Abstract

A scenario of $SO(10)$ grand unified theory (GUT) proposed by one of the authors is extended to $E_6$ unification. This gives realistic quark and lepton mass matrices. In neutrino sector, the model reproduces large mixing angle (LMA) MSW solution as well as large mixing angle for atmospheric neutrino anomaly. In this model, right-handed down quark and left-handed lepton of the first and second generation belong to a single multiplet $27$. This causes natural suppression of flavor changing neutral current (FCNC).
1. Introduction

We have a strong motivation for grand unified theories (GUT)\(^1\) in which the quarks and leptons are unified in several multiplets in a simple gauge group. They explain various matters which are not understood in the Standard Model; the miracle of anomaly cancellation between quarks and leptons, hierarchy of gauge couplings, charge quantization, etc. Three gauge groups in the standard model are unified to a simple gauge group at a GUT scale which is considered as just below the Planck scale. On the other hand, the GUT scale destabilizes the weak scale. One of the most promising way to avoid this is to introduce supersymmetry (SUSY).

However, it is not so easy to obtain a realistic SUSY GUT.\(^2\) It is difficult to obtain realistic fermion mass matrices. Especially unification of quarks and leptons gives strong constraints on the Yukawa couplings. Moreover, one of the hardest obstacles is “Doublet-triplet(DT) splitting problem”.

There are several attempts to evade the latter problem.\(^3,4\) Among them, the Dimopoulos-Wilczek mechanism may be a promising way to realize DT splitting in the \(SO(10)\) SUSY GUT.\(^4-7\)

For the fermion masses, recent progress on neutrino experiments\(^8\) provides important informations on the family structure. There are several impressing papers\(^9-13\) which reproduce the large neutrino mixing angles within GUT framework. It is now natural to examine \(SO(10)\) or higher gauge group because every quarks and leptons including right-handed neutrino can be unified in a single multiplet, which is important to discuss neutrino masses.

Recently one of the authors (N.M)\(^14\) proposed a scenario of \(SO(10)\) grand unified theory (GUT) with anomalous \(U(1)_A\) gauge symmetry, which has the following interesting features:

1. The doublet-triplet (DT) splitting is realized using Dimopoulos-Wilczek mechanism.
2. The proton decay via dimension five operator is suppressed.
3. Realistic quark and lepton mass matrices can be obtained in a simple way. Especially in neutrino sector, bi-large neutrino mixing is realized.
4. The symmetry breaking scales are determined by the anomalous \(U(1)_A\) charges.
5. The mass spectrum of the super heavy particles is fixed by the anomalous \(U(1)_A\) charges.

As a consequence of the above features, the fact that the GUT scale is smaller than the Planck scale is strongly connected to the improvement of the undesired GUT relation between the Yukawa couplings \(y_\mu = y_s\) \((y_e = y_d\) also\) while keeping \(y_e = y_b\). Moreover, it is remarkable that the interaction is generic, namely, all the interactions, which are allowed by
the symmetry, are taken into account. Therefore, once we fix the field contents with their quantum numbers, all the interactions are determined except the coefficients of order one.

There the anomalous $U(1)_A$ gauge symmetry,\textsuperscript{15} whose anomaly is cancelled by Green-Schwarz mechanism,\textsuperscript{16} plays an essential role in explaining the DT splitting mechanism at the unification scale as well as in reproducing Yukawa hierarchies\textsuperscript{17-19}. Also bi-large neutrino mixing is naturally obtained by choosing $10$ representation with an appropriate $U(1)_A$ charge in addition to the three family $16$ representations. This anomalous $U(1)_A$ is a powerful tool not only to reproduce DT splitting but also to determine the GUT breaking scales.

This paper aims to show further that, the above $SO(10)$ model is naturally extended to $E_6$ GUT in which the additional fields $10$ of $SO(10)$ is included in a chiral multiplet $27$ of $E_6$. In order to achieve this scenario it is important to introduce the concept of “twisting family structure” in $E_6$ unified model.\textsuperscript{11}

Under $SO(10)$ group, we know that $10$ and $\bar{5}$ of $SU(5)$ are combined into $16$ of $SO(10)$. Usually each family belongs to $16$. In this framework, however, it is not easy to reproduce the Maki-Nakagawa-Sakata (MNS)\textsuperscript{20} large mixing and small Cabbibo-Kobayashi-Maskawa (CKM)\textsuperscript{21} mixing. A promising way to reproduce this is to introduce other multiplets, $10$ of $SO(10)$ in addition to usual $3 \times 16$‘s.\textsuperscript{10} Since $10$ of $SO(10)$ is decomposed into $5(10)$ and $\bar{5}(10)$ of $SU(5)$, one of the fields $\bar{5}(16)$’s can be replaced by this $\bar{5}(10)$. Such a replacement is essential to reproduce large MNS mixing, preserving small CKM mixing. In the case of $E_6$, $16$ and $10$ of $SO(10)$ are naturally included in a single multiplet $27$ of $E_6$. The $E_6$ model automatically prepares such a replacement as we shall see in the next section. We call it “twisting mechanism”. This gives us a strong motivation to examine $E_6$ unification.

It is interesting that the above scenario in $SO(10)$ unification can be extended to $E_6$ unification while keeping the desirable features of $SO(10)$ unification. In this paper we will focus on the extension of the matter sector to $E_6$ unification, leaving the discussion of DT splitting in a separate paper,\textsuperscript{22} which is actually non-trivially extended to $E_6$ unification. Moreover, we show that the condition for the suppression of the flavor changing neutral current (FCNC) is automatically satisfied. This is essentially caused by the twisting mechanism and the unification of the matter fields in a single multiplet $27$, which guarantees that $\bar{5}(16)$ and $\bar{5}(10)$ have the same anomalous $U(1)_A$ charge. Then it can happen that the charge of the first generation of $\bar{5}$ becomes equivalent to that of the second generation of $\bar{5}$. This weakens the severe constraint from the FCNC. It is interesting that the selection of the anomalous $U(1)_A$ charge to realize bi-large neutrino mixing angles automatically realizes the above FCNC suppression.
§2. Twisting in $E_6$ Unification

Let us first recall the twisting mechanism, which has been proposed by one of the authors (M.B.).\textsuperscript{11)} The twisting family structure by this mechanism is peculiar to $E_6$ unification model and we here explain how it happens. In the case of $E_6$, $16$ and $10$ of $SO(10)$ are naturally included in a single multiplet $27$ of $E_6$. The fundamental representation of $E_6$ contains $16$ and $10$ of $SO(10)$ automatically: Under $E_6 \supset SO(10) \supset SU(5)$,

$$27 \rightarrow \left[ \begin{array}{c}
16 \\
10 \\
1
\end{array} \right] \left[ \begin{array}{c}
(16, 10) + (16, \bar{5}) + (16, 1) \\
(10, 5) + (10, \bar{5}) \\
(1, 1)
\end{array} \right]$$

where the representation of $SO(10), SU(5)$ are explicitly denoted in the above. As we have already seen, the $E_6$ model naturally has the freedom for replacing matter fields $(16, 5)$ by $(10, 5)$. So here let us explain how the twisting family structure arises in the $E_6$ unification. Let us take the following $E_6$ multiplets

1. Matter multiplets: $\Psi_i(27)$, whose $U(1)_A$ charges are denoted as $\psi_i^*(i=1,2,3)$,
2. Higgs field which breaks $E_6$ into $SO(10)$: $\Phi(27)$ ($\langle \Phi(1, 1) \rangle = v$),
3. Higgs field which breaks $SO(10)$ into $SU(5)$: $C(27)$ ($\langle C(16, 1) \rangle = v'$),
4. Higgs field which includes the Higgs doublets: $H(27)$.

Throughout this paper we denote all the superfields with uppercase letters and their anomalous $U(1)_A$ charges with the corresponding lowercase letters. Assigning negative R-parity to ordinary matter $\Psi_i(27)$ as usual and using a field $\Theta$ with charge $-1$, the $U(1)_A$ invariant superpotential for low energy Yukawa terms is,

$$W_Y = \left( \frac{\Theta}{M_P} \right)^{\psi_i + \psi_j + h} \psi_i \psi_j H$$

(2.2)

where we suppress the coefficients of order one and for the above we assume that $\psi_i + \psi_j + h \geq 0$ for each $i, j$ pair so that there appears no SUSY zero.\textsuperscript{***) After getting non-zero VEV $\langle \Theta \rangle = \lambda M_P$ ($\lambda \sim 0.2$) by D-flatness condition of the anomalous $U(1)_A$ gauge symmetry, hierarchical structure of the Yukawa couplings is realized. Since we need $3 \times [10 + \bar{5} + 1]$ in $SU(5)$ representations for three families, among the above three $27$ fields, three pairs of $(5, \bar{5})$ must get heavy.\textsuperscript{****) Indeed the Higgs fields $\Phi, C$ can give those masses: The superpotentials,

\textsuperscript{***) We assume that $\psi_1 > \psi_2 > \psi_3$

\textsuperscript{****) Note that if total charge of an operator is negative, the $U(1)_A$ invariance forbids operators in the action since the field $\Theta$ with negative charge cannot compensate the negative total charge of the operator (SUSY zero mechanism).

\textsuperscript{****) The neutrino candidates $\Psi_i(16, 1)$ and $\Psi_i(1, 1)$ also get heavy masses but here we concentrate ourselves on the family structure of $\bar{5}$.
which give heavy masses for \( (5, \bar{5}) \) pairs, are

\[
W = \left( \frac{\theta}{M_P} \right)^{\psi_i + \psi_j + c} \Psi_i \Psi_j C + \left( \frac{\theta}{M_P} \right)^{\psi_i + \psi_j + \phi} \Psi_i \Psi_j \phi, \tag{2.3}
\]

which we shall here analyse to see how \( \bar{5} \) fields acquire heavy masses. The VEV \( \langle \Phi(1, 1) \rangle = v \) gives \( 3 \times 3 \) mass matrix of \( \Psi_i(10, 5) \Psi_j(10, \bar{5}) \) pairs,

\[
\begin{pmatrix}
\Psi_1(10, 5) & \Psi_2(10, 5) & \Psi_3(10, 5)
\end{pmatrix}
\begin{pmatrix}
\lambda^{\psi_1} & \lambda^{\psi_1 + \psi_2} & \lambda^{\psi_2 + \psi_3} \\
\lambda^{\psi_1 + \psi_2} & \lambda^{2\psi_2} & \lambda^{\psi_2 + \psi_3} \\
\lambda^{\psi_1 + \psi_3} & \lambda^{\psi_2 + \psi_3} & \lambda^{2\psi_3}
\end{pmatrix}
\lambda^\phi v, \tag{2.4}
\]

while the VEV \( \langle C(16, 1) \rangle = v' \) gives mass terms of \( \Psi_i(16, \bar{5}) \) and \( \Psi_j(10, 5) \),

\[
\begin{pmatrix}
\Psi_1(10, 5) & \Psi_2(16, 5) & \Psi_3(16, 5)
\end{pmatrix}
\begin{pmatrix}
\lambda^{2\psi_1} & \lambda^{\psi_1 + \psi_2} & \lambda^{\psi_1 + \psi_3} \\
\lambda^{\psi_1 + \psi_2} & \lambda^{2\psi_2} & \lambda^{\psi_2 + \psi_3} \\
\lambda^{\psi_1 + \psi_3 + c v'} & \lambda^{\psi_2 + \psi_3} & \lambda^{2\psi_3}
\end{pmatrix}
\lambda^{c v'}. \tag{2.5}
\]

Then the full mass matrix is after all,

\[
\begin{pmatrix}
\Psi_1(16, 5) & \Psi_2(16, 5) & \Psi_3(16, 5) & \Psi_1(10, 5) & \Psi_2(10, 5) & \Psi_3(10, 5)
\end{pmatrix}
\begin{pmatrix}
\lambda^{2\psi_1 + r} & \lambda^{\psi_1 + \psi_2 + r} & \lambda^{\psi_1 + \psi_3 + r} & \lambda^{2\psi_1} & \lambda^{\psi_1 + \psi_2} & \lambda^{\psi_1 + \psi_3} \\
\lambda^{\psi_1 + \psi_2 + r} & \lambda^{2\psi_2 + r} & \lambda^{\psi_2 + \psi_3 + r} & \lambda^{\psi_1 + \psi_2} & \lambda^{2\psi_2} & \lambda^{\psi_2 + \psi_3} \\
\lambda^{\psi_1 + \psi_3 + r} & \lambda^{\psi_2 + \psi_3 + r} & \lambda^{2\psi_3 + r} & \lambda^{\psi_1 + \psi_3} & \lambda^{\psi_2 + \psi_3} & \lambda^{2\psi_3}
\end{pmatrix}
\lambda^\phi v, \tag{2.6}
\]

where we define a parameter \( r \) as

\[
\lambda^r \equiv \frac{\lambda^{c v'}}{\lambda^{\phi v}}, \tag{2.7}
\]

which we will use frequently in the following discussion. Note that some of the matrix elements become zero if the index becomes negative (SUSY zero). For a moment we assume that no such zero appears in the superpotential. In general we have three massless modes out of six \( \bar{5} \)'s by solving the above \( 3 \times 6 \) matrix. However, since the matrix has hierarchical structure we can easily classify the cases.

1. Under the condition that we have no SUSY zeros, it is evident that the heaviest mass is either \( M_{33} \) or \( M'_{33} \) whose ratio is \( M'_{33}/M_{33} = \lambda^r \).

2. \( 0 < r \) case: In this case \( M_{33} \) is larger than \( M'_{33} \) and the pair \((\Psi_3(10, \bar{5}), \Psi_3(10, 5))\) gets heavy. Next compare \( M_{23} \) and \( M_{22} \) whose ratio is \( M_{23}/M_{22} = \lambda^{r + \psi_1 - \psi_2}, \) so according to the sign of \( r + \psi_1 - \psi_2 \), we have different options.
3. $0 < r < \psi_2 - \psi_3$: $M'_{22} > M_{22}$ so that the pair $\Psi_3(16, \bar{5})\Psi_2(10, 5)$ gets heavy and at the same time the pair $\Psi_2(10, \bar{5})\Psi_1(10, 5)$ obtains heavy mass because $M'_{12}/M_{12} = \lambda' < 1$. $\Psi_1(10, 5)$ as well as $\Psi(16, \bar{5})$ and $\Psi_2(16, \bar{5})$ are left massless. This option is denoted as (1, 1′, 2). (In this paper, massless mode whose dominant component is $\Psi(16, \bar{5})$ ($\Psi_1(10, 5)$) is simply denoted by $i(i')$.)

4. $(0 <) \psi_2 - \psi_3 < r$: The pair $\Psi_2(10, \bar{5})\Psi_2(10, 5)$ becomes heavy. Further this case is divided into two cases according to the sign of $r + \psi_3 - \psi_1$.

5. $\psi_2 - \psi_3 < r < \psi_1 - \psi_3$: In this case, $M'_{13}$ is larger than $M_{11}$, thus $\Psi_3(16, \bar{5})\Psi_1(10, 5)$ gets heavy and this case also becomes the option (1, 1′, 2).

6. $(\psi_2 - \psi_3 <) \psi_1 - \psi_3 < r$: In this extreme case, $M'_{13} < M_{11}$, namely, $\Psi_1(10, \bar{5})\Psi_1(10, 5)$ gets heavy, hence all the $\Psi_i(10, \bar{5})$ are heavy states and those of $\Psi_i(16, \bar{5})$ are massless modes. This corresponds to the situation that three massless $\bar{5}$ fields (quarks and leptons) belong to $\Psi_i(16, \bar{5})$. This is just the option usually adopted in $SO(10)$ model. We may call this option as “parallel family structure.” Let us denote such option simply as (1, 2, 3).

7. $r < 0$ case is easily classified just exchanging the 10 representation by the 16 representation.

Thus we can classify all the cases as follows:

1. $\psi_1 - \psi_3 < r \rightarrow (1, 2, 3)$ type,
2. $0 < r < \psi_1 - \psi_3 \rightarrow (1, 1', 2)$ type,
3. $\psi_3 - \psi_1 < r < 0 \rightarrow (1, 1', 2')$ type,
4. $r < \psi_3 - \psi_1 \rightarrow (1', 2', 3')$ type.

If we use SUSY zero coefficients, various type of massless modes can be realized. For example, if $\psi_1 + \psi_3 + \phi < 0$, SUSY zero appears and the Yukawa terms $\Psi_3\Psi_i\Phi(i = 1, 2, 3)$ are forbidden and so the mass matrix $M$ becomes

$$
\begin{pmatrix}
\Psi_1(10, 5) & \Psi_2(10, \bar{5}) & \Psi_3(10, \bar{5}) \\
\Psi_1(10, 5) & \lambda^{2\psi_1} & \lambda^{\psi_1 + \psi_2} & 0 \\
\Psi_2(10, 5) & \lambda^{\psi_1 + \psi_2} & (\lambda^{2\psi_2}) & 0 \\
\Psi_3(10, 5) & 0 & 0 & 0
\end{pmatrix} \lambda^{\phi v}, \quad (2.8)
$$

and the massless mode $\Psi_3(10, \bar{5})$ does not mix via non diagonal mass matrix elements with any other $\bar{5}$ field. We shall call such a massless field as “isolated” field. There are various different patterns of massless modes, using the “isolated” fields. For example, if conditions $2\phi_2 + \phi \geq 0, 2\phi_3 + c \geq 0$ and $\lambda^{2\psi_1 + \phi v} > \lambda^{\psi_1 + \psi_2 + c v'}$ are satisfied in addition to the above
condition $\psi_1 + \psi_3 + \phi < 0$, we have the pattern $(1, 2, 3')$, i.e.,
\[
\begin{pmatrix}
\bar{\psi}_1 \\
\bar{\psi}_2 \\
\bar{\psi}_3
\end{pmatrix}
= 
\begin{pmatrix}
\psi_1(16, \bar{5}) + \cdots \\
\psi_2(16, \bar{5}) + \cdots \\
\psi_3(10, \bar{5})
\end{pmatrix},
\] (2.9)
which has been adopted in Ref.11. Note that $\bar{\psi}_3$ has no mixing with any other states (isolated field).

One must say that, in addition to the mixing of the matter contents, the massless Higgs doublets itself can in principle be mixed as
\[
H(\bar{5}) = H(10, \bar{5}) \cos \theta + H(16, \bar{5}) \sin \theta,
\] (2.10)
which is also determined by solving a whole mass matrix of the doublet Higgs fields. Note that the Yukawa couplings of $\Psi_i(16, 10)\Psi_j(16, \bar{5})H(\bar{5})$ ($\Psi_i(16, 10)\Psi_j(10, \bar{5})H(\bar{5})$ ) are proportional to $\cos \theta (\sin \theta)$.

§3. Feature of the Vacua in $U(1)_A$ Framework

In this section, we explain how the vacua of the Higgs fields are determined by the anomalous $U(1)_A$ quantum numbers.\(^{14}\)

First of all, the VEV of a gauge invariant operator with positive anomalous $U(1)_A$ charge must vanish. Otherwise, the mechanism of SUSY zero does not work since such a VEV can compensate the negative $U(1)_A$ charge of the term. Generically such an undesired vacuum is allowed, but as is shown in the appendix, on such a vacuum, Froggatt-Nielsen mechanism\(^{23}\) does not work. Therefore we are not interested in such a vacuum. Here we simply assume that we are on the vacuum where SUSY zero and Froggatt-Nielsen mechanism work, namely any VEV of a gauge invariant operator with positive anomalous $U(1)_A$ charge vanishes.

Next we show that the VEV of a gauge invariant operator $O$ is determined by its $U(1)_A$ charge $o$ as $\langle O \rangle = \lambda^{-o}$ if the $F$-flatness condition determines the VEV. For simplicity, we examine this relation using singlet fields $Z_i$ with the anomalous $U(1)_A$ charge $z_i$. The general superpotential is written as
\[
W = \sum_i \lambda^{z_i} Z_i + \sum_{i,j} \lambda^{z_i+z_j} Z_iZ_j + \cdots
\] (3.1)
\[
= \sum_i \tilde{Z}_i + \sum_{i,j} \tilde{Z}_i\tilde{Z}_j + \cdots,
\] (3.2)
where $\tilde{Z}_i \equiv \lambda^{z_i} Z_i$. The equations of $F$-flatness of $Z_i$ fields require
\[
\lambda^{z_i}(1 + \sum_j \tilde{Z}_j + \cdots) = 0,
\] (3.3)
which generically lead to solutions $\tilde{Z}_j \sim O(1)$ so that $\langle Z_i \rangle \sim \lambda^{-z_i}$ as announced above.

If an adjoint field $A$ has a VEV by the $F$-flatness condition, the VEV is determined as $\langle A \rangle \sim \lambda^{-a}$ because $A^2$ can be gauge invariant. Suppose that in addition to $\Phi$ and $C$, there are $\Phi(\overline{27})$ and $C(\overline{27})$. Since $\overline{\Phi}\Phi$ is also gauge invariant, the VEV of the operator is given by $\langle \overline{\Phi}\Phi \rangle \sim \lambda^{-\overline{\Phi}+\Phi}$ if the VEV is determined by the $F$-flatness condition. The $D$-flatness condition of $E_6$ gauge group requires

$$|\langle \overline{\Phi}\Phi \rangle| = |\langle \Phi \rangle| \sim \lambda^{-\overline{\Phi} + \Phi}/2.$$  \hspace{1cm} (3.4)

Note that these VEVs are also determined by the anomalous $U(1)_A$ charges but they are different from the naive expectation $\langle \Phi \rangle \sim \lambda^{-\overline{\Phi}}$. This is because the $D$-flatness condition plays an important role to fix the VEVs. The VEVs of $C$ and $\bar{C}$ are also determined by the anomalous $U(1)_A$ charges as

$$|\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-(c+\bar{c})}/2.$$  \hspace{1cm} (3.5)

By the above argument, it is now found that $v$ and $v'$ are determined by the anomalous $U(1)_A$ charges. Therefore the massless modes of $\overline{5}$, which are determined by the twisting mechanism, are also determined by the anomalous $U(1)_A$ charges.

§4. Quark and Lepton Masses in $E_6$ Unification

Now let us consider a simple model which can be consistent with realistic data. Consider the following minimal matter contents, fermion and Higgs chiral fields. Here in addition to $R$-parity, we introduce $Z_2$ parity, which plays an important role to solve the DT splitting problem as explained in separate papers.

1. Matter multiplet (odd $R$-parity): $\Psi_i(\overline{27}, +)$ $i=1,2,3,$
2. Higgs field which breaks $E_6$ into $SO(10)$: $\Phi(\overline{27}, +)$, $\overline{\Phi}(27, +)$, $\langle \Phi \rangle (=\langle \overline{\Phi} \rangle)$,
3. Adjoint Higgs field $A(\overline{78}, -)$, whose $SO(10)$ component $A(\overline{45})$ breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ by the VEV $\langle A(\overline{45}) \rangle_{B-L} = i\tau_2 \times \text{diag}(V, V, V, 0, 0)$. This Dimopoulos-Wilczek form of the VEV plays an important role to solve the DT splitting problem.
4. Higgs field which breaks $SU(2)_R \times U(1)_{B-L}$ into $U(1)_Y$: $C(\overline{27}, +)$, $\overline{C}(\overline{27}, +)$, by developing $\langle C \rangle (=\langle \overline{C} \rangle)$,
5. Higgs field which contains usual $SU(2)_L$ doublet: $H(\overline{27}, +)$,

where the signature $\pm$ indicates $Z_2$ parity of the fields. Here we introduce Higgs fields $H$ in addition to the other Higgs fields $\Phi, \overline{\Phi}, C$ and $\overline{C}$, but it might be the case where the Higgs
doublet can be a part of component of the other Higgs fields $\Phi$ and/or $C$. Even in that case, the following argument can be applied by taking $h = \phi$ or $h = c$.

In the following, we take the $U(1)_A$ charges of the matter fields $\Psi_i$ as $(\psi_1 = 3 + n, \psi_2 = 2 + n, \psi_1 = n)$, which has been determined in the previous papers to be consistent with the up type quark masses and mixings. Then the top Yukawa coupling of order one determines the anomalous $U(1)_A$ charge of the Higgs field $H$ as $h = -2n$.

Also in this paper we assume that the mixing angle $\sin \theta$ (defined in eq. (2.10)) is zero, i.e., the down type Higgs is purely $H(10,5)$. This assumption makes our following discussion much simpler. Of course, once we fix the model which realize the DT splitting, the Higgs mixing angle is also determined by the anomalous $U(1)_A$ charges. We shall discuss this point in a separate paper. Actually we shall find various DT splitting models which give $\sin \theta = 0$.

Now the Yukawa couplings are obtained by Froggatt-Nielsen mechanism as

$$W = \lambda^{\psi_1 + \psi_3 + h} \psi_1 \psi_2 H,$$

(4.1)

where the mass matrix of the up quark sector is uniquely determined since we have already fixed the $U(1)$ charges of the fields $\Psi_i$.

$$M_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H(10,5) \rangle,$$

(4.2)

while the twisting mechanism discussed in the section 2 makes the down quark mass matrix different from that of the up quark.

We examine the massless modes of $\mathbf{5}$ in the followings under the assumption $\sin \theta = 0$. Note that under such situation, component fields $\Psi_i(10,\mathbf{5})$ have no Yukawa couplings because the Yukawa terms $\Psi_i(10,\mathbf{5}) \Psi_i(16,10) H(10,\mathbf{5})$ are forbidden by $SO(10)$ gauge symmetry. This excludes the options which include isolated $\Psi_i(10,\mathbf{5})$ fields. Moreover, the options which include isolated $\Psi_i(16,\mathbf{5})$ fields should be discarded since we cannot obtain large neutrino mixing angle in such cases. Under the charge assignment $(\psi_1 = n + 3, \psi_2 = n + 2, \psi_3 = n)$, the classification discussed in section 2 applies even to the cases where SUSY zeros appear, provided that there are no isolated fields. Among the options $(1, 2, 3), (1, 1', 2), (1, 1', 2')$ and $(1', 2', 3')$, only the option $(1, 1', 2)$ gives realistic quark and lepton mass matrices.

Let us examine the option of $(1, 1', 2)$, i.e., $0 < r < 3$, with the constraints which forbid the existence of isolated state, $0 \leq \psi_1 + \psi_3 + c$ and $0 \leq \psi_1 + \psi_3 + \phi$. Here the parameter $r$,
which has been defined by $X = \lambda v'/(\lambda \phi v)$, is given by
\[ r = \frac{1}{2} [\bar{c} - \bar{c} - (\phi - \bar{\phi})], \] (4.3)
because the VEVs $\nu$ and $\nu'$ are fixed by the anomalous $U(1)_A$ charges. Note that even
if we take anomalous $U(1)_A$ charges integer, $r$ can be half integer. This fact plays an
important role to realize bi-large neutrino mixing angle as we shall see in the next section.
Taking this option we investigate which type of mixing pattern of $\bar{5}$'s can reproduce the
bi-large neutrino mixing. In order to see this, let us consider the options $(\bar{5}_1, \bar{5}_2, \bar{5}_3) =
(1, 1', 2)$ and $(\bar{5}_1, \bar{5}_2, \bar{5}_3) = (1, 2, 1')$ as phenomenologically viable patterns of massless three
fields of $(\bar{5}_1, \bar{5}_2, \bar{5}_3)$. Note that the correct expression of the massless states left in low
energy, $\bar{5}_i$, are obtained as mixed states of $\psi_j$ by solving the mass matrix of eq.(2.6). It
should be remarked that $\psi_1(10, \bar{5})$ itself does not have Yukawa coupling, so the field 1' really
can have Yukawa coupling only through the mixing with $\psi_i(16, \bar{5})$. In order to get the
exact mass matrix of down quarks as well as leptons, we should take account of the mixing
effects from the non dominant states. We first fix the three bases of the massless modes
$(\bar{5}_1, \bar{5}_2, \bar{5}_3)$ to $(\psi_1(16, \bar{5}), \psi_1(10, \bar{5}), \psi_2(16, \bar{5}))$. On this basis we can estimate the order of
mixing parameters with the heavy states $\psi_3(16, \bar{5}), \psi_2(10, \bar{5})$ and $\psi_3(10, \bar{5})$:

\[ \bar{5}_1 = \psi_1(16, \bar{5}) + \lambda^{\psi_1-\psi_3} \psi_3(16, \bar{5}) + \lambda^{\psi_1-\psi_3-r} \psi_2(10, \bar{5}) + \lambda^{\psi_1-\psi_3+r} \psi_3(10, \bar{5}), \] (4.4)
\[ \bar{5}_2 = \psi_1(10, \bar{5}) + \lambda^{\psi_1-\psi_3-r} \psi_3(16, \bar{5}) + \lambda^{\psi_1-\psi_2} \psi_2(10, \bar{5}) + \lambda^{\psi_1-\psi_3} \psi_3(10, \bar{5}), \] (4.5)
\[ \bar{5}_3 = \psi_2(16, \bar{5}) + \lambda^{\psi_2-\psi_3} \psi_3(16, \bar{5}) + \lambda^r \psi_2(10, \bar{5}) + \lambda^{\psi_2-\psi_3+r} \psi_3(10, \bar{5}), \] (4.6)

where the first terms of the right hand side are the main components of these massless modes
and the other terms are mixing terms with heavy states, $\psi_3(16, \bar{5}), \psi_2(10, \bar{5})$ and $\psi_3(10, \bar{5})$.
The order of these mixing parameters can be estimated by the ratios of the relevant mass
matrix elements. For example, the ratio of the mass matrix element $M'_{k_1} = \lambda^{\psi_1+\psi_k+c} v'$ to
$M'_{k_3} = \lambda^{\psi_3+\psi_k+c} v'$ becomes $M'_{k_1}/M'_{k_3} = \lambda^{\psi_1-\psi_3}$, which appears in the coefficient of the second
term of eq. (4.4). Note that this ratio is independent on the parameter $\psi_k$. Similarly, the
ratio of $M'_{k_1} = \lambda^{\psi_1+\psi_k+c} v'$ to $M_{k_3} = \lambda^{\psi_3+\psi_k+\phi} v$ becomes $M'_{k_1}/M_{k_3} = \lambda^{\psi_1-\psi_3+r}$, which appears
in the coefficient of the third term of eq. (4.4).

The mass matrices of down type quark and charged lepton can be obtained from the
above mixing pattern by introducing the RGE factor $\eta^{-1} \sim 2 - 3$;

\[ M_D = M_D^\frac{1}{\eta} = \begin{pmatrix} \psi_1(16, 10) & \psi_2(16, 10) & \psi_3(16, 10) \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^{6-r} & \lambda^5 \\ \lambda^5 & \lambda^{5-r} & \lambda^4 \\ \lambda^3 & \lambda^{3-r} & \lambda^2 \end{pmatrix} \langle H(10, 5) \rangle, \] (4.7)
which corresponds to the option \((1, 1', 2)\) when \(3-r > 2 \rightarrow 1 > r\).\(^3\) Note that in eq.\((4.5)\), the main modes of \(\mathbf{5}_2\) is \(\Psi_1(\mathbf{10}, \mathbf{5})\) which has no Yukawa coupling to \(H(\mathbf{10}, \mathbf{5})\), so the contribution from the mixing term \(\lambda \psi_1 \psi_3 \Psi_3(\mathbf{16}, \mathbf{5})\) determines the order of the Yukawa couplings. On the other hand, main modes of \(\mathbf{5}_1\) and \(\mathbf{5}_3\) determine the order of the Yukawa couplings, while the contribution of the mixing terms is of the same order.

Now that we have mass matrices for up and down quarks, we can estimate the CKM matrix\(^*\) as

\[
U_{CKM} = \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix},
\]

which is consistent with the experimental value if we take \(\lambda \sim 0.2\). Since the ratio of the Yukawa couplings of top and bottom quarks is \(\lambda^2\), \(\tan \beta \equiv \langle H(\mathbf{10}, \mathbf{5}) \rangle / \langle H(\mathbf{10}, \mathbf{5}) \rangle \sim m_t/m_b \cdot \lambda^2\) is predicted by these mass matrices. Yukawa matrix for charged lepton sector is the same as the transpose of \(M_d\) at this stage except an overall factor \(\eta\) induced by the renormalization group effect.

\section{5. Bi-Large Neutrino Mixing in \(E_6\) Unification}

Now we come to the neutrino masses and mixing. In order to do this, we must estimate the mixings in neutrino mass matrix, since the Maki-Nakagawa-Sakata (MNS) matrix\(^20\) is given by

\[
U_{MNS} = U_l U_\nu^\dagger,
\]

with the unitary matrices \(U_l\) and \(U_\nu\) which make the matrices \(U_E (M_E^\dagger M_E) U_E^\dagger\) and \(U_\nu^\dagger M_\nu U_\nu\) diagonal. The matrix \(M_\nu\) is the Majorana mass matrix of the light (almost) left-handed neutrinos, which is obtained from the Dirac masses and right handed Majorana masses. First the Dirac neutrino mass matrix is given by the \(3 \times 6\) matrix,

\[
\begin{pmatrix}
\Psi_1(\mathbf{16}, 1) & \Psi_2(\mathbf{16}, 1) & \Psi_3(\mathbf{16}, 1) & \Psi_1(\mathbf{1}, 1) & \Psi_2(\mathbf{1}, 1) & \Psi_3(\mathbf{1}, 1) \\
\mathbf{5}_1 & \lambda^6 & \lambda^5 & \lambda^3 & \lambda^{r+6} & \lambda^{r+5} & \lambda^{r+3} \\
\mathbf{5}_2 & \lambda^{6-r} & \lambda^{5-r} & \lambda^{3-r} & \lambda^6 & \lambda^5 & \lambda^3 \\
\mathbf{5}_3 & \lambda^5 & \lambda^4 & \lambda^2 & \lambda^{r+5} & \lambda^{r+4} & \lambda^{r+2} \\
\end{pmatrix} \langle H(\mathbf{10}, \mathbf{5}) \rangle \eta,
\]

\(^*\) If in the case \(1 < r\), the second family should be exchanged with the third family (the option \((1, 2, 1')\)).

\(^*\) Strictly speaking, if the Yukawa coupling originated only from the interaction \((4.1)\), the mixing concerning to the first generation becomes too small because of a cancellation. In order to get the expected value of CKM matrix as in eq. (4.8), non-renormalizable terms, for example, \(\Psi_i \Psi_j HCC\) must be taken into account.
or we simply express as

\[
M_N = \left( \begin{array}{cc}
\lambda^2 & \lambda^{r+2}
\end{array} \right) \otimes \left( \begin{array}{ccc}
\lambda^4 & \lambda^3 & \lambda \\
\lambda^{4-r} & \lambda^{3-r} & \lambda^{1-r} \\
\lambda^3 & \lambda^2 & 1
\end{array} \right) \langle H(10, \mathbf{5}) \rangle \eta. \tag{5.3}
\]

On the other hand, the right-handed Majorana masses come from the interaction:

\[
\lambda \psi_i \psi_j + 2 \tilde{\phi} \bar{\psi}_i \bar{\psi}_j \bar{\phi} + \lambda \psi_i \psi_j \bar{\psi}_j \bar{\psi}_i \tilde{C} + \lambda \psi_i \psi_j + 2 \bar{c} \psi_i \psi_j \bar{c} \tilde{C}. \tag{5.4}
\]

Then the \(6 \times 6\) matrix for \(\Psi_i(1, \mathbf{1}), \psi = 1, 2, 3; \Psi_k(\mathbf{16}, \mathbf{1}), k = 1, 2, 3\) right handed neutrinos, is expressed as

\[
M_R = \lambda \psi_i \psi_j + 2 \tilde{\phi} \psi_i(1, \mathbf{1}) \psi_j(1, \mathbf{1}) \langle \bar{\phi} \rangle^2 + \lambda \psi_i \psi_j + \bar{c} + 2 \bar{c} \psi_i \psi_j \bar{c} \tilde{C} + \lambda \psi_i \psi_j + 2 \bar{c} \psi_i \psi_j \bar{c} \tilde{C}. \tag{5.5}
\]

\[
= \lambda^{2n} \left( \begin{array}{ccc}
\lambda^{\delta-\phi} & \lambda^{(\delta-\phi+\bar{c}-\bar{\bar{c}})/2} \\
\lambda^{(\delta-\phi+\bar{c}-\bar{\bar{c}})/2} & \lambda^{\bar{c}-\bar{\bar{c}}}
\end{array} \right) \otimes \left( \begin{array}{ccc}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{array} \right), \tag{5.6}
\]

from which the neutrino mass matrix is given by seesaw mechanism\(^{24}\)

\[
M_\nu = M_N M_R^{-1} M_N^T = \lambda^{4-2n+c-\bar{c}} \left( \begin{array}{ccc}
\lambda^2 & \lambda^{2-r} & \lambda \\
\lambda^{2-r} & \lambda^{2-2r} & \lambda^{1-r} \\
\lambda & \lambda^{1-r} & 1
\end{array} \right) \langle H(10, \mathbf{5}) \rangle^2 \eta^2, \tag{5.7}
\]

where we use the relation (4.3).

Combining those charged lepton sector and neutrino sector, we finally obtain the Maki-Nakagawa-Sakata matrix\(^{\star}\),

\[
U_{MNS} = \left( \begin{array}{ccc}
1 & \lambda^r & \lambda \\
\lambda^r & 1 & \lambda^{1-r} \\
\lambda & \lambda^{1-r} & 1
\end{array} \right). \tag{5.9}
\]

Recent experiments on atmospheric neutrino have suggested a very large mixing angle between second and third generation, and thus \(r = 1/2, 1\) may be favorable (for the case of \(\star\) while in the case \(r > 1 (1, 2, 1')\), we get

\[
U_{MNS} = \left( \begin{array}{ccc}
1 & \lambda & \lambda^r \\
\lambda & 1 & \lambda^{r-1} \\
\lambda^r & \lambda^{r-1} & 1
\end{array} \right). \tag{5.8}
\]
(1,1',2), i.e., \( r \leq 1 \))\(^{**}\). It turns out that \( r = 1/2 \) actually leads to bi-large neutrino mixing angle, which has been already examined within the \( SO(10) \) model in Ref.14\(^{**}\). Indeed if we take \( r = 1/2 \), namely,
\[
c - \bar{c} = \phi - \bar{\phi} + 1, \tag{5-10}
\]
the MNS matrix is given by
\[
U_{MNS} = \begin{pmatrix}
1 & \frac{\lambda^{1/2}}{2} & \lambda \\
\frac{\lambda^{1/2}}{2} & 1 & \frac{\lambda^{1/2}}{2} \\
\lambda & \frac{\lambda^{1/2}}{2} & 1
\end{pmatrix}, \tag{5-11}
\]
which gives bi-large mixing angles for neutrino sector, since \( \lambda^{1/2} \sim 0.5 \). At the same time it predicts \( V_{e3} \sim \lambda \). It is interesting whether the future experiments will observe some evidence just below the CHOOZ upper limit \( V_{e3} \leq 0.15 \).\(^{25}\) For the neutrino masses, the model predicts that \( m_{\nu_\mu}/m_{\nu_\tau} \sim \lambda \), which is consistent with the experimental data: \( 1.6 \times 10^{-3}(\text{eV})^2 \leq \Delta m^2_{\text{atm}} \leq 4 \times 10^{-3}(\text{eV})^2 \) and \( 2 \times 10^{-5}(\text{eV})^2 \leq \Delta m^2_{\text{sol}} \leq 1 \times 10^{-4}(\text{eV})^2 \), which is the allowed region for the most probable MSW solution for the solar neutrino (LMA).\(^{8}\)

If we take the condition
\[
\phi - \bar{\phi} = 2n - 10 - l, \tag{5-12}
\]
the neutrino mass matrix is obtained as
\[
M_\nu = \lambda^{-(5+l)} \begin{pmatrix}
\lambda^2 & \lambda^{2-r} & \lambda \\
\lambda^{2-r} & \lambda^{2-2r} & \lambda^{1-r} \\
\lambda & \lambda^{1-r} & 1
\end{pmatrix} \langle H(10,5) \rangle^2 \eta^2, \tag{5-13}
\]
where we use the relation (5-10). From the above equation, we obtain
\[
\lambda^l = \lambda^{-5} \langle H(10,5) \rangle^2 \eta^2 \frac{m_{\nu_e}}{M_P}. \tag{5-14}
\]
We are supposing that the cutoff scale \( M_P \) is in a range \( 10^{16}(\text{GeV}) < M_P < 10^{20}(\text{GeV}) \), which allows us to take \(-2 \leq l \leq 2\). If we take \( l = 0 \), the neutrino masses are given by \( m_{\nu_e} \sim \lambda^{-5} \langle H(10,5) \rangle^2 \eta^2 / M_P \sim m_{\nu_\mu} / \lambda \sim m_{\nu_\tau} / \lambda^2 \). If we take \( \eta \langle H(10,5) \rangle = 100 \text{ GeV}, \) \( M_P \sim 10^{18} \text{ GeV} \) and \( \lambda = 0.2 \), then we get \( m_{\nu_e} \sim 3 \times 10^{-2} \text{ eV}, m_{\nu_\mu} \sim 6 \times 10^{-3} \text{ eV} \) and \( m_{\nu_\tau} \sim 1 \times 10^{-3} \text{ eV} \). From such a rough estimation, we can obtain almost desirable values

\(^{**}\) In the case of \((1,2,1')\), the parameter value \( r = 3/2 \) can be a candidate for reproducing the large mixing indicated by atmospheric neutrino experiments.

\(^{***}\) When \( r = 1 \), the fermion mass matrices becomes so-called lopsided type. It seemingly gives small mixing angle solution for solar neutrino problem. However, recently it has been pointed out that taking account of \( O(1) \) coefficients, the lopsided type mass matrices can reproduce even large mixing angle solution for solar neutrino problem.
for explaining the experimental data from the atmospheric neutrino and large mixing angle (LMA) MSW solution for solar neutrino problem.\textsuperscript{26} This LMA solution for the solar neutrino problem gives the best fitting to the present experimental data.\textsuperscript{27}

Finally we would like to make a comment on an interesting feature of this scenario which is seen also in $SO(10)$ model.\textsuperscript{14} In addition to the eq.(4.1), interactions

$$\lambda^{\psi_i + \psi_j + 2a + h} \psi_i A^2 \psi_j H$$

(5.15)

also contribute to the Yukawa couplings after $A$ develops non-vanishing VEV. Here only $A^2$ appears because of its odd $Z_2$ parity. Since $\langle A \rangle$ is proportional to the generator of $B-L$, the contribution to lepton Yukawa coupling is nine times larger than that to the quark Yukawa couplings. If one takes $a = -2$, the additional matrices are

$$\Delta M_u^{\langle H(10,5) \rangle} = \frac{V^2}{4} \begin{pmatrix} \lambda^2 & \lambda & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta M_d^{\langle H(10,5) \rangle} = \frac{V^2}{4} \begin{pmatrix} \lambda^2 & 0 & \lambda \\ \lambda & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

(5.16)

$$\Delta M_e^{\langle H(10,5) \rangle} = \frac{9V^2}{4} \begin{pmatrix} \lambda^2 & \lambda & 0 \\ 0 & 0 & 0 \\ \lambda & 1 & 0 \end{pmatrix}.$$  

(5.17)

Note that the additional terms contribute mainly to the lepton sector. It is interesting that this modification essentially changes the mass eigenvalues of only the first and second generation, so it is natural to expect that the realistic mass pattern can be obtained by this modification: It changes the unrealistic prediction $m_\mu = m_s$ at the GUT scale, by keeping the beautiful prediction $m_b = m_\tau$ at GUT scale (GUT relation)\textsuperscript{∗}). This enhancement factor $2 \sim 3$ of $m_\mu$ can be enough to improve the unwanted situation of the lepton quark relation of the second family.

Remarkably enough, this charge assignment of $A$ determines the scale of $\langle A \rangle \sim \lambda^2$. This strong correlation of the unification scale, which is a bit smaller than the Planck scale, and the improvement of the undesired GUT prediction $m_\mu = m_s$ is indeed a consequence of $U(1)_A$. It is also interesting that the SUSY zero plays an essential role again. When $z, \bar{z} \geq -4$, the terms $\lambda^{\psi_i + \psi_j + a + z + h} Z \psi_i A \psi_j H + \lambda^{\psi_i + \psi_j + 2z + h} Z^2 \psi_i \psi_j H$ also contribute to the fermion mass matrices, though only to the first generation.

\section*{§6. SUSY Breaking and FCNC}

\textsuperscript{*} Strictly speaking, terms, which are forbidden by SUSY zero mechanism, are generically induced by integrating out heavy fields which are introduced for solving the DT splitting problem. These terms may give the small correction to the GUT relation $m_b = m_\tau$.  

15
Finally we discuss SUSY breaking. Since we should assign the anomalous $U(1)_A$ charges dependent on the flavor to produce the hierarchy of Yukawa couplings, generically the non-degenerate scalar fermion masses are induced through the anomalous $U(1)_A$ D-term.\textsuperscript{**) Various experiments on the FCNC processes give strong constraints to the off-diagonal terms $\Delta$ in the sfermion mass matrices on the basis on which the flavor changing terms appear only in the non-diagonality of the sfermion propagators as in Ref.30. The sfermion propagators can be expanded in terms of $\delta = \Delta/\tilde{m}^2$ where $\tilde{m}$ is an average sfermion mass. As long as $\Delta$ is sufficiently smaller than $\tilde{m}^2$, it is enough to take the first term of this expansion and, then, the experimental information concerning FCNC and CP violating phenomena is translated into upper bounds on these $(\delta_{ij}^F)_{XX}$'s, where $F = U, D, N, E$, the chirality index $X, Y = L, R$ and the generation index $i, j = 1, 2, 3$. For example, the experimental value of $K^0 - \bar{K}^0$ mixing gives

\[
\sqrt{|\text{Re}(\delta_{12}^D)_{LL}(\delta_{12}^D)_{RR}|} \leq 2.8 \times 10^{-3} \left( \frac{\tilde{m}_q (\text{GeV})}{500} \right),
\]

(6.1)

\[
|\text{Re}(\delta_{12}^D)_{LL}|, |\text{Re}(\delta_{12}^D)_{RR}| \leq 4.0 \times 10^{-2} \left( \frac{\tilde{m}_q (\text{GeV})}{500} \right),
\]

(6.2)

with $\tilde{m}_q$, an average value of squark masses.\textsuperscript{*}) The $\mu \to e\gamma$ process gives

\[
|(\delta_{12}^E)_{LL}|, |(\delta_{12}^E)_{RR}| \leq 3.8 \times 10^{-3} \left( \frac{\tilde{m}_l (\text{GeV})}{100} \right)^2,
\]

(6.3)

where $\tilde{m}_l$ is an average mass of scalar leptons. In the usual anomalous $U(1)_A$ scenario, $\Delta$ can be estimated as

\[
(\Delta_{ij}^F)_{XX} \sim \lambda^{\lfloor f_i - f_j \rfloor} (|f_i - f_j|) \langle D_A \rangle,
\]

(6.4)

since the mass difference is given by $(f_i - f_j) \langle D_A \rangle$, where $f_i$ is the anomalous $U(1)_A$ charge of $F_i$. Here the reason for appearing the coefficient $\lambda^{\lfloor f_i - f_j \rfloor}$ is that the unitary diagonalizing matrices are given by

\[
\begin{pmatrix}
1 & \lambda^{\lfloor f_i - f_j \rfloor} \\
-\lambda^{\lfloor f_i - f_j \rfloor} & 1
\end{pmatrix},
\]

(6.5)

\textsuperscript{**) The large SUSY breaking scale can avoid the flavor changing neutral current (FCNC) problem,\textsuperscript{28,29} but in our scenario it does not work because the anomalous $U(1)_A$ charge of the Higgs $H$ is inevitably negative to forbid the Higgs mass term in tree level.

\textsuperscript{*}) The CP violation parameter $\epsilon_K$ gives about one order severer constraints on the imaginary part of $(\delta_{12}^F)_{XY}$ than the real part. We here concentrate ourselves only on the constraints from the real part of $K^0\bar{K}^0$ mixing, since under the other experimental constraints to the CP phase originated from SUSY breaking sector, which are mainly given by electric dipole moment, we may expect that the CP phases are small enough to satisfy the constraints from the imaginary part of the $K^0\bar{K}^0$ mixing.
In our scenario, the anomalous $U(1)_A$ charge of $\bar{5}_1$ is the same as that of $\bar{5}_2$, namely the sfermion masses of $\bar{5}_1$ and $\bar{5}_2$ are almost degenerate, which weakens the constraints from these FCNC processes. This is because the constraints from the $K^0 - \bar{K}^0$ mixing and the CP violation to the product $(\delta_{12})_{LL} \times (\delta_{12})_{RR}$ are much stronger than those to $(\delta_{12})^2_{LL}$ or $(\delta_{12})^2_{RR}$ as shown in eq. (6-1) and (6-2). Therefore suppression of $(\Delta_{12})_{RR}$ makes the constraints much weaker. Now that the constraints from the $K^0 - \bar{K}^0$ mixing (and the CP violation) become weaker as discussed above, we have larger region in the parameter space, where the lepton flavor violating processes like $\mu \rightarrow e\gamma$ are appreciable. Actually, if the ratio of the VEV of $D_A$ to the gaugino mass squared at the GUT scale is given by

$$R \equiv \frac{\langle D_A \rangle}{M_{1/2}^2},$$

(6.6)

the scalar fermion mass square at the low energy scale is estimated as

$$\tilde{m}_{F_i}^2 \sim f_i RM_{1/2}^2 + \eta_F M_{1/2}^2,$$

(6.7)

where $\eta_F$ is a renormalization group factor. Therefore in our scenario, the eq.(6-2) for $(\delta_{12})_{LL}$ becomes

$$(\delta_{12})_{LL} \sim \lambda \frac{(\psi_1 - \psi_2)RM_{1/2}^2}{(\eta_{D_L} + \frac{\psi_1 + \psi_2}{2} R)M_{1/2}^2} = \lambda \frac{(\psi_1 - \psi_2)R}{(\eta_{D_L} + \frac{\psi_1 + \psi_2}{2} R)}$$

$$\leq 4.0 \times 10^{-2} \left( \frac{(\eta_{D_L} + \frac{\psi_1 + \psi_2}{2} R)^{1/2} M_{1/2} (\text{GeV})}{500} \right),$$

(6.9)

which is rewritten

$$M_{1/2} \geq 1.25 \times 10^4 \frac{\lambda (\psi_1 - \psi_2) R}{(\eta_{D_L} + \frac{\psi_1 + \psi_2}{2} R)^{3/2} (\text{GeV})}.$$  

(6.10)

Though the main contribution to $(\delta_{12})_{RR}$ vanishes, through the mixing in eq. (4-4) and (4-5), $(\delta_{12})_{RR}$ is estimated as

$$(\delta_{12})_{RR} \sim \lambda \frac{1}{2} \frac{\lambda^2 (-\psi_2) R}{\eta_{D_R} + \psi_1 R},$$

(6.11)

where the mixing $\lambda^2 \frac{1}{2}$ is different from the naively expected value $1 = \lambda^{\psi_1 - \psi_1}$. From the eq.(6-1) for $\sqrt{(\delta_{12})_{LL}^2 + (\delta_{12})_{RR}^2}$, the constraint to the gaugino mass $M_{1/2}$ is given by

$$M_{1/2} \geq 1.8 \times 10^5 \frac{\lambda^{1.75} R \sqrt{\psi_2 (\psi_1 - \psi_2)}}{(\eta_{D} + \psi_1 R)^{1.5}}.$$  

(6.12)

On the other hand, the eq.(6-3) for $(\delta_{12})_{RR}$ leads to

$$M_{1/2} \geq 1.6 \times 10^3 \frac{(\lambda (\psi_1 - \psi_2) R)^{1/2}}{\eta_{D_R} + \frac{\psi_1 + \psi_2}{2} R} (\text{GeV}).$$

(6.13)

Taking probable values, $\psi_1 = 5$, $\psi_2 = 4$, $\eta_{D_L} \sim \eta_{D_R} \sim 6$ and $\eta_{E_R} \sim 0.15$, the lower limits of the gaugino mass are roughly estimated as in Table.1.
Note that when $R = 0.1$, the $\mu \to e\gamma$ process gives the severest constraint in these FCNC processes.\textsuperscript{31} Therefore the lepton flavor violating processes\textsuperscript{31, 32} might be seen in future.

The reason for suppression of $(\Delta D_{12})_{RR}$ is that the anomalous $U(1)_{A}$ charge of $\bar{5}_2$ becomes the same as that of $\bar{5}_1$ because the fields $\bar{5}_1$ and $\bar{5}_2$ are originated from a single field $\tilde{\Psi}_1$. This is a non-trivial situation. The massless mode of the second generation $\bar{5}_2 = \psi_1(10, \bar{5}) + \lambda^{5/2}\tilde{\psi}_3(16, \bar{5})$ has Yukawa couplings through the second term $\lambda^{5/2}\tilde{\psi}_3(16, \bar{5})$. However, for SUSY breaking term which is proportional to the anomalous $U(1)_{A}$ charge, the contribution from the first term dominates the one from the second term, which realizes the degenerate SUSY breaking terms between the first and the second generation. It is obvious that the twisting mechanism in $E_6$ unification plays an essential role to realize this non-trivial structure. Note that such a structure is realized only when $(\bar{5}_1, \bar{5}_2) = (1, 1')$ case,\textsuperscript{4)} in which bi-large neutrino mixing angle is also realized. It is suggestive that the requirement to reproduce the bi-large mixing angle in neutrino sector leads to this non-trivial structure, which suppresses the FCNC processes.\textsuperscript{5)} In this way, such a non-trivial structure is automatically obtained in $E_6$ model, which is much different from the $SO(10)$ model in which the condition can be satisfied only by hand.

<table>
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<td>$</td>
<td>370</td>
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Table 1. Lower bound of gaugino mass $M_{1/2}$ at GUT scale (GeV).

\textsuperscript{4)} The option $(1, 1', 2')$ is not realistic to reproduce the large mixing angle indicated by atmospheric neutrino experiment.

\textsuperscript{5)} We should comment on $D$-term contribution to the scalar fermion masses. Generically such $D$-term has non-vanishing VEV\textsuperscript{33} when the rank of the gauge group is reduced by the symmetry breaking and SUSY breaking terms are non-universal. In our scenario, when $E_6$ gauge group is broken to $SO(10)$ gauge group, the $D$-term contribution gives different values to the sfermion masses of $16$ and $10$ of $SO(10)$, which destroys the natural suppression of FCNC in the $E_6$ unification. However, if SUSY breaking parameters become universal by some reason, the VEV of $D$ can become negligible. Actually, the condition $m^2_\phi = m^3_\phi$ makes the VEV of the $D$ much suppressed. Therefore in principle, we can control the $D$-term contribution, though it is dependent on the SUSY breaking mechanism.
breaking scales at high energy and also the hierarchical structure of the Yukawa couplings, are determined without any ambiguity. Only the exception is the SUSY breaking scale and the electroweak breaking scale, which we here adjusted from the experimental W masses. Even if the SUSY breaking scale is introduced by hand, we have to explain why the SUSY Higgs mass parameter $\mu$ is around the SUSY breaking scale (the $\mu$ problem). One of the possible solution for the $\mu$ problem, has recently been examined in Ref.34. Here we recapitulate the essence. The SUSY Higgs mass which is forbidden by the SUSY zero mechanism can be induced when SUSY is broken. Thus the parameter $\mu$ must be proportional to a SUSY breaking parameter, and its coefficient is determined by anomalous $U(1)_A$ charges. Let us introduce the GUT gauge singlets $S$ with positive $s$ and $Z$ with negative $z$ with the mass term $\lambda^sSZ$ in the superpotential. Since $S$ has positive charge, it has vanishing VEV in SUSY vacua (see Appendix). When SUSY is broken, generically tadpole term $\lambda^sAS$ ($A$ is a SUSY breaking parameter) is induced in the SUSY breaking potential $V_{SB}$. As the result, the $S$ field develops non-vanishing VEV as

$$
\langle S \rangle = \lambda^{-s-2z}A. \tag{7.1}
$$

Using this VEV shift, we can generically obtain the $\mu$ term proportional to the SUSY breaking parameter $A$. In our $E_6$ scenario, introducing the superpotential

$$
W = \lambda^{s+\phi+2h}S\Phi H^2, \tag{7.2}
$$

the SUSY Higgs mass $\mu$ is obtained as

$$
\mu \sim \lambda^{2(h-z)+\frac{1}{2}(\phi-\bar{\phi})}A. \tag{7.3}
$$

Therefore, if

$$
-1 \leq 2(h-z) + \frac{1}{2}(\phi - \bar{\phi}) \leq 1, \tag{7.4}
$$

the $\mu$ parameter becomes naturally around the SUSY breaking scale. Moreover, $E_6$ gauge singlet fields $S$ and $Z$ can be identified to composite operators, for example, we can take $Z \sim \bar{C}C$ or $Z \sim \bar{\Phi}\Phi$.

In our $E_6$ case the minimal field contents are, in addition to $\Theta$, three matter multiplets, $\Psi_i(27)$, a pair of Higgs fields ($\Phi(27)\bar{\Phi}(27)$) and a pair of Higgs fields, $C(27)$, $\bar{C}(\bar{27})$, which needs for the breaking $E_6 \rightarrow SO(10) \rightarrow SU(5)$ standard gauge groups, together with an adjoint field $A(78)$ which also provides a natural D-T splitting mechanism as explained in $SO(10)$ model in separate papers,$^{14,22}$ and leaves light Higgs doublets $H$. Among those minimal contents of matters and Higgs fields, we have nine charges, $(\psi_1, \psi_2, \psi_3, \bar{\phi}, \bar{\phi}, (c, \bar{c}), a, h)$, which determine the main features of the mass matrices of quarks and leptons. First the
CKM mixing angle almost fixes the charges of the matters $\psi_i = (n+3, n+2, n)$ and the doublet Higgs $h = -2n$, and in order to get bi-large mixing angle for neutrino sector we need a constraint on the charges of Higgs fields, $r = 1/2$, i.e., $c - \bar{c} = \phi - \bar{\phi} + 1$, and also we have the constraint $\phi - \bar{\phi} = 2n - 10 - l$ ($-2 \leq l \leq 2$) in order to reproduce the neutrino masses. If we take the charge $a = -2$ in order to realize that the GUT relation between the masses of down type quark and charged lepton is workable only for third generation, the remaining freedom is now just three. Moreover, it may be possible to build DT splitting models in which the light Higgs can be identified to the components of $\Phi$ or $C$. Actually, it is naturally realized that $\Phi$ can play a role of $H$, so in that case, $\phi = h$.\textsuperscript{22} It means that the remaining freedom is only two. If we further impose the condition for solving the $\mu$ problem (7.4), the freedom is only one. It is completely non-trivial fact that there is a set of charge assignment which can realize all the above features. Actually, if we take $n = 2, \phi = h = -4, \bar{\phi} = 0, c = -4, \bar{c} = -1$, all the above conditions are satisfied for $l = -2$. The charge assignment $n = 2, \phi = h = -4, \bar{\phi} = 3, c = -6, \bar{c} = 0, l = 1$ is quite interesting because the composite operator $\bar{\Phi}\Phi$ can even play the same role of $\Theta$ field when $\xi^2 \sim \lambda$. Namely $\langle \bar{\Phi}\Phi \rangle \sim \lambda \equiv \xi^2$. In this case, we have the minimum model where there are $\Psi_1, \Psi_2, \Psi_3, \Phi, \bar{\Phi}, C, \bar{C}$ and $A$, where all the charges are uniquely determined.

What is interesting in $E_6$ unified model is that the condition for suppression of FCNC is automatically satisfied. The essential point is that the first and second generation fields of $\bar{5}$ have the same anomalous $U(1)_A$ charge because these fields are originated from a single field $\Psi_1$.

The aspect of family structure which has been recently made clear by the neutrino experiments gives really a leading guide to investigate the origin of the family. The scenario discussed here is quite impressive and makes us to expect the possibility that we can find “the real GUT” in near future.

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Appendix A

In this appendix, we explain how the vacua of the Higgs fields are determined by the anomalous $U(1)_A$ quantum numbers.

First of all, we show that none of the field with positive anomalous $U(1)_A$ charge gets nonzero VEV if the Froggatt-Nielsen (FN) mechanism works well in the vacuum. Let the gauge singlet fields be $Z_i^\pm (i = 1, 2, \cdots n_\pm)$ with charges $z_i^\pm$ with $z_i^+ > 0$ and $z_i^- < 0$. From the $F$ flatness conditions of the superpotential we get $n = n_+ + n_-$ equations plus one $D$-flatness condition,

$$\frac{\delta W}{\delta Z_i} = 0, \quad D_A = g_A \left( \sum_i z_i |Z_i|^2 + \xi^2 \right) = 0,$$

where $\xi^2 = \frac{g_A^2 Q_A^2}{192 \pi^2} (\equiv \lambda^2 M_P^2)$. At a glance, these look to be over determined. However, the $F$ flatness conditions are not independent because the gauge invariance of the superpotential $W$ leads to a relation$^1$

$$\frac{\delta W}{\delta Z_i} z_i Z_i = 0.$$  \hfill (A.2)

Therefore, generically SUSY vacuum with $\langle Z_i \rangle \sim M_P$ exists (Vacuum a), because the coefficients of the above conditions are generally of order 1. However, if $n_+ \leq n_-$, we can take another vacuum (Vacuum b) with $\langle Z_i^+ \rangle = 0$, which automatically satisfy the $F$-flatness conditions $\frac{\delta W}{\delta Z_i} = 0$. Then $\langle Z_i^- \rangle$ are determined by $F$-flatness conditions $\frac{\delta W}{\delta Z_i} = 0$ with a constraint (A.2) and $D$-flatness condition $D_A = 0$. Note that if $\lambda < 1$ (i.e., $\xi < 1$), the VEVs of $Z_i^-$ are less than the Planck scale, that can lead to Froggatt-Nielsen mechanism. If we fix the normalization of $U(1)_A$ gauge symmetry so that the maximal value of $z_i^-$ equals 1, then the VEV of the field $Z_i^-$ is determined from $D_A = 0$ as $\langle Z_i^- \rangle = \lambda$, which breaks $U(1)_A$ gauge symmetry. (The field $Z_i^-$ was introduced in the previous section as $\Theta$.) On the other hand, other VEVs are determined by $F$-flatness conditions of $Z_i^+$ as $\langle Z_i^- \rangle \sim \lambda^{-z_i^+}$, which is shown below. Since $\langle Z_i^+ \rangle = 0$, it is sufficient to examine the terms linear in $Z_i^+$ in the superpotential in order to determine $\langle Z_i^- \rangle$. Therefore, in general the superpotential can be written

$$W = \sum_{i}^n W_{Z_i^+},$$  \hfill (A.3)

$$W_{Z_i^+} = \lambda^{z_i^+} Z_i^+ \sum_{j}^n \lambda^{z_j^-} Z_j^- + \sum_{j,k}^n \lambda^{z_j^- + z_k^-} Z_j^- Z_k^- + \cdots$$  \hfill (A.4)

$^1$ We thank H. Nakano for pointing out this relation.
\[ \sum_i^{n_+} \tilde{Z}_i^+ (\sum_j^{n_-} \tilde{Z}_j^- + \sum_{j,k}^{n_-} \tilde{Z}_j^- \tilde{Z}_k^- + \cdots), \quad (A.5) \]

where \( \tilde{Z}_i \equiv \lambda^{z_i} Z_i \). The \( F \)-flatness conditions of \( Z_i^+ \) fields require

\[ \lambda^{z_i} (1 + \sum_j \tilde{Z}_j^- + \cdots) = 0, \quad (A.6) \]

which generally lead to solutions \( \tilde{Z}_j \sim O(1) \) if these \( F \)-flatness conditions determine the VEVs. Thus the \( F \)-flatness condition demands,

\[ \langle Z_j \rangle \sim O(\lambda^{-z_j}). \quad (A.7) \]

Here we have examined the VEVs of singlets fields, but generally the gauge invariant operator \( O \) with negative charge \( o \) has non-vanishing VEV \( \langle O \rangle \sim \lambda^{-o} \) if the \( F \)-flatness conditions determine the VEV.

If the vacuum \( a \) is selected, the anomalous \( U(1)_A \) gauge symmetry is broken at the Planck scale and the FN mechanism does not work. Therefore, we cannot know the existence of the \( U(1)_A \) gauge symmetry from the low energy physics. On the other hand, if the vacuum \( b \) is selected, the FN mechanism works well and we can understand the signature of the \( U(1)_A \) gauge symmetry from the low energy physics. Therefore, it is natural to assume that the vacuum \( b \) is selected in our scenario, in which the \( U(1)_A \) gauge symmetry plays an important role for the FN mechanism. Namely, the VEVs of the fields \( Z_i^+ \) vanish, that guarantee that the SUSY zero mechanism works well.

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