The principle of teleportation is used to perform a quantum computation even before its quantum input is defined. The basic idea is to perform the quantum computation at some earlier time with qubits which are part of an entangled state. At a later time a generalized Bell state measurement is performed jointly on the then defined actual input qubits and the rest of the entangled state. This projects with a certain probability the output state onto the correct one.

The quantum computer [1] – a new type of machine that exploits the quantum properties of information – could perform certain types of calculations with exponential speedup over any foreseeable classical computer. Quantum teleportation [2] – one of the most basic information procedures in quantum mechanics – enables transmission and reconstruction of a general quantum state over arbitrary distances. Here we show that the principle of teleportation can be used to perform a quantum computation even before its quantum input is defined. This allows a certain probability to obtain the output of an arbitrary long quantum computation immediately after its input is given.

Imagine that an engineer is given a certain problem of such a complexity that in order to obtain its solution within reasonable time she uses a quantum computer. Suppose that the conditions on the quantum computation are the following:

1. At time $t_1$ the engineer is given an input to the quantum computation in an arbitrary quantum state unknown to her.
2. The engineer is required to give the output of her computation at time $t_2$. If however she is not sure that her output is the right one (e.g., because her computer has not finished the computation before $t_2$) she is allowed to not give any state. Such a situation is denoted by "no answer".
3. The engineer is strongly advised not to overestimate her computational resources. By this we mean that the engineer’s choice to give no answer is not evaluated negatively, and that an incorrect result is evaluated more negatively than the correct one is evaluated positively (e.g., one may imagine that she obtains $P$ positive points for the correct result, 0 points for no answer, and $N$ negative points for an incorrect result, where $N$ is much larger than $P$).

For the purpose of evaluating the preciseness of the engineer’s computation, one may imagine that whenever the engineer decides to give the output of her computation it is subjected to a kind of "check-measurement" in the basis in which one of the basis states is the right output state. Denote the outcome corresponding to the right output by $O$. Then, only if the measurement gives the result $O$ does the engineer gain $P$ points, otherwise she loses $N$ points.

Normally, after the engineer gets the input $n$ qubits (qubits 1) for the quantum computation, she feeds it into her computer and starts the quantum computation. Now assume the quantum computation is very
time-consuming, so that this procedure is not fast enough and the quantum computer does not terminate before the deadline at \( t_2 \), as illustrated in Fig. 1a. In such a situation the engineer, for instance, can decide not to give an answer, which results in a total of zero points. Alternatively, she can choose any state at random and give this as the output of her computation. This however leads to a high negative score because the probability of \( 1 - (1/2)^n \) not to obtain the result \( O \) in the check-measurement is higher than the one of \( 1/2^n \) to obtain this result for a \( n \)-qubit state \((n > 1)\) chosen at random.

We will now show that there is an alternative strategy where the engineer can obtain the exact output state of an arbitrarily long quantum computation with some probability instantaneously. This strategy is based on quantum teleportation. Quantum teleportation is the transmission and reconstruction over arbitrary distances of the state of a quantum system. During teleportation, an initial system in the state that is to be transferred and one of a pair of entangled subsystems are subjected to a Bell state measurement, such that the second subsystem of the entangled pair acquires the state of the initial system. The later subsystem is brought into the state of initial system by an accordingly chosen transformation after receiving via classical communication channel the information which of the Bell-state results was obtained.

Now imagine that the engineer has \( n \) entangled pairs of qubits 2 and 3. She can feed members of the entangled pairs (qubits 3) into her quantum computer long before \( t_1 \), when the actual input for her computation is given to her, as illustrated in Fig. 1b. This means that during the computation the qubits 3 in her quantum computer are entangled to the qubits 2. At some point her computation will terminate and output qubits 3.

As soon as she obtains the input qubits 1 the engineer performs the (generalized) Bell state measurement on qubits 1 and 2. In \( (1/4)^n \) cases the whole state of qubits 3 is projected onto the state resulting from the correct input and she does not have to perform any additional transformation on qubits 3. In the remaining \( 1 - (1/4)^n \) cases, the result of the engineer’s Bell state analysis will not be the right one. Yet it is obvious that situations can exist where it is of enormous advantage to have the correct output state for a problem at a very early time even if only with small probability. Our schemes clearly achieves this goal for the fraction \( (1/4)^n \) of all cases [3].

We now consider also the remaining cases. In the usual teleportation procedure, the engineer would have to perform a unitary transformation on her qubits 3. But now she has already fed them into her quantum computer. Because the set of unitary transformations necessary to finish the usual teleportation procedure and her computation do not commute in general, she has to invert the full quantum computation performed so far, perform the unitary transformation required by the teleportation procedure, and start the quantum computation again. This results in a total computational time twice as long as the time needed for the conventional computation (if we neglect the time required for the unitary transformations). Note however that often this composite transformation can be performed in a time shorter than if the three transformations are performed successively [4].

This means that in \( 1 - (1/4)^n \) cases in general our scheme will not help the engineer to meet her deadline. In those cases the best strategy for her is to give no answer. But in the successful cases, depending on the length of her computation, the scheme for instantaneous computation may constitute an enormous gain in time, which might be decisive in certain situations. In the present paper we specified one such case through conditions 1-3 listed above. Following the evaluation criterion 3, the averaged point score gained in our scheme is

\[
S_{\text{inst}} = P(1/4)^n,
\]

which exceeds both the score of \( S_{\text{no}} = 0 \) if the engineer constantly provides no answer, and the score of \( S_{\text{rand}} = P(1/2^n) - N(1 - 1/2^n) \), if she constantly chooses the output at random.

We would like to make some comments on the scheme just presented. Firstly, it was here assumed that the input of the computation is a genuine quantum input, i.e.

\[
i.e. \text{it can be in any state from the entire Hilbert space (for example this is a common assumption for quantum simulations [5])}.
\]

Secondly, in our scheme the engineer is allowed to perform the computation only once before the actual input is given to her. Any other alternative scheme which is to be compared with ours should therefore be considered under this condition. Indeed it can be shown that other schemes are possible where this condition is fulfilled, which do not involve quantum teleportation, and still have the fidelity of the output state (the square overlap between engineer’s output state and the correct one) larger than by simple random choice [6]. However, in contrast with our scheme, in such an alternative scheme the engineer cannot infer with certainty whether her output state is the correct one or not. Therefore, if the engineer decides to pass on the output obtained in this alternative scheme, there will always be a certain probability not to obtain the result \( O \) in the check-measurement, which consequently leads to a negative average point score. Clearly such a scheme can never be better than ours for \( N \) sufficiently larger than \( P \).

An interesting observation is that our scheme can also be applied in cases where, for some reason, parts of a quantum computation are performed at distant locations [7]. Imagine two people, Alice and Bob, in two distant locations, each of them performing part of a common quantum computation under conditions 1-3. Suppose that the output qubits (identified with qubits 1 in our scheme) of Alice’s quantum computer are an essential input for Bob’s computation. Suppose also that Bob’s part of computation is very time-consuming.

Imagine that entangled pairs of qubits (identified with
At some point after the European Union. FWF, Project No. F1506 and by the QIPC Program has been supported by the Austrian Science Foundation Tyc for helpful comments and discussions. This work presented by a quantum state without reading the state derives from the possibility to process information represented by a quantum state before its input is defined. We suggest that the ability to perform computational time, i.e. immediately after its quantum computation even before its input is defined. We propose that the one can obtain the output of an arbitrary long computation with non-zero probability in zero computational time, i.e. immediately after its quantum input is defined. We suggest that the ability to perform a quantum computation even before its input is defined derives from the possibility to process information represented by a quantum state without reading the state beforehand.

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[3] Evidently one can have very complicated entangled states as the shared entanglement resources for the teleportation, which are given to the engineer. Then she has to make recourse to appropriate generalizations of Bell state measurements. Yet the basic procedures remain the same with possible change of the efficiencies discussed above.

[4] An interesting observation here is that there is a group $C_2$ of gates (Clifford group) including CNOT-gates, Hadamard-gates and $\pi/2$-shifts, which map Pauli operators $C_1$ into Pauli operators under conjugation, i.e. $C_2 \equiv \{U|UC_1U^\dagger \subseteq C_1\}$ (See M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 2000, UK). This means that the composite transformation of first running these gates backwards, performing a single-qubit transformation (Pauli operators) and then running these gates forwards is equivalent to the performance of a simple single-qubit transformation.


[6] We give an example for an alternative scheme for the case of an one-qubit computation. At a time earlier than $t_1$, the engineer has taken any state, say state $|0\rangle$, as an input of her computation and has performed the computation on it. At the later time $t_1$ when the actual input is given to her, she measures it in the basis $\{|0\rangle, |1\rangle\}$. If the result is $0$, she passes the output state of her computation and if the result is $1$, she passes the state which is obtained by applying the optimal universal NOT (V. Bužek, M. Hillery, R. F. Werner, Phys. Rev. A 60, R2626 (1999)) on the output. The engineer uses the optimal universal NOT because she does not know the state of her output.