in 9 dimensions, here one got a magnetic R-R vector directly in 10 dimensions.
Like in the NS-NS Melvin case (4), the strength of the the R-R 1-form is approximately constant near the core, decaying away from the core. However, in contrast to (2) here the dilaton grows away from the core, so the theory is weakly-coupled only at small $r \ll b^{-1}$ [7, 13].

In the other special case $b = 0$ the $9 - 11$ symmetry of (12) implies that here the two “U-dual” reductions are formally equivalent, leading to the same (pure NS-NS) background (7) (with the dilaton decreasing away from the core).

4 Perturbative and non-perturbative type 0 – type II relations

The above considerations suggest [13, 8] non-perturbative dualities relations between type II and type 0 strings in magnetic backgrounds. Let us first review the well-known perturbative flat space orbifold relation between these two string theories.

Type 0 closed string theory [10] which is a “symmetric” $(-1)^{F_L + F_R}$ modular-invariant projection of the NSR string may be interpreted also as $(-1)^{F_S}$ ($F_S$ is space-time fermion number) orbifold of type II superstring theory. The orbifolding projects out all space-time fermions and introduces extra twisted sector states (in particular, the tachyon and another copy of massless R-R bosons). This relation between type II and type 0 string theories may be expressed by saying that type 0 string is (a limit of) type II string compactified on a circle with antiperiodic boundary conditions for space-time fermions [9]. More precisely, the two theories are the limits of the same interpolating “9-dimensional” string theory [11] – $\Sigma^R$ orbifold of type II theory. $\Sigma^R$ stands for $(S^1)_R/[(−1)^{F_S} \times S]$, where $S$ is half-shift along the circle ($X_9 \rightarrow X_9 + \pi R$). Type IIA on $\Sigma^R \rightarrow \infty$ is type IIA theory in flat $d = 10$ and type IIA on $\Sigma^R \rightarrow 0$ is type 0A theory on $(S^1)_{R \rightarrow 0}$ (or T-dual type 0B theory on $(S^1)_{R \rightarrow \infty}$). Thus the $\Sigma^R$ orbifold type IIA theory (which has massive fermions in its spectrum) continuously interpolates between supersymmetric type IIA and non-supersymmetric type 0B ten-dimensional theories [11].

It is possible to replace the orbifolding procedure by the K-K Melvin background with $b R = 1$. Indeed, using that type II superstring in the $d = 10$ K-K Melvin NS-NS background (6) (with $x_9 \equiv x_9 + 2 \pi R$) is, for $b R = 1$, equivalent to type II theory on $R^9 \times (S^1)_R$ with antiperiodic boundary conditions for the space-time fermions along the $x_9$ circle [4], one may notice [13] that it is equivalent to the above orbifold of type II on $\Sigma^R$ with the radius $R' = 2 R$ (extra 2 is related to the shift $S$). This NS-NS Melvin model thus describes,

4 The role of $S$ is to mix $(-1)^{F_S}$ orbifold with compactification on the circle, allowing for the interpolation [11]. Invariant states under $S$ have even momentum quantum numbers $m$ and integer winding numbers $w$, while twisted sector states have odd $m$ and half-integer $w$. The action of $S$ is irrelevant for $R \rightarrow \infty$, but is crucial for reproducing type 0 spectrum for $R \rightarrow 0$.

5 This model at $b R = 1$ is essentially the same as a limit ($T^2 \rightarrow R^2$) of the twisted 3-torus $T^2 \times S^1$ model of [3] or a Wick rotation of the finite temperature superstring theory [9], and is closely related to Scherk-Schwarz compactification in string theory [22] (see also [23]).

6 In the Melvin model with $b R = 1$ the shift $x_9 \rightarrow x_9 + 2 \pi R$ is equivalent to $2 \pi$ rotation in the 2-plane under which space-time fermions change sign. In the orbifold, the corresponding shift $S$ is by $\pi R'$. 
in the limit $R \to 0$, the weakly coupled type 0 string on $R^{1,8} \times (S^1)_{R \to 0}$ (or type 0B on $R^{1,8} \times (S^1)_{R \to \infty}$).

To summarize, the type II orbifold on $\Sigma_{2R}$ is the same as type II theory in Melvin background at $bR = 1$ and thus the latter model interpolates between type II and type 0 theories in infinite $d = 10$ space. In general [13], type IIA(B) theory in the Kaluza-Klein NS-NS Melvin background with parameters $(b, R)$ is equivalent (has the same perturbative spectrum and thus the same torus partition function) to the orbifold of type IIA(B) on $\Sigma_{R'}$ in this NS-NS Melvin background with parameters $(b' = b - R^{-1}, R' = 2R)$. Since the K-K Melvin model (6) has the same spectrum as the T-dual model (7), a similar statement is true also for the type II and type 0 strings in this dual curved background [8].

Returning now back to the case of the trivial flat background, the above compactification of type IIA theory on $\Sigma_R \equiv \Sigma_{R_0}$ corresponds to M-theory on $(S^1) R_{11}' \times (S^1) R_0 / [(-1)^{F_2} \times S]$, where $R_0$ and $R_{11}'$ are taken to be small. Making the “9-11” flip (i.e. exchanging the roles of the two circles, assuming that there is indeed an interpolating coordinate-invariant 11-d M-theory) one may then conjecture [12] that one should get an equivalent description of this theory as an ordinary (i.e. with periodic fermions) $S^1$ compactification of the $d = 10$ theory obtained from M-theory on $\Sigma_{R_{11}'}$ (in the limit $R_{11}' \to 0$). Then type 0A theory may be interpreted as M-theory on $\Sigma_{R_{11}'}$, or, essentially, as M-theory on $S^1$ with radius $\frac{1}{2} R_{11}'$ with periodic boundary conditions for the bosons and antiperiodic for the fermions.\(^7\)

Lifting the flat K-K Melvin metric (6) to 11 dimensions, replacing $x_9 \leftrightarrow x_{11}$ in (11), and reducing it down to 10 dimensions along $x_{11}$ gives a non-supersymmetric R-R Melvin 7-brane in type IIA theory (14). Combining this fact with the above observations, it was suggested in [13] that type IIA and type 0A theories in the $d = 10$ R-R Melvin flux 7-brane background (14)\(^8\) are non-perturbatively dual to each other, with parameters related by $b_0 = b - R_{11}^{-1}$, $g_{s0} = \frac{R_{11}'}{2\sqrt{\alpha'}} = \frac{R_{11}^{-1}}{\sqrt{\alpha}}$. For example, for $b R_{11} = 1$ the $d = 11$ Melvin theory has antiperiodic fermions on the circle $R_{11}$, while, according to [12], type 0 theory is M-theory with antiperiodic fermions on circle $\frac{1}{2} R_{11}'$. That means, in particular, that starting with type 0 theory and increasing the value of the R-R magnetic parameter $b_0$, the tachyon should disappear at strong enough magnetic field [13] when the theory becomes strongly coupled – its weakly-coupled description is in terms of stable weakly coupled type II theory in R-R Melvin background with small $b$.\(^9\)

Replacing the starting point of the above discussion – K-K Melvin space (6) (and thus also its $d = 11$ counterpart (11)) by the perturbatively equivalent T-dual background (7) (with the associated $d = 11$ background (12)) we are able to give an alternative formulation [8] of the type II – type 0 duality conjecture. Indeed, there are two magnetic type II models which are equivalent to type II superstring theory in flat space with fermions obeying antiperiodic

\(^7\)The factor of $1/2$ is again due to the half-shift $S$. It suggests that the same factor should be present in the relation between string coupling constants. However, it is not clear how to decide this unambiguously since the relation to perturbative type 0 theory based on 9-11 flip applies only in the limit of zero radius, i.e. at zero coupling.

\(^8\)Any bosonic solution of type II supergravity can be embedded into type 0 theory provided the fields of the twisted sector (tachyon and second set of R-R bosons) are set equal to zero [16].

\(^9\)As explained in detail in [8], one indication that, as in the weakly-coupled $AdS_5 \times S^5$ example in [16], the R-R flux may shift the value of the tachyon mass follows from the presence of a non-minimal $T^2 F_{mn}^2$ coupling [16] of the tachyon to the R-R background.
boundary conditions in the $x_9$ direction: (a) the model with $\tilde{b} = 0$, $b_R = 1$; (b) the model with $b R = 1$, $b = 0$. They are equivalent (T-dual) as weakly-coupled type II superstring theories compactified on a circle. T-duality equivalence is believed to hold also for finite coupling, so one expects that M-theory compactified along $x_9$, $x_{11}$ in the background (11) with $b R = 1$ and in (12) with $\tilde{b} R = 1$ should be equivalent not only in the weak-coupling limit $R_{11} \to 0$ but also for finite $R_{11}$. Thus it is natural to conjecture that M-theory in the background (12) with $\tilde{b} R = 1$ compactified on the $(x_9, x_{11})$ torus with $(-, +)$ (periodic) boundary conditions for the fermions is equivalent to M-theory in flat space compactified on the 2-torus with the $(+, +)$ fermionic boundary conditions. One may thus propose the following description of type 0A string theory: as M-theory in the background (12) with periodic $x_9$ and $\tilde{b} R = 1$. Because of the 9-11 symmetry of (12) one may consider also the equivalent model with $\tilde{b} R = 1$.

While the reduction of (11) to 10 dimensions gives string theory in R-R F7 background, the reduction of (12) gives apparently much simpler string theory in the NS-NS background (7) the spectrum of which (in the weakly-coupled regime) we know. However, the 9-11 flip necessary to relate the type II and type 0 theories implies that, e.g., a weakly coupled ($R_{11} \ll R_9$) type II theory is mapped into a strongly coupled ($R_9 \ll R_{11}$) type 0 theory or vice versa. That precludes one from drawing immediate conclusions about the presence or absence of tachyons in type 0 theory in the NS-NS Melvin background directly from the known weakly coupled string spectrum.\footnote{In particular, the weakly coupled type 0 theory in the NS-NS Melvin background is certainly unstable for any value of magnetic field parameter (because of its own tachyon present already in flat space) but it is expected to become stable at strong coupling and critical magnetic field since it should then become equivalent to a weakly coupled type II string in NS-NS background with a small magnetic field parameter.}

According to the proposal of [13], type IIA and type 0A theories in the $\tilde{b} = 0$ R-R Melvin background (14) are equivalent, being related by a shift of the R-R field strength parameter: $b_0 = b - R_{11}^{-1}$. More generally, the discussion in [8] implies (assuming again the validity of the 9-11 flip) the following generalization of this relation to the case of the 2-parameter magnetic Melvin model ($R = R_{11}$): Type IIA in $(b, \tilde{b})$ Melvin = Type 0A in $(b - R^{-1}, \tilde{b})$ Melvin = Type 0A in $(b, \tilde{b} - \tilde{R}^{-1})$ Melvin.

It was conjectured in [12] that the type 0A tachyon originates from a massive mode (in twisted sector of Σ-orbifold) in microscopic M-theory, i.e. a mode which is absent in the $d = 11$ supergravity spectrum. The magnetic instability (related to type 0 tachyon) appears in different ways in the two models. In the first case the tachyon is associated with a winding string mode (or winding membrane state in M-theory, see [8]). In the second M-theory description of type 0A theory – as M-theory in the background (12) at the critical magnetic field $b R = 1$ the type 0A tachyon corresponds to an unstable mode in the momentum part of the spectrum which may be thus seen directly in $d = 10$ or $d = 11$ supergravity spectra in the corresponding curved backgrounds (7) and (12).\footnote{While K-K Melvin background is stable as a supergravity solution (the string instability is associated with a winding mode), the curved T-dual background (7) is unstable already at the supergravity level.} This setting thus allows one to interpret the type 0A tachyon as a particular fluctuation mode of $d = 11$ supergravity expanded near (12). The solution of the relevant Laplace operator has the following structure [8]

$$M^2 = p_9^2 + p_{11}^2 + \tilde{b}^2 J^2 + 2\tilde{b}\sqrt{p_9^2 + p_{11}^2}(l_L + l_R + 1 - S_R + S_L),$$

(15)
where $J = l_L - l_R + S_L + S_R$ and $p_9 = m/R_9$, $p_{11} = n/R_{11}$ ($R_9 > R_{11}$). The tachyon appears for $l_L = l_R = 0$, $S_R = -S_L = 1$, $m = \pm 1$, $n = 0$. The corresponding tachyonic mode in the “dual” description of type 0 theory as M-theory on the flat background (11) may be interpreted as a particular wound membrane state [8]

$$M^2 = (4\pi^2 w R_9 R_{11} T_2)^2 + b^2 J^2 + 8\pi^2 bw R_9 R_{11} T_2 (l_L + l_R + 1 - S_R + S_L),$$

(16)

where $T_2 = (4\pi^2 R_{11} \alpha')^{-1}$. The first term represents the mass of a membrane of tension $T_2$ wrapped on a 2-torus (wound $w$ times around $x_9$ and once around $x_{11}$). The spectrum is similar to (15), as expected from the T-duality relation between the corresponding ten-dimensional models. The counterpart of the first two terms in (15) is now a winding term; the gyromagnetic interaction in (15) is traded for an analogous gyromagnetic term where the charge is now the winding number $w$. In this way we determine part of the spectrum for the $d = 11$ flat model (11) which is relevant for the study of instabilities. The winding membrane state that becomes tachyonic at sufficiently large $b$ has quantum numbers $S_R = 1 = -S_L$, $l_L = l_R = 0$.

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**References**


