Jet Collimation by Small-Scale Magnetic Fields

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ABSTRACT

A popular model for jet collimation is associated with the presence of a large-scale and predominantly toroidal magnetic field originating from the central engine (a star, a black hole, or an accretion disk). Besides the problem of how such a large-scale magnetic field is generated, in this model the jet suffers from the fatal long-wave mode kink magnetohydrodynamic instability. In this paper we explore an alternative model: jet collimation by small-scale magnetic fields. These magnetic fields are assumed to be local, chaotic, tangled, but are dominated by toroidal components. Just as in the case of a large-scale toroidal magnetic field, we show that the “hoop stress” of the tangled toroidal magnetic fields exerts an inward force which confines and collimates the jet. The magnetic “hoop stress” is balanced either by the gas pressure of the jet, or by the centrifugal force if the jet is spinning. Since the length-scale of the magnetic field is small (< the cross-sectional radius of the jet ≪ the length of the jet), in this model the jet does not suffer from the long-wave mode kink instability. Many other problems associated with the large-scale magnetic field are also eliminated or alleviated for small-scale magnetic fields. Though it remains an open question how to generate and maintain the required small-scale magnetic fields in a jet, the scenario of jet collimation by small-scale magnetic fields is favored by the current study on disk dynamo which indicates that small-scale magnetic fields are much easier to generate than large-scale magnetic fields.

Subject headings: galaxies: jets — stars: winds, outflows — magnetic fields — MHD
1. Introduction

Jets are ubiquitous in astronomy. They exist in many astronomical systems, e.g., quasars, galactic nuclei, stellar binaries, young stellar objects (Begelman, Blandford, & Rees 1984; Ferrari 1998; Livio 1999; Wiita 2001), and young pulsars (Gotthelf 2001). Recently it has been suggested that jets also exist in gamma-ray bursts (Piran 1999; Berger et al. 2001, and references therein). A remarkable feature of jets is that they are highly collimated on a super-large length-scale compared to their cross-sectional radius (Begelman, Blandford, & Rees 1984; Ferrari 1998; Livio 1999) except at the very beginning (Junor, Biretta, & Livio 1999; Eislöffel et al. 2000). Though more than eighty years have passed since the first jet was discovered in the center of the galaxy M87 (Curtis 1918), it is still a mystery how jets are generated and collimated (Ferrari 1998; Krolik 1999; Sikora 2001; Wiita 2001).

A popular model for jet collimation is associated with the existence of a large-scale and predominantly toroidal magnetic field which originates from the central engine: a black hole, a star, or an accretion disk (Benford 1978; Blandford & Payne 1982; Begelman, Blandford, & Rees 1984; Ferrari 1998; Krolik 1999, and references therein). In this model, the jet is assumed to be launched from the central object [a star (Mestel 1999, and references therein), or a black hole (Blandford & Znajek 1977; Macdonald & Thorne 1982; Phinney 1983)], or an accretion disk (Blandford 1976; Lovelace 1976; Blandford & Payne 1982; Lovelace, Berk, & Contopoulos 1991; Li, Chiueh, & Begelman 1992; Ferreira & Pelletier 1993a,b, 1995; Shu et al. 1994). An initially poloidal magnetic field threading the central object and the accretion disk extracts angular momentum and energy from the central object and the accretion disk through the magnetosphere (corona) above them. At large radii, the inertia of plasma particles becomes important and bends the magnetic field lines so as to increase the toroidal component of the magnetic field. From the simple argument of the magnetic flux conservation, it is easy to show that at large radii the toroidal magnetic field always dominates the poloidal magnetic field (Begelman, Blandford, & Rees 1984). The dominant toroidal magnetic field produces an inward force (the “hoop stress”) which confines the outflow into a collimated jet. In this model, the appearance of a super-large-scale magnetic field, which extends from the central engine into the body of the jet, is crucial for the jet collimation.

The association of jets with magnetized accretion disks or magnetized central objects (stars or black holes) is strongly supported by the recent observations of HST, Chandra, and VLA (or VLBA) on jets in galactic nuclei, protostellar systems, and young pulsars (Ford et al. 1994; Harms 1994; Junor, Biretta, & Livio 1999; Eislöffel et al. 2000; Weisskopf et al. 2000; Gotthelf 2001; Gaensler, Pivovaroff, & Garmire 2001). However, though the
scenario of jet collimation by a large-scale magnetic field is simple and attractive, it has the following problems: (1) The origin of the large-scale magnetic field is unknown. It is unknown if such a large-scale magnetic field, which extends from the central engine into the body of the jet, could exist; and, if it exists, how the large-scale magnetic field is generated (Livio 1999). In contrast, small-scale, chaotic, and tangled magnetic fields seem to be easily generated by dynamo processes in an accretion disk (Galeev, Rosner, & Vaiana 1979; Tout & Pringle 1992; Balbus & Hawley 1998; Miller & Stone 2000, and references therein). Though some effort has been made (Tout & Pringle 1996), it is still unclear if large-scale magnetic fields can be generated from small-scale magnetic fields. (2) In this scenario, the jet is essentially continuously accelerated and collimated by the central engine, since the large-scale magnetic field is assumed to eventually connect to the central engine. But it is hard to understand how the central engine can continue controlling the dynamics of the jet far beyond the fast magneto-sonic point where the jet loses its causal connection to the central engine (Begelman 1995; Paczyński 2000). (3) A large-scale magnetic field dominated by toroidal components is globally unstable (Bateman 1978; Freidberg 1987). In particular, for a cylindrically shaped magnetic field with both poloidal and toroidal components, the long-wave mode kink instability (the so-called screw instability) sets in when the Kruskal-Shafranov criterion is satisfied (Kadomtsev 1966; Bateman 1978; Freidberg 1987)

\[
\frac{2\pi RB_\parallel}{LB_\perp} < 1,
\]

where \(L\) is the length of the cylinder, \(R\) is the radius of the cylinder, \(B_\parallel\) is the poloidal component of the magnetic field which is parallel to the axis of the cylinder, and \(B_\perp\) is the toroidal component of the magnetic field which is perpendicular to the axis of the cylinder. Equation (1) is equivalent to the statement that the magnetic field is unstable when the magnetic field lines make more than one turn about the axis over the cylinder. For a jet we have \(L \gg R\), equation (1) implies that the magnetic field is unstable when \(B_\perp > B_\parallel\). Thus, a jet collimated by a large-scale toroidal magnetic field (which implies \(B_\perp > B_\parallel\)) suffers from the long-wave mode kink instability and thus its global structure is expected to be twisted and disrupted quickly (Eichler 1993; Spruit, Foglizzo, & Stehle 1997; Begelman 1998; Mestel 1999; Li 2000a; Kersalé, Longaretti, & Pelletier 2000).

Jet collimation by a large-scale poloidal magnetic field has also been proposed (Blandford 1994; Spruit 1994, 1996; Spruit, Foglizzo, & Stehle 1997), but that raises the question of what collimates the poloidal magnetic field.

In this paper, we explore an alternative scenario: jet collimation by small-scale magnetic fields. In this scenario, we assume the jet is threaded by small-scale, chaotic, and tangled magnetic fields which are dominated by toroidal components. The length-scale of the magnetic field lines is assumed to be \(<\) the cross-sectional radius of the jet \(\ll\) the length of the
jet. This scenario is motivated by the investigation on disk dynamo which shows that chaotic and tangled magnetic fields dominated by toroidal components, and having typical length-scales equal to the disk thickness, are efficiently generated in the disk (Galeev, Rosner, & Vaiana 1979; Tout & Pringle 1992; Balbus & Hawley 1998; Miller & Stone 2000). Bubbles of magnetic fields may be driven into the disk corona by magnetic buoyancy (Galeev, Rosner, & Vaiana 1979; Chakrabarti & D’Silva 1994; Miller & Stone 2000), which may eventually form a MHD outflow. The dominance of the toroidal components of the magnetic field in the disk is due to the differential rotation of the disk. If in the outflow the magnetic fields are still dominated by toroidal components — the recent numerical simulation (Miller & Stone 2000) seems to indicate that this is true, then the outflow can be confined and collimated into a jet by the “hoop stress” of the toroidal magnetic fields, just as in the case of a large-scale magnetic field. But the problems that appear in the case of a large-scale magnetic field are eliminated or alleviated due to the small scale concerned in this scenario.

2. Jet Collimation by Small-Scale Magnetic Fields

Motivated by the disk dynamo investigation, we assume the jet is threaded by small-scale, chaotic, and tangled magnetic fields. The magnetic fields are dominated by toroidal components. The typical length of the magnetic field lines is \( l < R \ll L \), where \( R \) is the cross-sectional radius of the jet, \( L \) is the length of the jet. This scenario naturally arises if the jet is launched from a magnetized disk or a magnetized star. Though it is unclear if such a scenario can exist for a jet originating from a central black hole, with magnetic connection to a disk a rapidly rotating black hole may power a jet launched from the disk through pumping energy and angular momentum into the disk (Blandford 1998, 2000; Li 2000b,c, and references therein). Begelman (1995) and Heinz & Begelman (2000) have presented a model which shows how jets may be accelerated by tangled magnetic fields. As in their model, we assume that in the jet

\[
\langle B_r \rangle = \langle B_\phi \rangle = \langle B_z \rangle = \langle B_r B_\phi \rangle = \langle B_\phi B_z \rangle = \langle B_z B_r \rangle = 0, \tag{2}
\]

but

\[
\langle B_r^2 \rangle \neq 0, \quad \langle B_\phi^2 \rangle \neq 0, \quad \langle B_z^2 \rangle \neq 0, \tag{3}
\]

where \( r, \phi, \) and \( z \) are cylindrical coordinates with the \( z \)-axis oriented along the jet axis, \( B_r, B_\phi, \) and \( B_z \) are the components of the magnetic field in the \( r, \phi, \) and \( z \) directions, respectively, \( \langle \rangle \) denotes suitable statistical average over space and time. In addition to their assumptions, we assume that

\[
\langle B_\phi^2 \rangle \gg \langle B_r^2 \rangle, \quad \langle B_\phi^2 \rangle \gg \langle B_z^2 \rangle. \tag{4}
\]
which assure that the magnetic fields are dominated by toroidal components. Unlike Begelman (1995) and Heinz & Begelman (2000), here we work in an inertial frame (instantly) comoving with the motion of the jet along its axis. The cylindrical coordinates and the magnetic fields are defined in such an inertial frame.

The special relativistic momentum equation for an ideal MHD fluid in an inertial frame is given by Begelman (1998). If we choose the inertial frame to be (instantly) comoving with the translation motion of the jet along its axis, the momentum equation can be written as

$$\Gamma^2 \left( \rho + \frac{p}{c^2} \right) \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\left( \nabla p + \frac{\mathbf{v}}{c^2} \frac{\partial p}{\partial t} \right) + \frac{1}{4\pi} \langle (\nabla \times \mathbf{B}) \times \mathbf{B} \rangle,$$

where \(\mathbf{v}\) is the flow velocity, \(\Gamma = (1 - v^2/c^2)^{-1/2}\) is the Lorentz factor, \(p\) and \(\rho\) are the proper pressure and the proper mass density, respectively, \(\mathbf{B}\) is the magnetic field, \(c\) is the speed of light\(^1\). All quantities are defined in the inertial frame, except that the pressure and the mass density are defined in the frame comoving with the fluid which is different from the inertial frame if the jet rotates or expands radially. Since the magnetic field discussed in this paper is assumed to be chaotic, tangled, and have small scale, in equation (5) we average the term \((\nabla \times \mathbf{B}) \times \mathbf{B}\) over space and time. The scale over which the average is made is chosen to be larger than the scale of the micro-structure of the magnetic field but smaller than the macro-scale over which observations are taken.

Assuming the jet is approximately symmetric about the rotation around the jet axis and the translation along the jet axis, then the average gradients of the magnetic field in the \(\phi\) and \(z\) directions are small and can be neglected. Then, with the assumptions in equations

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\(^1\)For a perfectly conducting fluid the electric field in the comoving frame (i.e., the rest frame of the fluid) is vanishing. Through the Lorentz transformation, an electric field is induced in a non-comoving inertial frame, which is related to the magnetic field in the same inertial frame by \(\mathbf{E} = -\frac{\mathbf{v}}{c^2} \times \mathbf{B}\) where \(\mathbf{v}\) is the velocity of the fluid in the inertial frame. The electric field induces a Goldreich-Julian charge density in the inertial frame through the Gauss law (Goldreich & Julian 1969): \(\rho_e = \frac{1}{4\pi} \nabla \cdot \mathbf{E}\). The magnetic field induces a current density in the inertial frame through the Ampere law including the displacement current: \(\mathbf{j} = \frac{1}{c} \left( \nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial}{\partial t} \mathbf{E} \right)\). For relativistic motion \((v \approx c)\), the electric field may be comparable to the magnetic field, but can never dominate the magnetic field since \(\mathbf{B}^2 - \mathbf{E}^2\) is frame-independent and in the comoving frame the electric field is vanishing. However, here we choose the inertial frame to be comoving with the translation motion of the jet along its axis, and in this inertial frame the only possible relativistic motion of the jet is the rotation around its axis. And, in this inertial frame, we assume the magnetic field is predominantly toroidal [eq. (4)], which means that \(\mathbf{v}\) is almost parallel to \(\mathbf{B}\). Thus, in the frame comoving with the axi-translation motion of the jet, we always have \((\mathbf{E}^2) \ll (\mathbf{B}^2)\). Therefore, in equation (5) we have ignored the electric forces.

If the translation velocity of the jet along its axis varies with radius, we choose the inertial frame to comove with the translation velocity at a radius \(r\). Then equation (5) applies to the neighborhood of \(r\) in such an inertial frame.
(2–4), we have

\[
\langle (\nabla \times \mathbf{B}) \times \mathbf{B} \rangle = \langle \mathbf{B} \cdot \nabla \mathbf{B} \rangle - \frac{1}{2} \nabla \langle B^2 \rangle \approx -\hat{r} \left( \frac{\langle B^2_\phi \rangle}{r} + \frac{1}{2} \frac{\partial}{\partial r} \langle B^2_\phi \rangle \right) = -\frac{\hat{r}}{2r^2} \frac{\partial}{\partial r} \left( r^2 \langle B^2_\phi \rangle \right),
\]

where \( \hat{r} \equiv r/r \). In the right hand side of the second line of equation (6), the first term represents the “hoop stress” which points inward toward the jet axis, and is nonzero even when the gradient of the magnetic pressure (the second term) is zero. From equation (6), the jet can be self-collimated only if

\[
\frac{\partial}{\partial r} \left( r^2 \langle B^2_\phi \rangle \right) > 0.
\]

Since \( \langle B^2_\phi \rangle = 0 \) at the jet axis, equation (7) must hold close to the jet axis. At large radii, \( \langle B^2_\phi \rangle \) decreases with \( r \), equation (7) is violated if \( \langle B^2_\phi \rangle \) decreases faster than \( r^{-2} \). Then, if \( \langle B^2_\phi \rangle \) decreases faster than \( r^{-2} \) at large radii, there must exist a radius \( r_0 \) where

\[
\frac{\partial}{\partial r} \left( r^2 \langle B^2_\phi \rangle \right) \bigg|_{r=r_0} = 0.
\]

Jet collimation can only happen in the region \( r < r_0 \). For \( r > r_0 \), the gradient of the magnetic pressure must dominate the “hoop stress”, the flow cannot be confined by the magnetic “hoop stress”. Thus, outside \( r = r_0 \) the flow must expand radially and eventually merge into the surrounding medium where the magnetic pressure is balanced by the gas pressure of the surrounding medium. This indicates that around the collimated jet there exists an uncollimated thin “corona” which has a high ratio of the magnetic pressure to the gas pressure. This scenario is consistent with the speculation of Li & Shu (1996) of the coexistence of a wide-angle component surrounding the well-collimated jet in young stellar objects.

3. A Toy Model: a Self-Collimated Jet Supported by the Centrifugal Force

In this section, we consider a jet which has \( p \ll \rho c^2 \). For simplicity, we assume the jet is steady, non-expanding, axisymmetric, and translation-symmetric along the jet axis. Then, in the inertial frame comoving with the jet motion along its axis, we have \( \mathbf{v} = \Omega r \mathbf{\hat{\phi}} \) and \( \mathbf{v} \cdot \nabla \mathbf{v} = -\Omega^2 r \hat{r} \), where \( \mathbf{\hat{\phi}} \) is the azimuthal unit vector, \( \Omega = \Omega(r) \) is the spinning angular
velocity of the jet. Inserting equation (6) into equation (5), we obtain a one-dimensional equation

\[ \Gamma^2 \rho \Omega^2 r \approx \frac{dp}{dr} + \frac{1}{8\pi r^2} \frac{d}{dr} \left( r^2 \langle B^2_\phi \rangle \right), \]  

(9)

which describes the dynamical equilibrium of the jet in the radial direction.

In a thin shell close to the surface of the jet \((R - \delta < r < R, \delta \ll R)\) we assume \(\frac{d}{dr} \langle B^2_\phi \rangle \approx 0\) and the jet is cold (thus \(p \approx 0\)). Then, in this surface shell, the jet is supported by the centrifugal force, and from equation (9) we have

\[ \Gamma^2 \Omega^2 R^2 \approx \frac{\langle B^2_\phi \rangle}{4\pi \rho}, \]

(10)

where \(\Gamma^2 = (1 - \Omega^2 R^2/c^2)^{-1}\). In the non-relativistic limit, equation (10) corresponds to an equipartition between the rotational energy and the magnetic energy. One can check that the equilibrium between the centrifugal force and the magnetic “hoop stress” is stable: if the radius \(R\) increases a little bit, the “hoop stress” becomes greater than the centrifugal force, making \(R\) decrease and return to the original value in the equilibrium state; on the other hand, if \(R\) decreases a little bit, the “hoop stress” becomes smaller than the centrifugal force, making \(R\) increase and return to the original value in the equilibrium state.

We can express equation (10) with conserved quantities. The rest mass of the jet per unit length is \(M_0 \approx \pi \rho R^2\). The angular momentum of the jet per unit length is \(J_\phi \approx \frac{1}{2} M \Omega R^2\), where \(Mc^2\) is the total energy of the jet per unit length which is also a conserved quantity. If the jet is perfectly conducting as we have assumed, the magnetic field lines are frozen in the jet flow. Though for a chaotic magnetic field the macroscopic net magnetic flux through a macroscopic surface is always zero, we can define a conserved absolute magnetic flux by \(\Psi \equiv \int |B \cdot dS|\), where \(dS\) is the area element. The absolute magnetic flux \(\Psi\) represents the total number of magnetic field lines passing through a surface with an area \(S\), regardless of the direction of the magnetic field lines. For the toroidal magnetic field, we have \(\Psi_\phi \approx \langle B^2_\phi \rangle^{1/2} R\) for per unit length of the jet. Inserting \(M_0, J_\phi, \) and \(\Psi_\phi\) into equation (10), we obtain

\[ R \approx \frac{2J_\phi}{Mc} \left( 1 + \frac{4M_0c^2}{\Psi_\phi^2} \right)^{1/2}, \]

(11)

We expect the jet to spin since the jet can carry angular momentum from the central engine.

Due to the complex MHD turbulence, in a real jet thermal dissipation must happen so the gas pressure is not negligible. Thus, a real jet does not seem to be supported purely by the centrifugal force. The toy model presented here is mainly used to show, as the simplest example, how the jet collimation by small-scale magnetic fields works in principle.
and

\[ \Omega \approx \frac{M c^2}{2 J \phi} \left(1 + \frac{4 M_0 c^2}{\Psi_\phi^2}\right)^{-1}. \]  

The spin velocity on the surface of the jet is

\[ V = \Omega R \approx \frac{c}{\sqrt{1 + \frac{4 M_0 c^2}{\Psi_\phi^2}}} \],

which as expected can approach but cannot exceed the speed of light.

In the non-relativistic limit (i.e., \( \Psi_\phi^2 \ll M_0 c^2 \)), we have \( M \approx M_0 \), \( R \approx 4 J \phi / (M_0^{1/2} \Psi_\phi) \), \( \Omega \approx \Psi_\phi^2 / (8 J \phi) \), and \( V \approx \Psi_\phi / (2 M_0^{1/2}) \). In the relativistic limit (i.e., \( \Psi_\phi^2 \gg M_0 c^2 \)), we have \( R \approx 2 J \phi / (M c) \), \( \Omega \approx M c^2 / (2 J \phi) \), and \( V \approx c \).

4. Discussion and Conclusions

As in the case of a large-scale and predominantly toroidal magnetic field, small-scale and predominantly toroidal magnetic fields can produce a “hoop stress” which confines and collimates the jet. Compared to the scenario that jets are collimated by a large-scale magnetic field, the scenario that jets are collimated by small-scale magnetic fields has the following features: (1) Small-scale magnetic fields are likely to be easier to create through disk dynamo processes (Galeev, Rosner, & Vaiana 1979; Tout & Pringle 1992; Balbus & Hawley 1998; Miller & Stone 2000). (2) Once the outflow of small-scale magnetic bubbles leaves the central engine (e.g., a disk), the outflow is disconnected from the central engine. The jet formed from such outflows is completely self-collimated — in the sense that the small-scale magnetic fields in the jet do not connect to the central engine so the dynamics of the jet is not controlled by the central engine. (3) Since the magnetic fields are assumed to exist only on small-scales, any MHD instability would happen only on a length-scale \( \ll \) the radius of the jet \( \ll \) the length of the jet. Such small-scale MHD instabilities are not expected to destroy the global structure of the jet unless the relevant dissipation and evolution processes are so strong that the small-scale toroidal magnetic fields are eliminated so that the conditions in equation (4) are violated. In other words, a jet collimated by small-scale magnetic fields is immune to the large-scale MHD instability.

Almost all models of disk dynamo predict that the chaotic and tangled small-scale magnetic fields produced in the disk are dominated by toroidal components because of the differential rotation of the disk. Thus, if a jet originates from such a disk, at the beginning
of the jet the magnetic fields are already dominated by toroidal components. In the disk the centrifugal force is balanced by the gravity of the central object (a black hole or a star). As bubbles of small-scale magnetic fields leave the disk, the gravity decreases. So it is reasonable to assume that at the beginning of the jet the centrifugal force prevails and makes the outflow expand radially. Due to the conservation of the absolute magnetic flux which is defined in the last section, we have \( \langle B^2_r \rangle \propto r^{-2} \), \( \langle B^2_\phi \rangle \propto r^{-2} \), and \( \langle B^2_z \rangle \propto r^{-4} \) as \( r \) — the radius of the flow — increases. Therefore, the expansion of the outflow increases the ratio \( \langle B^2_\phi \rangle / \langle B^2_z \rangle \) but keeps the ratio \( \langle B^2_\phi \rangle / \langle B^2_r \rangle \). As \( r \) increases, the density of the rotational energy decreases as \( \rho \Omega^2 r^2 \propto r^{-4} \), since \( \rho \propto r^{-2} \) due to the conservation of mass, and \( \Omega \propto r^{-2} \) due to the conservation of angular momentum. Thus, the radius will increase to a value \( R \) when the magnetic “hoop stress” balances the centrifugal force (and the pressure gradient if the jet is not cold), i.e. when equation (10) is satisfied. Then the jet will stop expanding and be collimated at the radius \( R \) since the equilibrium between the magnetic “hoop stress” and the centrifugal force is stable. In this picture, the magnetic field in the jet is dominated by the toroidal components at the birth of the jet, and this feature is kept at large distance in the jet if the magnetic flux is conserved. However, we point out that even if initially in the jet flow the toroidal magnetic field is comparable to the poloidal magnetic field, at large radii we also expect \( \langle B^2_\phi \rangle > \langle B^2_z \rangle \) if the magnetic flux is conserved, just as in the case of a large-scale magnetic field.

In this paper we have not addressed the problem how jets are produced and accelerated. With three-dimensional MHD simulations, Miller & Stone (2000) have shown that the dynamo process associated with the MHD turbulence driven by the magneto-rotational instability (MRI) (Balbus and Hawley 1991; Balbus & Hawley 1998) works very effectively in a weakly magnetized accretion disk. About 25% of the magnetic energy generated by the MRI in the disk escapes due to buoyancy, producing a strongly magnetized corona above the disk. They have shown that in both the disk and the corona the magnetic fields are dominated by toroidal components. However, due to the limits of the length-scales in the simulation, Miller & Stone (2000) have not addressed the problem if a MHD outflow is produced. If a MHD outflow is produced from such a disk and corona, we expect that in the outflow the magnetic fields are chaotic, tangled, dominated by toroidal components, and have small length-scales. As the outflow gets far from the black hole, the magnetic “hoop stress” becomes dominant in the flow dynamics, confines and collimates the outflow into a jet. We have also not considered the dissipation and evolution of the magnetic fields in jets. Small-scale and tangled magnetic fields tend to reverse signs frequently, thus magnetic reconnection must happen; this will annihilate magnetic fields, and lead to energy dissipation. Jets are likely to spin differentially. Due to the differential spin and the large velocity gradient near the surface of the jet, small-scale shear instabilities tend to generate ripples.
and vortices as shown by numerical simulations (Matthews & Scheuer 1990a,b; Cerqueira & de Gouveia Dal Pino 2001), which will make the structure of the jet much more disordered than assumed here. Furthermore, the shear motion introduced by the large velocity gradient near the surface of the jet will generate strong poloidal magnetic fields which tend to weaken the self-confinement of the jet (Begelman, Blandford, & Rees 1984; Matthews & Scheuer 1990a,b; Kersalé, Longaretti, & Pelletier 2000). These processes are important for the dissipation and evolution of the magnetic fields in jets, but it is unclear if they are strong enough to destroy the collimation mechanism provided by the small-scale magnetic fields.

We note that Begelman (1995) and Heinz & Begelman (2000) have considered the problem of jet acceleration by tangled magnetic fields, but in their model the jet is assumed to be collimated by the thermal pressure from an external medium. Begelman (1995) has argued that, since tangled magnetic fields include large pressure gradients as well as tension forces, it is doubtful if a local equilibrium can be reached. However, we point out that, since we assume the length-scales of the magnetic fields are small, all large pressure gradients and tension forces exist only on small scales which are comparable to the length of the field lines. On macroscopic scales, the average pressure gradients and tension forces need not be large. To see this, let us consider a simple example: \( B_r = 0 \), \( B_\phi = A(r/\rho_1) f(r) \sin(m \phi) \cos(kz) \), \( B_z = f(r) \cos(m \phi) \sin(kz) \), where \( A, \rho_1, m, \) and \( k \) are constants. Since \( B(\phi + 2\pi) = B(\phi) \), \( m \) must be an integer. To satisfy \( \nabla \cdot \mathbf{B} = 0 \), we must have \( k = -mA/\rho_1 \). Assuming \( m \) and \( k \) are large, then \( B_\phi \) and \( B_z \) reverse signs frequently. If we choose the length-scale on which the average is taken to be larger than the periods of the magnetic field, then we have \( \langle B_\phi \rangle = \langle B_z \rangle = \langle B_\phi B_z \rangle = 0 \), \( \langle B_\phi^2 \rangle = A^2(r/\rho_1)^2(f^2/4) \), and \( \langle B_z^2 \rangle = f^2/4 \). Then, if \( r > \rho_1 \) and \( A^2 \gg 1 \), we have \( \langle B_\phi^2 \rangle \gg \langle B_z^2 \rangle \). Since \( k \) is large, \( dB_\phi^2/dz \propto k \) is also large. However, we have \( d\langle B_\phi^2 \rangle/dz = d\langle B_z^2 \rangle/dz = \langle \sin(kz) \cos(kz) \rangle = 0 \). This is because within the macroscopic average scale the pressure gradients change signs rapidly thus they cancel each other. Certainly the local fluctuation of magnetic fields must be accompanied by the corresponding local fluctuation of gases, so that the force balance is maintained. Begelman (1995) has also argued that the MHD turbulent motion in the jet tends to make the overall stress less anisotropic which will in turn weaken the self-collimation of the jet, but how important this effect will be remains to be shown by numerical simulations. Tangled magnetic fields in jets have also been discussed by Laing (1980, 1981), who claims that the emission of highly polarized synchrotron radiation by a source does not necessarily imply a highly ordered magnetic field. Laing (1980) has also shown how the shear motion in jets can compress an initially random field into an anisotropic configuration.

In summary, we have presented an interesting model for jet collimation alternative to the popular model with a large-scale magnetic field. We have shown that small-scale and predominantly toroidal magnetic fields are promising for confining and collimating jets.
Almost all the problems associated with the case of a large-scale magnetic field are eliminated or alleviated for the case of small-scale magnetic fields, though many other problems remain to be addressed, e.g. the production and the acceleration of jets by small-scale magnetic fields, and the dissipation and evolution of small-scale magnetic fields in jets.

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