Unified Supersymmetric Model of Naturally Small Dirac Neutrino Masses and the Axionic Solution of the Strong CP Problem

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Abstract

Using the particle content of the fundamental $27$ supermultiplet of $E_6$, naturally small Dirac neutrino masses are obtained in the context of $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$, where $U(1)_\chi$ comes from the decomposition $E_6 \rightarrow SO(10) \times U(1)_\psi$, then $SO(10) \rightarrow SU(5) \times U(1)_\chi$. New observable consequences are predicted at the TeV scale. An axionic solution of the strong CP problem may be included at no extra cost.
With the present experimental evidence [1, 2, 3] on neutrino oscillations, the notion that neutrinos should be massive is no longer in dispute. The next question is whether neutrino masses are Majorana or Dirac. Experimentally, the nonobservation of neutrinoless double beta decay at the 0.2 eV level [4] is unable to settle this issue, but there are very strong and convincing theoretical reasons to believe that neutrino masses should be Majorana. On the other hand, if the theoretical context is changed, naturally small Dirac neutrino masses are possible, as shown below.

To obtain a Dirac mass, the left-handed neutrino $\nu_L$ must be paired with a right-handed singlet $N_R$. Two problems arise immediately. (i) There is no symmetry to prevent $N_R$ from acquiring a large Majorana mass. (ii) Even if such a symmetry (such as additive lepton number) is imposed, an extremely small Yukawa coupling (less than $10^{-11}$) is still needed to satisfy the experimental bound $m_\nu < \text{a few eV}$. The usual resolution of these problems is to take advantage of (i) to make $m_N$ very large, so that the famous canonical seesaw mechanism [5] makes $m_\nu = m_D^2/m_N$. Now (ii) is also not a problem because the Yukawa coupling for the Dirac mass $m_D$ is no longer required to be very small.

In this paper a new scenario is proposed where (i) $N_R$ is naturally prevented from having a Majorana mass and (ii) $m_D$ is small without having a small Yukawa coupling [6, 7]. This is possible because the theoretical framework used will be that of superstring-inspired $E_6$ [8]. As a bonus, the axionic solution [9] of the strong CP problem may also be included.

The starting point is the gauge group $E_6$ and its decomposition $E_6 \rightarrow SO(10) \times U(1)_\psi$, then $SO(10) \rightarrow SU(5) \times U(1)_\chi$. It is often assumed that at TeV energies, a linear combination of $U(1)_\psi$ and $U(1)_\chi$ remains [10] in addition to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$. It is usually also assumed that three complete matter supermultiplets of the fundamental 27 representation of $E_6$ are present at these energies, which include the known three families of quarks and leptons as well as other new particles. Under the subgroup $SU(5) \times U(1)_\psi \times U(1)_\chi$,
the particle content of each supermultiplet is given by

\[ 27 = (10; 1, -1)[(u, d), u^c, e^c] + (5^*; 1, 3)[d^c, (\nu_e, e)] + (1; 1, -5)[N^c] \]
\[ + (5; -2, 2)[h, (E^c, N^c_E)] + (5^*; -2, -2)[h^c, (\nu_E, E)] + (1; 4, 0)[S], \]

where the U(1) charges refer to \(2\sqrt{6}Q_\psi\) and \(2\sqrt{10}Q_\chi\). Note that the known quarks and leptons are contained in \((10; 1, -1)\) and \((5^*; 1, 3)\), and the two Higgs scalar doublets are represented by \((\nu_E, E)\) and \((E^c, N^c_E)\). Since \(N^c\) and \(S\) are singlets under \(SU(5)\), one linear combination will be trivial under the assumed low-energy gauge group, i.e. \(SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha\) with \(Q_\alpha = Q_\psi \cos \alpha + Q_\chi \sin \alpha\). For the choice \(\tan \alpha = \sqrt{1/15}\), the \(U(1)_N\) model [11, 12] is obtained, for which \(N^c\) is trivial, thus allowing it to acquire a large Majorana mass. Combining this with the existing term \((\nu_e N^c_E - e E^c) N^c\), the usual seesaw Majorana neutrino mass may then be obtained.

Consider now the case \(\sin \alpha = 1\), i.e. the \(U(1)_\chi\) model. This allows \(S\) to have a large Majorana mass, but not \(N^c\). Hence the only apparent way that \(\nu_e\) may become massive is to pair up with \(N^c\) to form a Dirac neutrino with mass proportional to the vacuum expectation value (VEV) of the scalar component of \(N^c_E\). If the latter is of the order of the electroweak symmetry breaking scale, i.e. \(10^2\) GeV, then an extremely small Yukawa coupling is required. This is in fact the prevailing working ansatz of all \(U(1)_\alpha\) models except \(U(1)_N\). However, there is a very simple and natural solution. If \(\tilde{N}^c_E\) has \(m^2 > 0\) with \(m\) large, then its VEV can be very small [6, 7]. This is precisely the case in the \(U(1)_\chi\) model, where \(\nu_E N^c_E - E E^c\) is an allowed term.

There are 11 generic terms [13] in the superpotential of such \(E_\alpha\) models. They are

\[(1) \quad \hat{Q} \hat{u}^c \hat{E} = (\hat{u} \hat{N}^c_E - \hat{d} \hat{E}^c) \hat{u}^c, \]
\[(2) \quad \hat{Q} \hat{d} \hat{E} = (\hat{u} \hat{E} - \hat{d} \hat{\nu}_E) \hat{d}^c, \]
\[(3) \quad \hat{L} \hat{e}^c \hat{E} = (\hat{\nu}_e \hat{E} - \hat{e} \hat{\nu}_E) \hat{e}^c, \]
To prevent rapid proton decay, some terms must be absent. This is usually accomplished by
the imposition of an exactly conserved discrete symmetry, such as the well-known $R$ parity.
Here the choice is $Z_3 \times U(1)_{PQ}$. (13)

Under $Z_3$ with $\omega^3 = 1$, $\hat{E}_{1,2}, \hat{E}_{1,2}$ transform as $\omega; \hat{u}^c, \hat{d}^c, \hat{e}^c$ as $\omega^2$; and all other superfields as 1. Under $U(1)_{PQ}$, the only superfields with nonzero charges are $\hat{h}, \hat{h}^c, \hat{S}_1, \hat{S}_2, \hat{S}_3$ with charges $1/2, 1/2, -1, 2, -2$ respectively. This means that the terms (7) to (11) are all forbidden, the
term (6) involves only $\hat{E}_3$, the term (5) involves only $\hat{S}_1$, and the term (4) is forbidden, but
since $\hat{S}$ is trivial under $U(1)_\chi$, the soft term

$$\hat{E}\hat{E} = \hat{\nu}_e\hat{N}_E - \hat{E}\hat{E}^c$$

by itself is allowed. Note that only one term, i.e. $\hat{E}_3\hat{E}_3$, is invariant under $Z_3$. All other $\hat{E}\hat{E}$
terms will break $Z_3$ but only softly.

The superpotential of this model is then given by

$$\hat{W} = \mu_{ij}\hat{E}_i\hat{E}_j + f^{(u)}_{(1,2)ij}\hat{Q}_i\hat{u}^c_j\hat{E}_{1,2} + f^{(d)}_{(1,2)ij}\hat{Q}_i\hat{d}^c_j\hat{E}_{1,2} + f^{(e)}_{(1,2)ij}\hat{L}_i\hat{e}^c_j\hat{E}_{1,2}$$

$$+ f^{(N)}_{ij}\hat{L}_i\hat{N}_j^c\hat{E}_3 + f^{(h)}_{ij}\hat{S}_i\hat{h}_j\hat{h}^c + m_2\hat{S}_2\hat{S}_3 + f_2\hat{S}_2\hat{S}_1\hat{S}_1.$$ (15)
The anomalous global $U(1)_{PQ}$ is spontaneously broken at the high scale $m_2$ so that an “invisible” axion will emerge to solve the strong CP problem. The $U(1)_{PQ}$ charges of $\hat{S}_{1,2,3}$ are chosen so that $S_1$ may acquire a large VEV ($\sim m_2 > 10^9$ GeV) without breaking the supersymmetry of the entire theory at that scale. Details are contained in Ref.[14]. Because the usual quarks and leptons here do not transform under $U(1)_{PQ}$, the axion of this model is of the KSVZ type [15], whereas that of Ref.[14] is of the DFSZ type [16].

Note that $U(1)_{PQ}$ here serves the dual purpose of solving the problem of rapid proton decay as well. Note also that the choice of $U(1)_\chi$ as the extra gauge symmetry is the only one which allows that to work. It also serves the purpose of allowing the term $\hat{E}\hat{\bar{E}}$ and the choice of $Z_3$ allows only $\hat{E}_3$ to couple to $\hat{N}^c$, with a large mass for $\hat{E}_3\hat{E}_3$. The Dirac mass linking $\nu_e$ to $N^c$ is proportional to the VEV of the scalar component of $\hat{E}_3$, which may then be very small [6, 7], as shown below.

Consider the following Higgs potential of 4 scalar doublets $H_{1,2,3,4}$ representing the scalar components of $\hat{E}_1, \hat{E}_1, \hat{E}_3, \hat{E}_3$ respectively [17] (assuming that $\hat{E}_2$ and $\hat{\bar{E}}_2$ have no VEV):

$$V = \sum_i m_i^2 H_i^\dagger H_i + [m_{12}^2 H_1 H_2 + m_{14}^2 H_1 H_4 + m_{32}^2 H_3 H_2 + m_{34}^2 H_3 H_4 + h.c.]$$

$$+ \frac{1}{2} \left( \frac{g_1^2}{4} + \frac{g_2^2}{10} \right) [-H_1^\dagger H_1 + H_2^\dagger H_2 - H_3^\dagger H_3 + H_4^\dagger H_4]^2 + \frac{1}{2} g_2^2 \sum_\alpha \sum_i H_i^\dagger \tau_\alpha H_i |^2, \quad (16)$$

where $\tau_\alpha (\alpha = 1, 2, 3)$ are the usual SU(2) representation matrices. Let the VEV’s of $H_i$ be $v_i$, then the minimum of $V$ is

$$V_{\text{min}} = \sum_i m_i^2 v_i^2 + 2m_{12}^2 v_1 v_2 + 2m_{14}^2 v_1 v_4 + 2m_{32}^2 v_2 v_3 + 2m_{34}^2 v_3 v_4$$

$$+ \frac{1}{8} \left( g_1^2 + g_2^2 + \frac{2g_2^2}{5} \right) (v_1^2 - v_2^2 + v_3^2 - v_4^2)^2, \quad (17)$$

where all parameters have been assumed real for simplicity. The 4 equations of constraint are

$$0 = m_1^2 v_1 + m_{12}^2 v_2 + m_{14}^2 v_4 + \frac{1}{4} \left( g_1^2 + g_2^2 + \frac{2g_2^2}{5} \right) v_1(v_1^2 - v_2^2 + v_3^2 - v_4^2), \quad (18)$$
0 = m_2^2 v_2 + m_{12}^2 v_1 + m_{32}^2 v_3 - \frac{1}{4} \left( g_1^2 + g_2^2 + \frac{2g_\chi^2}{5} \right) v_2 (v_1^2 - v_2^2 + v_3^2 - v_4^2), \quad (19)

0 = m_3^2 v_3 + m_{32}^2 v_2 + m_{34}^2 v_4 - \frac{1}{4} \left( g_1^2 + g_2^2 + \frac{2g_\chi^2}{5} \right) v_3 (v_1^2 - v_2^2 + v_3^2 - v_4^2), \quad (20)

0 = m_4^2 v_4 + m_{14}^2 v_1 + m_{34}^2 v_3 - \frac{1}{4} \left( g_1^2 + g_2^2 + \frac{2g_\chi^2}{5} \right) v_4 (v_1^2 - v_2^2 + v_3^2 - v_4^2). \quad (21)

Since m_3^2 and m_4^2 are the only parameters which have contributions from the square of the large mass of \( \hat{E}_3 \hat{\tilde{E}}_3 \), i.e. \( \mu_{33} \), it is clear that Eqs. (20) and (21) have the solution

\[ v_3 \simeq -\frac{m_{32}^2 v_2}{m_3^2}, \quad v_4 \simeq -\frac{m_{14}^2 v_1}{m_4^2}. \quad (22) \]

They may then be of order 0.1 eV if \( m_{3,4} \sim \mu_{33} \sim 10^9 \text{ GeV} \) [as suggested by the fact that the axion scale \( m_2 \) and \( \mu_{33} \) are the only two allowed large masses in Eq. (15) and should be similar in magnitude] and \( m_{32}^2, m_{14}^2 \sim M_{SUSY}^2 \sim (\text{TeV})^2 \). Setting \( v_3 = v_4 = 0 \) in Eqs. (18) and (19), the usual conditions of the minimal supersymmetric standard model are obtained except for the additional terms due to \( g_\chi \).

Now \( U(1)_\chi \) also undergoes spontaneous symmetry breaking through the VEV of one linear combination of the 3 \( \tilde{N}^c \)'s. As a result, there appear a new massive neutral gauge boson \( Z' \), the corresponding scalar boson \( \sqrt{2} Re\tilde{N}^c \), and the Dirac fermion which comes from the pairing of \( \tilde{z}' \) and \( N^c \), all having the mass \( (\sqrt{5}/2)g_\chi \langle \tilde{N}^c \rangle \) [18]. Hence only 2 \( (N^c)'s \) remain and they combine with 2 of the 3 \( \nu \)'s to form 2 light Dirac neutrinos. The remaining \( \nu \) gets a negligible Majorana mass from the allowed supersymmetry-breaking soft Majorana mass of \( \tilde{z}' \). A satisfactory framework is thus established for describing the oscillations of 2 light Dirac neutrinos and 1 essentially massless Majorana neutrino.

At the TeV energy scale, this model is verifiable experimentally by its many unique predictions. First, there must be a \( Z' \) gauge boson with couplings to quarks and leptons according to Eq. (1). In particular, it will have invisible decays to neutrinos given by

\[ \frac{\Gamma(Z' \rightarrow \bar{\nu}\nu + \tilde{N}^c N^c)}{\Gamma(Z' \rightarrow l^+ l^-)} = \frac{77}{30}. \quad (23) \]
There are likely to be 4 Higgs doublets, instead of 2, and definitely not 6. There should not be exotic quarks (i.e. \( h \) and \( h^c \)) because they are predicted to be very heavy with masses at the axion scale. The axion itself is of course very light and very difficult to detect [19]. Its partners, the saxion and the axino, are likely to be at or below the TeV scale and may also be components of the dark matter of the Universe. Lepton number is violated through \( \langle \tilde{N}^c \rangle \), but since \( \tilde{N}^c \) only appears in Eq. (15) with the very heavy \( \tilde{E}_3 \), this violation is highly suppressed. Thus my proposed model evades the general conclusion of Ref.[12] regarding \( E_6 \) subgroups that only \( U(1)_N \) [11] and the skew left-right model [20] do not have lepton-number violating interactions at the TeV scale which would erase any preexisting lepton or baryon asymmetry of the Universe.

In conclusion, a new unified supersymmetric model has been proposed which has the following desirable properties.

(1) Its particle content comes from 3 complete fundamental \( 27 \) representations of \( E_6 \), which may be the remnant of an underlying superstring theory.

(2) Its low-energy gauge group is \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi \), where \( U(1)_\chi \) comes from \( E_6 \to SO(10) \to SU(5) \times U(1)_\chi \).

(3) It has the additional symmetry \( Z_3 \times U(1)_{PQ} \) which serves many purposes, including that of preventing rapid proton decay. \( Z_3 \) is softly broken; \( U(1)_{PQ} \) is spontaneously broken.

(4) Naturally small Dirac neutrino masses [21, 22] come from the \( \hat{L} \hat{N}^c \hat{E}_3 \) term of Eq. (15) because \( \hat{E}_3 \) has a very small VEV, using the mechanism [6, 7] of a large positive \( m^2 \) for \( \tilde{E}_3 \), as shown by Eq. (22).

(5) The 3 singlet superfields \( \hat{S}_{1,2,3} \), which do not transform under \( U(1)_\chi \), are chosen [14] to obtain an axionic solution of the strong CP problem, such that \( f_a >> M_{SUSY} \).

(6) The 2 large masses of the superpotential of Eq. (15), i.e. \( m_2 \) and \( \mu_{33} \), are likely to be
similar in magnitude, in which case \( m_\nu \sim M^2_{3USY} M_W / f_a^2 \), as shown by Eq. (22).

(7) This model predicts a definite supersymmetric particle structure associated with the extra \( U(1)_\chi \) gauge symmetry at the TeV scale, which should be accessible in near-future high-energy accelerators.

(8) It is the only model to date which incorporates naturally small Dirac neutrino masses with the axionic solution of the strong CP problem in a comprehensive theoretical framework of all particle interactions.

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References


