Neutron stars, just after their formation, are surrounded by expanding, dense, and very hot envelopes which radiate thermal photons. Protons can be accelerated in the wind zones of such energetic pulsars to very high energies. These protons lose energy efficiently in collisions with thermal photons and with the matter of the envelope, mainly via pion production. When the temperature of radiation inside the envelope of supernovae drops below $\sim 3 \times 10^6$ K, these pions decay before losing energy and produce high energy neutrinos. We estimate the flux of muon neutrinos emitted during such an early phase of the pulsar - supernova envelope interaction. We find that a $0.1 \text{ km}^2$ neutrino detector should be able to detect on the order of hundreds of neutrinos above 1 TeV within about one year after the explosion from a supernova in our Galaxy. This result holds if these pulsars have surface magnetic fields typical of those observed for radio pulsars, and for initial periods on the order of a few milliseconds when the pulsar is formed.
1. Introduction

The production of neutrinos with different energies during supernova explosions has been discussed extensively during the last several years, mainly in the context of gamma-ray bursts (GRBs). For example, neutrinos with energies \( > 100 \) TeV can be produced in the interactions of protons accelerated by a fireball shock with GRB photons (e.g. Waxman & Bahcall 1997, Vietri 1998). TeV neutrinos can arise in interactions of protons with radiation when the fireball jet breaks through the stellar envelope (Meszaros & Waxman 2001). GeV neutrinos can also be produced in the interactions of protons with neutrons which can be present in the fireball models discussed by Derishev et al. (1999) and Meszaros & Rees (2000).

Of course, the acceleration of particles to high energies is also expected in the case of classical supernovae. Recently, Waxman & Loeb (2001) estimated the neutrino flux from a Type II supernova when the shock breaks out of its progenitor star. Berezinsky & Prilutsky (1978) and Bednarek, Protheroe & Luo (1998) have estimated the flux of neutrinos produced by particles accelerated by the young pulsar during the early phase of the supernova explosion (see also the calculations for the Crab Nebula case in Bednarek & Protheroe 1997).

In this paper we show that neutrinos can also be produced soon after the pulsar formation inside the supernova envelope and that these neutrinos are detectable by current neutrino detectors. As shown by Blasi, Epstein & Olinto (2000) or De Goubeia Dal Pino & Lazarian (2000), among others, particles can be accelerated to very high energies within the “pleirion-like” region of a supernova shortly after the formation of the neutron star. We consider the scenario in which protons accelerated above the light cylinder of the pulsar (Blasi, Epstein & Olinto 2000) interact: first, with the thermal radiation of the expanding supernova envelope, and later, with the matter of the envelope. Acceleration of hadrons in the pulsar wind zone has been discussed in the context of the production of high energy cosmic rays since shortly after the discovery of pulsars (e.g. Gunn & Ostriker 1969, Karakula, Osborne & Wdowczyk 1974). For likely parameters of pulsars at birth, we predict that the flux of muon neutrinos can be observable even by medium-sized neutrino detectors during about one year after supernova explosion.

2. The physical scenario

We consider type Ib/c supernovae, whose progenitors are Wolf-Rayet type stars. Such stars evolve from massive stars with \( M \geq 35 \) M\(_\odot\), and create iron cores surrounded by relatively light envelopes of the order of a few solar masses. We use the models for the evolution of such stars and their explosions as published by Woosley et al. (1993). As an
example, we concentrate on their model 60 WRA. The iron core collapses to a very hot proto-neutron star which cools to the neutron star during about $t_{\text{NS}} \approx 5 - 10$ s from the collapse (Burrows & Lattimer 1986, Wheeler et al. 2000). The rest of the mass of the presupernova (the envelope) is expelled with the velocity at the inner radius of the order of $v_1 = 3 \times 10^8$ cm s$^{-1}$. However because of the density gradient, the outer parts of the envelope move faster. We approximate the velocities of matter in the envelope by the profile

$$v(R) = v_1 (R/R_1)^b,$$

where the parameter $b = 0.5$ is obtained from an approximation of the velocity profile in the expanding envelope shown in Fig. 9 in Woosley et al. (1993), and we use $R_1 = 3 \times 10^8$ cm as the inner radius of the envelope at the moment of explosion. The density of matter in the envelope just before the collapse of the iron core can be approximated by the profile

$$n(R) = n_1 (R/R_1)^{-a},$$

where the density at $R_1$ is $n_1 = 1.2 \times 10^{31}$ cm$^{-3}$, and the parameter $a = 2.4$ is obtained from Fig. 1 in Khokhlov et al. (1999) by interpolation of the profiles for the radius and density versus the mass of the presupernova star (see also Fig. 3c in Woosley et al. 1993). The initial column density decreases with time, $t$, due to the expansion of the envelope according to

$$\rho(t) = \int_{R_1}^{R_2} n(R) \left( \frac{R}{R + v(R) t} \right)^2 dR,$$

where $R_2 = 3 \times 10^{10}$ cm is the outer radius of the envelope at the moment of explosion, and $v(R)$ and $n(R)$ are given by Eqns. 1 and 2, respectively. Just after the collapse of the iron core the temperature at the bottom of the envelope is $T_0 \approx 3 \times 10^8$ K at $R_1$ (see Fig. 8 in Woosley et al. 1993). Therefore the volume above the pulsar and below the expanding envelope is filled with thermal radiation which is not able to escape because of the high optical depth of the envelope. We apply that the temperature of this radiation drops with time during the expansion of the envelope according to

$$T(t) = T_0 \left( \frac{R_1}{R_1 + v_1 (t_{\text{NS}} + t)} \right)^{3/4}.$$

At the moment of the neutron star’s formation (after $\sim 10$ s), the temperature in the region below the envelope already drops to $\sim 5 \times 10^8$ K.

We assume that at this early age, the pulsar loses energy only via electromagnetic radiation. Therefore, its period changes according to the formula $P_{\text{ms}}^2(t) = 1.04 \times 10^{-9} t B_{12}^2 + P_{0,\text{ms}}^2$, where $P_{0,\text{ms}}$ $P_{\text{ms}}$ are the initial and present periods of the pulsar (in milliseconds),
and $B_{12}$ is the pulsar surface magnetic field in units $10^{12}$ G. Note that after about 1 yr, the pulsar may also lose efficiently energy on emission of gravitational radiation due to the r-mode instabilities which are excited in cooling neutron star (e.g. Andersson 1998, Lindblom, Owen & Morsink 1998). During this time, it is likely that the pulsar period changes suddenly reaching the value $\sim 10 - 15$ ms at about 1 yr after formation.

During the first year after explosion the rate of rotational energy loss by the pulsar is too low to influence the initial expansion velocity of the envelope (Ostriker & Gunn 1971), assuming that the pulsar has been born with the surface magnetic field typical for the classical radio pulsars (e.g. the Crab pulsar). Only pulsars with millisecond periods and super-strong magnetic fields (magnetars) can significantly accelerate the envelope at short time intervals after the explosion. Since we discuss here only classical pulsars, it is assumed that the initial kinetic energy of the envelope is conserved during the early expansion phase.

In the next section, we consider the acceleration of protons in the pulsar magnetosphere above the light cylinder and below the expending envelope, adopting the above model for the pulsar formation and expansion of the supernova envelope. Our aim is to find out if the acceleration and radiation processes inside the expanding envelope can produce an observable flux of neutrinos during the early phase of supernova explosion.

3. Acceleration of protons

Following the recent work by Blasi, Epstein & Olinto (2000) we assume that a part of the magnetic energy in the pulsar’s wind zone accelerates protons. The magnetic field in the inner pulsar magnetosphere drops as $B(r) \propto B_s (r_s/r)^3$ and in the wind zone as $B(r) \propto (r_{LC}/r)$, where $B_s$, $r_s$, $r_{LC}$ are the pulsar’s surface magnetic field, surface radius and the radius of the light cylinder, respectively. The maximum energy that protons can reach in the wind zone does not depend on the distance from the star but only on the pulsar parameters, so that

$$E_{p,max} = \frac{B^2(r_{LC})}{8\pi n_G J(r_{LC})} \approx 1.8 \times 10^{11} \beta B_{12} P_{ms}^{-2} \text{GeV},$$

where $\beta$ is the coefficient describing the efficiency of acceleration. However, the available effective acceleration potentials in the wind zone are the strongest close to the light cylinder. They can be estimated from

$$V_{acc} = E_{p,max} r_{LC}^{-1} \approx 3.7 \times 10^{13} \beta B_{12} P_{ms}^{-3} \text{V cm}^{-1}.$$ 

Even during acceleration, protons lose energy via $e^\pm$ pair and pion production in collisions with the thermal radiation which fills the region beneath the expanding envelope of the
supernova. These energy losses can saturate the acceleration of protons if the temperature of the radiation in the acceleration region is high enough. For the acceleration mechanism described by the effective potential (Eq. 6), the acceleration of protons is saturated by the pion production losses, but not by the $e^\pm$ pair production losses, for potentials greater than (Bednarek & Kirk 1995)

$$V_{\text{loss}}^{\text{e}^{\pm}/\pi} \approx 2 \times 10^{-8} T^2 \text{ V cm}^{-1},$$

where $T$ is the temperature which depends on time according to Eq. 4. By comparing Eqs. 6, and 7, we estimate the temperature of radiation below which the energy of relativistic protons is transferred mainly to pions

$$T \approx 4.3 \times 10^{10} (\beta B_{12} P_{\text{ms}}^{-3})^{1/2} \text{ K.}$$

We calculate the energy losses of protons on $e^\pm$ pair production (following Chodorowski, Zdziarski & Sikora 1991) and pion production (following Stecker 1968) in order to determine the equilibrium energies of accelerated protons as a function of temperature in the acceleration region. For low enough temperatures, the energy losses are not able to saturate the acceleration of protons, which are thus injected into the radiation field with the maximum possible energies.

The temperature of the radiation field below the envelope of supernova and the energies of accelerated protons are shown as a function of time in Fig. 1, in the case of the pulsar with parameters $B_{12} = 4$, $P_{\text{ms}} = 3$ and $\beta = 0.1$. For these parameters, the acceleration of protons is saturated by their energy losses on pion production at times shorter than $\sim 3 \times 10^3$ s after explosion. At later times, protons are injected with the maximum energies allowed by the acceleration mechanism (see Eq. 5) and cool mainly on pion production (see Eq. 8 and Fig. 1).

We obtain the spectrum of protons accelerated in the pulsar wind zone close to its light cylinder by following general prescription given by Blasi, Epstein & Olinto (2000). In this simple model the number of protons accelerated to maximum energy, $E_{p,\text{max}}$, is a part of the Goldreich & Julian density, $n = \eta n_{\text{GJ}}$, where $n_{\text{GJ}} = B(r_c)/(2ecP)$ is the Goldreich & Julian (1969) density at the light cylinder. However, contrary to that work, we assume that the pulsar with specific parameters ($B_{12}$ and $P_{\text{ms}}$) injects particles within some range of energies, due to the fact that the magnetic field at different parts of the light cylinder radius is different. The magnetic field strength at the height $h$, measured from the plane containing the pulsar and perpendicular to the light cylinder radius, can be expressed by $B(r) \approx B(r_{\text{LC}}) \cos \alpha$, where $\cos \alpha = r_{\text{LC}}/(r_{\text{LC}}^2 + h^2)^{1/2}$. Therefore, the density of particles at the light cylinder $n(h) = n_{\text{GJ}} \cos^3 \alpha$, and their energies, $E = E_{p,\text{max}} \cos^3 \alpha$ depend on $h$. We calculate the number of particles injected at the height $h$ per unit time from $dN/dt = 2\pi r_{\text{LC}} cn(h) dh$. 

Using the above formulae, we replace $dh$ by $dE$ and obtain the differential spectrum of protons injected by the pulsar at the fixed age of the pulsar, $t$,

$$\frac{dN}{dEdt} = \frac{2\pi c \eta \eta_{GJ}^2 (r_{LC}) (E_{p,\text{max}} E^2)^{-1/3}}{3 [(E_{p,\text{max}}/E)^{2/3} - 1]^{1/2}} \approx \frac{8 \times 10^{31} \eta (B_{12} P_{-2}^{-2} E^{-1})^{2/3}}{[(E_{p,\text{max}}/E)^{2/3} - 1]^{1/2}} \text{ protons} \text{ s GeV}.$$  \hspace{1cm} (9)

In this paper we discuss only the consequences of proton acceleration at a relatively early phase after the supernova explosion, i.e. up to about 1 yr from pulsar formation. During this time, the radiation field inside the expanding supernova envelope, and thereafter, the column density of the envelope, are high enough to provide a target for relativistic protons. As we have already noted, the period of the neutron star can be significantly influenced by gravitational energy losses after about 1 year from the time of the explosion, when the neutron star cools enough.
Fig. 1.— The dependences of the maximum energies of accelerated protons (full curves), the temperature of thermal radiation in the acceleration region (dotted curve), the ratio $\gamma_{\pi} < \epsilon > / m_{\pi}$ for pions produced in proton photon collisions (dashed curve), and the column density of matter in the envelope (thick dot-dashed curve) are shown as a function of time which is measured from the supernova explosion. Protons are accelerated by the pulsar with initial parameters: $B_{12} = 4$, $P_{\text{ms}} = 3$ and the acceleration efficiency $\beta = 0.1$. The vertical lines mark the times: after which pions decay before losing energy on ICS (thin dot-dashed), after which the envelope becomes transparent to thermal radiation (dot-dot-dot-dashed), and at which the envelope becomes transparent and the parameters of the neutron star may change drastically due to the gravitational energy losses (long dashed).
4. Production of neutrinos

Accelerated protons move in the pulsar wind almost at rest in the wind reference frame (Blasi, Epstein & Olinto 2000). Therefore they do not lose significant energy by synchrotron emission. As we have noted, however, these protons will interact with the strong thermal radiation field in the supernova cavity, creating $e^\pm$ pairs and pions. For plausible parameters of the pulsar and the acceleration region, we have shown that the protons lose energy mainly via pion production. The pions then decay into high energy neutrinos if their decay distance scale $\lambda_\pi \approx 780 \gamma_\pi$ cm, is shorter than their characteristic energy loss mean free path. The Lorentz factors of pions, $\gamma_\pi$, are comparable to the Lorentz factors of their parent protons, so they move similarly in the pulsar wind and their synchrotron losses should not dominate over their inverse Compton losses (ICS) in the thermal radiation. Pions lose energy on ICS process mainly in the Klein-Nishina (KN) regime, but not very far from the border with the Thomson regime (see Fig. 1). Therefore we can estimate the ICS losses of pions in the KN regime by

$$P^{ICS}_{KN} \approx \frac{4}{3\pi} \sigma_T U_{rad} (m_e/m_\pi)^2 \gamma_{KN/T}^2,$$

where $\sigma_T$ is the Thomson cross section, $U_{rad}$ is the energy density of radiation, $m_e$, $m_\pi$ are the masses of electron and pion, and $\gamma_{KN/T} \approx 5 \times 10^{11}/T$ is the Lorentz factor at the transition between the KN and T regimes. We estimate the mean free path for pion energy losses via ICS

$$\lambda_{ICS} \approx \frac{m_\pi \gamma_\pi}{P^{ICS}_{KN}} \approx 10^{16} \gamma_\pi/T^2 \text{ cm.}$$

$\lambda_{ICS}$ is comparable to the pion decay distance, $\lambda_\pi$, only for temperatures of radiation $T \leq 3 \times 10^6$ K.

The temperature of the radiation inside the envelope drops to $T \leq 3 \times 10^6$ K at about $t_{dec} \sim 10^4$ s after the supernova explosion (see Eq. 4, and the limit marked by dot-dashed vertical line in Fig. 1). Note that at this time the acceleration of protons is already not saturated by their energy losses. Therefore, protons are injected with the maximum allowed energies and cool in collisions with thermal radiation mainly by pion production. However, when the optical depth through the expanding envelope drops below $\sim 10^3$, the radiation is not further confined in the region below the envelope and its temperature drops rapidly. Based on Eqs. 1, 2, and 3, we have found that this happens at the time $t_{conf} \sim 2 \times 10^6$ s after the explosion (see the thick dot-dashed curve and thin dot-dot-dot-dashed line in Fig. 1). Therefore, we conclude that protons are able to cool efficiently in the thermal radiation and produce pions, which then decay into muon neutrinos, but only from $t_{dec} \approx 10^4$ s up to $t_{conf} \approx 2 \times 10^6$s after the explosion. At later times, the relativistic protons interact with the
matter of the envelope whose density is already low enough so that pions produced by that interaction are able to decay into neutrinos and muons.
Fig. 2.— Spectra of muon neutrinos and antineutrinos produced in interactions of pulsar accelerated protons with the thermal radiation field inside the supernova envelope ($p\gamma \rightarrow \pi \rightarrow \nu_\mu$) and with the matter of the envelope ($pp \rightarrow \pi \rightarrow \nu_\mu$). The surface magnetic field of the pulsar is taken $4 \times 10^{12}$ G, the density factor $\eta = 1$, and the initial periods and acceleration efficiencies are: $P_{\text{ms}} = 3$ and $\beta = 0.1$ (full histograms), $P_{\text{ms}} = 3$ and $\beta = 0.01$ (dashed), and $P_{\text{ms}} = 10$ and $\beta = 0.1$ (dotted).
We now compute the differential spectra of muon neutrinos produced in the interaction of protons: (1) with the radiation field below the envelope during the period \(1 \times 10^4 - 2 \times 10^6\) s after the supernova explosion; and (2) with the matter of the envelope during the period from \(2 \times 10^6 - 3 \times 10^7\) s after the explosion, assuming that the protons cool to the lowest energies allowed by the column densities of photons and matter, respectively. In this calculation, we assume that pions are produced in p-\(\gamma\) collisions with the Lorentz factors comparable to their parent protons. In the case of p-p interactions, we apply the pion multiplicities given in Orth & Buffington (1976). As an example we show the results of these calculations (Fig. 2) for three pulsars with the surface magnetic field \(B_{12} = 4\), and with initial parameters: \(P_{ms} = 3\) and \(\beta = 0.1\) (full histogram), \(P_{ms} = 3\) and \(\beta = 0.01\) (dashed histogram), and \(P_{ms} = 10\) and \(\beta = 0.1\) (dotted histogram). The periods of pulsars with these parameters do not change drastically during the first year after the supernova explosion. However, the numbers of neutrinos produced in these two processes do differ significantly, due to the act that pion multiplicities in p-\(\gamma\) and p-p interactions are different. Therefore the neutrino fluxes from p-p interactions are up to an order of magnitude higher than the neutrino fluxes from p-\(\gamma\) interactions. Note also that the column density of the envelope after \(\sim 2 \times 10^6\) s drops rapidly with time and only protons injected earlier than \(\sim 10^7\) s can undergo multiple interactions with matter.

Just after the collapse of the iron core, the column density of matter is high enough to absorb the neutrinos with energies above \(\sim 0.1\) GeV. However, the column density drops quickly with time \((\propto t^{-2}\text{, see Eq. 4})\) and the cross-section for neutrino interaction increases with energy \(\propto E_\nu\) and \(\propto E_\nu^{0.4}\) below and above \(E_\nu \sim 1\) TeV, respectively (e.g. Hill 1997). Therefore the optical depth becomes less than one at times \(\sim 10^4\) s even for the highest energy neutrinos produced.

5. Conclusion

For a supernova inside our Galaxy at a distance, \(D, = 10\) kpc, we estimate the expected flux of muon neutrinos produced (in proton-photon interactions during \(1 \times 10^4 - 2 \times 10^6\) s after the explosion and produced in proton-proton collisions during \(2 \times 10^6 - 3 \times 10^7\) s after the explosion) by integrating the neutrino spectra shown in Fig. 2. The likelihood of detecting these neutrinos using the AMANDA detector with a surface area of \(10^5\) m\(^2\) can be obtained using the probability of neutrino detection given by Gaisser & Grillo (1987). The results of our calculations are shown in Table 1 for the case of neutrinos arriving from directions close to the horizon, i.e. not absorbed by the Earth (H), and for neutrinos which arrive moving upward from the nadir direction and are partially absorbed (N) (for absorption
coefficients see Gandhi 2000). The results presented in Table 1 concern neutron stars with a surface magnetic field $B = 4 \times 10^{12}$ G, the density factor $\eta = 1$, and different initial periods and acceleration efficiencies.

It is evident that if a neutron star is formed in a supernova explosion with a period $\leq 10$ milliseconds and an acceleration efficiency, $\beta \geq 0.01$, then even the present operating configuration of the AMANDA neutrino detector with a surface area $\sim 2 \times 10^4$ m$^2$ (Andres et al. 2000) should be able to detect at least several muon neutrinos from p-$\gamma$ and p-p interactions during about one year after the SN explosion. Such detection (or lack, thereof) will put constraints on the recent models of extremely high energy cosmic ray production in supernova explosion with formation of very energetic pulsar (Blasi, Epstein & Olinto 2000; De Goubeia Dal Pino & Lazarian 2000).

In fact, neutrinos can also be produced in later stages of supernova explosions, when the capturing of relativistic protons by the supernova envelope is efficient. Such scenarios have been considered in the case of proton acceleration in the pulsar’s wind zone (Berezinsky & Prilutsky 1978) and in the case of hadron acceleration in the pulsar inner magnetosphere (Bednarek & Protheroe 2001, Protheroe, Bednarek & Luo 1998). However, such a production of neutrinos is less certain, since it is expected that neutron stars can lose energy very efficiently via gravitational waves at about one year after explosion due to the r-mode instabilities.

It has been argued that r-mode instabilities are not excited in the neutron stars with the surface magnetic fields typical for magnetars, i.e. $B >> 10^{13}$ G (Rezzolla, Lamb & Shapiro 1999). Detection of neutrinos at these later times might militate against the r-mode instabilities as a means of the production of gravitational radiation. This in turn could have implications for the likelihood of detection of gravitational radiation associated with neutron star production. On the other hand, a sharp cutoff in the detected neutrino flux from a supernova could corroborate the existence r-mode instabilities and lend credence to the possibility of gravity wave generation and detection. Such a detection is of preeminent theoretical interest.

A more detailed analysis of the model, including a discussion of the consequences of neutron stars with the super-strong magnetic fields, will appear in a future paper.
Table 1: Expected number of detected $\nu_\mu$

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{ms}} = 3$ $\beta = 0.1$</th>
<th>$P_{\text{ms}} = 3$ $\beta = 0.01$</th>
<th>$P_{\text{ms}} = 10$ $\beta = 0.1$</th>
</tr>
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<tbody>
<tr>
<td>$p\gamma \rightarrow \nu_\mu$ (H)</td>
<td>320</td>
<td>84</td>
<td>16</td>
</tr>
<tr>
<td>$pp \rightarrow \nu_\mu$ (H)</td>
<td>3230</td>
<td>465</td>
<td>85</td>
</tr>
<tr>
<td>$p\gamma \rightarrow \nu_\mu$ (N)</td>
<td>59</td>
<td>21</td>
<td>4</td>
</tr>
<tr>
<td>$pp \rightarrow \nu_\mu$ (N)</td>
<td>912</td>
<td>210</td>
<td>39</td>
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