Mutual Constraints Between Reionization Models and Parameter Extraction From Cosmic Microwave Background Data

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ABSTRACT

Spectroscopic studies of high-redshift objects combined with increasingly precise data on the cosmic microwave background (CMB) are beginning to place strong bounds on the epoch of hydrogen reionization. Parameter estimation from current CMB data continues, however, to be subject to several degeneracies. Here, we focus on those degeneracies in CMB parameter forecasts related to the optical depth to reionization. We extend earlier work on the mutual constraints that such analyses of CMB data and a reionization model may place on each other to a more general parameter set, and to the case of data anticipated from the MAP satellite. A reionization model provides useful complementary information for cosmological parameter extraction from the CMB, particularly for the degeneracies between the optical depth and either of the amplitude and scalar index of the primordial power spectrum, which are still present in the most recent data. Alternatively, by using a reionization model, known limits on astrophysical quantities can reduce the forecasted errors on cosmological parameters. Forthcoming CMB data also have the potential to constrain the sites of early star formation, as well as the fraction of baryons that participate in it, if reionization were caused by stellar activity at high redshifts. Finally, we examine the implications of an independent, e.g., spectroscopic, determination of the epoch of reionization for the determination of cosmological parameters from the CMB.

Subject headings: cosmic microwave background—cosmological parameters—cosmology: theory—intergalactic medium

1. Introduction

The rapid progress in detector technology has led to the successful operation of many ground- and balloon-based experiments in the last few years for measuring the anisotropies in the CMB. Analyses of the recent data from experiments such as Boomerang (de Bernardis et al. 2001), MAXIMA-1 (Stompor et al. 2001), and DASI (Pryke et al. 2001) have confirmed the adiabatic cold dark matter (CDM) paradigm for describing the development of structure and the properties of the power spectrum of the CMB. They have also revealed that the universe is close to being
spatially flat, and have begun to place tight constraints, in advance of satellite CMB experiments, on the cosmological parameters that describe our universe. Analyses of present data (see papers above, and those of, e.g., Tegmark et al. (2001) and Wang et al. (2001)) indicate, however, that strong degeneracies are still present in parameter extraction from the CMB, so that techniques to break these degeneracies continue to be valuable at present. Many of these degeneracies had been anticipated on theoretical grounds, and several methods to break them using observations of Type Ia SNe (Efstathiou et al. 1999), weak lensing (Hu 2001), redshift surveys (Eisenstein et al. 1999; Wang et al. 1999; Popa et al. 2001), or combinations of these (Efstathiou & Bond 1999) have been proposed. Ongoing and future CMB observations\(^1\) should provide markedly improved constraints on degenerate parameters through detection of polarization in the CMB at large angular scales, and through dramatically increased sky coverage in the case of satellite experiments such as MAP\(^2\) or Planck\(^3\). The latter is especially important for overcoming cosmic variance for CMB multipoles, \(l \lesssim 100\). Current CMB data on the temperature anisotropy at degree and sub-degree scales provide an upper limit of about 0.3 for the optical depth to reionization, which may be translated to a model-dependent constraint on the redshift of hydrogen reionization, \(z_{\text{reion}} \lesssim 25\) (Wang et al. 2001).

Spectroscopic studies of high-\(z\) quasars and galaxies blueward of Ly\(\alpha\) have revealed the lack of a H I Gunn-Peterson (GP) trough, implying that the intergalactic medium (IGM) is highly ionized up to \(z \sim 6\) (Fan et al. 2000; Dey et al. 1998; Hu et al. 1999). Recently, Djorgovski et al. (2001) presented observations of quasars at \(z \gtrsim 5.2\) indicating a steady increase in the opacity of the Ly\(\alpha\) forest for \(z \sim 5.2\)–5.7, while Becker et al. (2001) presented a detection of the GP trough in the spectrum of the highest-redshift quasar known to date at \(z \sim 6.3\) (Fan et al. 2001). Together, these data may be an indication of the epoch of H I reionization occurring not far beyond \(z \sim 6\). As these authors have taken care to note, the detection of the GP trough in a single line-of-sight is not definitive evidence of \(z_{\text{reion}} \sim 6\); it may, however, be probing the end of the gradual process of inhomogeneous reionization coinciding with the disappearance of the last neutral regions in the high-\(z\) IGM. This would be consistent with the lower end of the range of redshifts, \(z \sim 6\)–20, predicted by theoretical models for H I reionization, either semi-analytic (Tegmark et al. 1994; Giroux & Shapiro 1996; Haiman & Loeb 1997; Valageas & Silk 1999; Miralda-Escudé et al. 2000) or based on numerical simulations (Cen & Ostriker 1993; Gnedin 2000; Ciardi et al. 2000; Benson et al. 2001a). Recent reviews of reionization may be found in Shapiro (2001), and Barkana & Loeb (2001). In this work, reionization is always meant to refer to that of H I, rather than He II for which the data indicate reionization at \(z \sim 3\) (see, e.g., Kriss et al. (2001)).

In this paper, we focus on those degeneracies in CMB parameter forecasts that involve the

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\(^1\)Compilations of and links to various CMB experiments may be found at: http://www.hep.upenn.edu/~max/cmb/experiments.html, and http://background.uchicago.edu/~whu/cmbex.html

\(^2\)http://map.gsfc.nasa.gov.

\(^3\)http://astro.estec.esa.nl/SA-general/Projects/Planck.
optical depth to reionization, $\tau$, based on methods developed in a previous work (Venkatesan
(2000); henceforth Paper I) that examined the valuable complementary information provided by a
reionization model. Typically, in CMB parameter extraction, the universe is assumed to reionize
abruptly, leading to discretized values of $\tau$ in the multi-dimensional grid of models being tested
in likelihood analyses of the data. This does not utilize, however, the strong sensitivity of $z_{\text{reion}}$, and hence $\tau$, to specific parameters such as the scalar spectral index of the primordial power
spectrum. As we noted in Paper I, $\tau$ is unique by definition amongst the set of standard cosmological
parameters extracted from CMB data, being the only quantity which is not determined purely by
the physics prior to the first few minutes after the Big Bang. Thus, it can potentially provide
information on post-recombination astrophysical processes, if the other (cosmological) parameters
which affect $\tau$ are well-constrained. We extend Paper I here to a larger parameter set in a $\Lambda$CDM
cosmology; in the spirit of timeliness, we specifically consider the constraints anticipated from the
data from the recently launched $MAP$ satellite, and we also include in our analysis the implications
of an independent, e.g., spectroscopic, determination of $z_{\text{reion}}$. Other improvements are detailed in
the next section.

The plan of this paper is as follows. In §2, we review the reionization model that we consider and
the formalism related to CMB parameter estimation. In §3, we present our results on the projected
parameter yield from $MAP$, and we detail how a reionization model may improve constraints on
cosmological parameters determined from the CMB, and vice versa. We conclude in §4.

2. Overview of the Reionization Model and CMB Analysis

The analysis in this paper essentially follows the methods developed in Paper I, which is
extended here for a $\Lambda$CDM model; the points of departure and improvements here are described
below.

We assume that stars are responsible for reionization, and use the semi-analytic stellar reion-
ization model developed by Haiman & Loeb (1997), with the modifications described in Paper I. We take the primordial matter power spectrum of density fluctuations to be, $P(k) \propto k^n T^2(k)$, where $n$ is the scalar index of the power spectrum, and the matter transfer function $T(k)$ is taken
from Eisenstein & Hu (1998). We normalize $P(k)$ to the present-day rms density contrast over spheres of radius $8 \, h^{-1} \, \text{Mpc}$, $\sigma_8$.

We track the fraction of all baryons in star-forming halos, $F_B$, by the Press-Schechter formalism,
allowing star formation only in halos of virial temperature $\gtrsim 10^4$ K, corresponding to the mass
threshold for the onset of hydrogen line cooling. The details of the adopted stellar spectrum of
ionizing photons and of the solution for the growth of ionization regions around individual halos
may be found in Paper I. We define reionization as the epoch of overlap of individual H II regions,
i.e., when the volume filling factor of ionized hydrogen, $F_{\text{H II}} = 1$. We include the effects of
inhomogeneity in the IGM through a clumping factor, $c_L$ (Shapiro & Giroux 1987), rather than
assuming a smooth IGM as in Paper I. The optical depth to reionization from electron scattering
is then given by:

\[ \tau \simeq 0.057 \Omega_b h \int_0^{z_{\text{reion}}} \frac{(1 + z)^2 \left[ 1 - f_* F_B(z) \right] F_{\text{HII}}(z)}{\sqrt{\Omega_\Lambda + (1 + z)^2(1 - \Omega_\Lambda + \Omega_m z)}} \, dz. \]  

(1)

The optical depth to reionization depends upon a number of parameters, \( \tau = f(\sigma_8, \Omega_b, h, n, \Omega_\Lambda, \Omega_M, f_*, f_{\text{esc}}) \), where \( f_* \) is the fraction of baryons in each galaxy halo forming stars, \( f_{\text{esc}} \) is the escape fraction of H I ionizing photons from individual halos, \( h \) is the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\), and the other symbols have their usual meanings. We fix \( \Omega_K = 1 - \Omega_M - \Omega_\Lambda \). We also set \( f_{\text{esc}} = 0.1 \) (Dove et al. 2000; Leitherer et al. 1995), so that it is no longer a free parameter as it was in Paper I, for the following reasons. First, the mass threshold scale for the Press-Schechter evolution of halos in our model corresponds to halos of virial temperature \( \gtrsim 10^4 \) K. The baryons in such halos are likely to be collisionally ionized, at least partly, so that \( f_{\text{esc}} \ll 1 \) is unlikely. The values of \( f_{\text{esc}} \) in low-mass systems (masses \( \lesssim 10^7 \) M\(_{\odot}\)) at high-\( z \) have been studied by Ricotti & Shull (2000). Second, as shown in Haiman & Loeb (1997) and Paper I (see in particular Table 1 and the associated discussion), \( \tau \) is not very sensitive to the chosen values of \( f_{\text{esc}} \), once they exceed a few percent. Third, limits on \( \tau \) from the CMB, being a single number, can be translated to a constraint on any one non-cosmological parameter that determines \( \tau \); recall, for example, that in Paper I, both \( f_* \) and \( f_{\text{esc}} \) could not be constrained simultaneously from the CMB. Hence we choose to retain \( f_* \) as the primary astrophysical input parameter, as \( \tau \) is most sensitive to it in our chosen reionization model.

To be complete, we note that observations of Lyman-continuum emission from Lyman-break galaxies at \( z \sim 3.4 \) by Steidel et al. (2001) indicate values of \( f_{\text{esc}} \) exceeding 0.5. Also, some simulations of reionization by stars often appear to require or imply similarly high values for \( f_{\text{esc}} \) (Gnedin 2000; Benson et al. 2001b) in order to have \( z_{\text{reion}} \) exceed \( \sim 7 \). The large derived value for \( f_{\text{esc}} \) in the former case arises partly from the definition itself of \( f_{\text{esc}} \); as Steidel et al. (2001) noted, their chosen observational procedure normalized the escape fraction of 900 Å photons to that of 1500 Å photons. Data from the local universe (Deharveng et al. 2001; Leitherer et al. 1995), especially of high-mass systems, generally do not support values of \( f_{\text{esc}} \) exceeding about 10%.

Reionization affects the CMB through the Thomson scattering of CMB photons from free electrons in the IGM. This leads to an overall damping of the primary temperature and polarization anisotropies in the CMB, except at the largest angular scales (small \( l \)), and the generation of a new feature in the polarization power spectrum. The first effect can be distinguished from CMB anisotropies with slightly lower peak amplitudes (corresponding to a lower \( \sigma_8 \) in our model) only at the lowest \( ls \), but cosmic variance obscures the difference at such scales. This is the origin of the amplitude–reionization degeneracy in the CMB temperature power spectrum. However, the reionized IGM creates a linear polarization signal which peaks at the horizon size at \( z_{\text{reion}} \), so that the amplitude and angular location of this new feature are comparatively direct probes of the values of \( \tau \) and \( z_{\text{reion}} \) respectively (Zaldarriaga 1997). A detection of polarization in the CMB can therefore
constrain $\tau$ far more accurately than can temperature data alone, and has the potential to break
the above degeneracy. In practice, it may prove difficult to measure, given that the polarization
anisotropy is expected to be only at the $\sim 10\%$ level relative to that in the CMB's temperature, and
that for late reionization the above feature has an extremely small amplitude (see next section).
Additionally, foregrounds are likely to complicate the extraction of a polarization signal at low $l$.
As we do not consider tensor contributions to the primordial matter power spectrum, polarization
here refers to the E-channel type.

Parameter extraction from the CMB is based on the methods outlined in Paper I. For cases
involving $\tau$ and a set of cosmological parameters, we follow the industry-tested Fisher matrix
formalism in, e.g., Knox (1995), Jungman et al. (1996), and Bond et al. (1997). If we expand the
angular power spectrum of the CMB in terms of its multipole moments $C_l$, assume Gaussian initial
perturbations, and that the $C_l$ are determined by a fiducial set of parameters describing the “true”
universe, then we can quantify the behavior of the likelihood function of observing any set of $C_l$s
near its maximum, given the fiducial parameter set, in terms of the Fisher information matrix,
$F_{ij}$. If we further assume that the likelihood function has a Gaussian form near its maximum, the
elements of $F_{ij}$ can be expressed as the product of pairs of derivatives of the $C_l$ with respect to the
appropriate parameters. The Fisher matrix represents the best accuracy with which parameters in
the chosen “true” model can be estimated from a CMB data set. The inverse of $F_{ij}$ is the covariance
matrix between the parameters; the minimum $1\sigma$ error in a parameter $P_i$ is given by
$\sqrt{(F^{-1})_{ii}}$.

The reionization model, as described above, yields $\tau = \tau(\sigma_8, \Omega_b, h, n, \Omega_\Lambda, \Omega_M, f_*) = \tau(P_{\text{cosmo}}, f_*)$, while the CMB data determines $[P_{\text{cosmo}}, \tau(P_{\text{cosmo}}, f_*)]$. We can therefore use a reionization
model to relate and mutually constrain $(P_{\text{cosmo}}, f_*)$. In such cases, the derivatives of the CMB
multipoles, $C_l$, that are used to construct the Fisher matrix become (Paper I):

$$\frac{\partial C_l}{\partial P_{\text{cosmo}}} = \frac{\partial C_l}{\partial P_{\text{cosmo}}} \bigg|_{\tau} + \frac{\partial C_l}{\partial \tau} P_{\text{cosmo}} \frac{\partial \tau}{\partial P_{\text{cosmo}}} \quad (2)$$

$$\frac{\partial C_l}{\partial f_*} = \frac{\partial C_l}{\partial \tau} \frac{\partial \tau}{\partial f_*} \quad (3)$$

In this work, we focus specifically on the constraints anticipated from the data from the MAP
satellite. We include the effects of instrumental noise, rather than assuming cosmic variance limited
cases as in Paper I. We take experimental specifications and the method of constructing $F_{ij}$ from
Eisenstein et al. (1999), and assume that foregrounds can be effectively subtracted from MAP data
(Tegmark et al. 2000). Parameter estimation is performed using theoretical CMB power spectra
generated by CMBFAST [version 4.0; Seljak & Zaldarriaga (1996)]. This version of CMBFAST
corrects a bug in previous versions related to some models with non-zero values of $\tau$, and includes
an improved treatment of recombination based on the work of Seager et al. (2000). In all the figures
below, the error ellipses, where displayed, represent 68% joint confidence regions.
3. Results

We now discuss the constraints that a stellar reionization model and CMB parameter forecasts may place on each other. We define our standard model (SM) as described by the 7-parameter set, \([\sigma_8, \Omega_b, h, n, \Omega_\Lambda, \Omega_M, \tau/f_\star] = [1.0, 0.04, 0.7, 1.0, 0.7, 0.3, 0.048/0.05] \). We set the clumping factor \(c_L = 30\), which, together with our choice of \(f_{\text{esc}} = 0.1\) (§2), leads to \(\tau \sim 0.048\) for the SM in this work, corresponding to a reionization epoch of \(z_{\text{reion}} = 8\). Our choice of parameters for the SM, though well-motivated and in concordance with a variety of observations, is deliberately constructed to generate late reionization, given the recent observational claim of detecting the last stages of reionization at \(z \sim 6.3\). The semi-analytic treatment here defines reionization as the overlap of H II regions, which somewhat precedes the disappearance of the GP trough in a homogenous IGM (Haiman & Loeb 1999).

As a reference, we show in Figure 1 the angular power spectrum of the CMB for the SM defined above, for both temperature and polarization. As we noted earlier, the main effect of the reionization of the IGM is an overall damping of the primary CMB temperature and polarization anisotropies; it also generates a new feature in the CMB polarization spectrum corresponding to the horizon size associated with \(z_{\text{reion}}\). For the late reionization in our SM, this corresponds to the polarization bump at \(l \lesssim 5\). The signal associated with this unique probe of reionization has an extremely small value, being less than the temperature anisotropy by over two orders of magnitude at these scales.

3.1. Using a Reionization Model to Improve Constraints from the CMB

Using the techniques in §2, we can use a reionization model to constrain cosmological parameters beyond the limits obtained from CMB data alone, through \(\tau\) or \(f_\star\). Let us first focus on the former case. Certain combinations of parameters are well known to be degenerate in CMB parameter extraction, such as \(\tau - \sigma_8^2\) and \(\tau - n\) (see, e.g., the recent analyses by the DASI, MAXIMA-1 and Boomerang collaborations). A reionization model can provide complementary information, as \(\tau\) is itself a function of cosmological parameters, and break such degeneracies. We display this in Figures 2 and 3, for the above combinations of degenerate parameters, where we marginalize only over the respective two-dimensional spaces and keep all the other parameters fixed at their SM values. The dark outer and light inner ellipses correspond to the 1 \(\sigma\) constraint from MAP’s temperature (T), and temperature plus polarization (T+P) data. The thin solid line represents the functional dependence of \(\tau\) on \(n\) or \(\sigma_8^2\) from the reionization model for \(f_\star = 0.05\), and the dashed lines represent the possible range for \(\tau\), given the uncertainty in the value of \(f_\star\). This possible range for \(f_\star\) of \(\sim 0.01-0.15\) comes from the results of numerical simulations and from arguments of avoiding excessive metal pollution of the IGM at late redshifts (Paper I, and references therein); it represents the astrophysical uncertainty in our chosen reionization model, given the choice to set those \(P_{\text{cosmo}}\) other than \(n\) or \(\sigma_8^2\) to their values in the SM. Figures 2 and 3 show that the reionization
model can be valuable in breaking degeneracies in CMB parameter analyses, even given the range in the potential values of $f_\star$. The main source of the dependence of $\tau$ on $n$ and $\sigma_8^2$ is $z_{\text{reion}}$, and to a lesser extent, the term $f_\star F_B$ in eqn. 1, which is never more than a 2% effect in the value of $\tau$ for the SM. Ideally, we would like to characterize $\tau$ as a function of $P_{\text{cosmo}}$, in order to eliminate its dependence on the astrophysical details of reionization. If we neglect the term $f_\star F_B$, eqn. 1 is considerably simplified as $F_{\text{HII}} = 1.0$ along the line-of-sight from the present ($z = 0$) to $z = z_{\text{reion}}$. The problem now reduces to parametrizing $z_{\text{reion}}$ in terms of $P_{\text{cosmo}}$ alone; in reality, however, $z_{\text{reion}}$ is a non-unique function of various cosmological parameters as well as the specific (astrophysical) reionization scenario. The analysis of Griffiths et al. (1999), while having the advantage of being fitted to the available observations at the time, encountered the same problem of being unable to uniquely relate $z_{\text{reion}}$ to the cosmological parameters that they considered ($h$, $n$, $\Omega_0$); the fit provided by them for $\tau$ as a function of these three parameters was purely empirical but not based on any model of the reionizing sources. Thus, the only way to utilize the valuable sensitivity of $\tau$ to $n$ and $\sigma_8^2$ is via a reionization model. The importance of retaining the information contained in $z_{\text{reion}}$, particularly for the lower bound on $n$, was noted in Covi & Lyth (2001), where they pointed out that the choice
Fig. 2.— Constraints from the reionization model and projected data from MAP in the $\tau-n$ plane, after 2-D marginalization over the $[\tau, n]$ space with all other parameters fixed at their SM values. The thin solid line displays $\tau$ as a function of $n$ from the reionization model with $f_*=0.05$, and the dashed lines represent the astrophysical uncertainty in $\tau$, given the permitted range of 0.01–0.15 in the value of $f_*$. The dark outer and light inner ellipses correspond to the 1 $\sigma$ joint confidence regions from MAP’s temperature, and temperature plus polarization data. Note the strong dependence of $z_{\text{reion}}$, and hence $\tau$, on $n$ through the reionization model (thin solid line): for $n = 0.98$–1.02, $z_{\text{reion}} \sim 7.75$–8.2. The thick solid line represents the constraint from a hypothetical independent measurement of $z_{\text{reion}} = 6.5$.

to leave $z_{\text{reion}}$ as a free parameter, e.g., in the analysis of Tegmark et al. (2001), could lead to an artificially lowered value of $n$ from CMB data.

What if, however, there were an independent limit on $z_{\text{reion}}$? One possible method, which involves relating $z_{\text{reion}}$ directly to the fraction of baryons in star-forming halos, $F_B$, has been explored by Covi & Lyth (2001) and Tegmark et al. (1994). Subject to theoretical uncertainties, this is well motivated, as regardless of the details of the nature and the sources of reionization, one requires in the end a certain number of IGM-ionizing photons per baryon in collapsed structures. Another possibility, which may shortly be upgraded to reality, would be a spectroscopic detection of $z_{\text{reion}}$ through the GP effect in the absorption-line spectra of the highest-$z$ sources (see §1). The great advantage of this second kind of independent determination of $z_{\text{reion}}$ is that one may safely bid farewell to the pesky details of “gastrophysics” in parametrizing $\tau$ for CMB parameter extraction!
If we drop the term $f_\star F_B$ in eqn. 1, we can now relate $\tau$ to the $P_{\text{cosmo}}$ other than $n$ and $\sigma_8^2$. This leads to a unique value of $\tau$ in the 2-D space of Figures 2 and 3, which is depicted as the thick solid line for a hypothetical measured value of 6.5 for $z_{\text{reion}}$. Such a detection can be useful in breaking parameter degeneracies, but without, in principle, the uncertainty associated with the astrophysical details of reionization. Note that a detection of $z_{\text{reion}}$ cannot be translated to a unique prior on $\tau$ for multi-parameter marginalization, as the latter is also determined by cosmological parameters such as $\Omega_\Lambda$, $\Omega_b$, etc. Thus, an independent determination of $z_{\text{reion}}$ is best utilized in the 2-D spaces of parameter combinations that are degenerate with $\tau$, such as the examples in Figures 2 and 3.

We now move on to the second case defined at the beginning of this section, where one may translate astrophysical limits to constrain cosmology. We marginalize over the 7-D space of $[f_\star, P_{\text{cosmo}}]$ rather than $[\tau, P_{\text{cosmo}}]$, by using the reionization model to relate them via $\tau$ (eqns. 2 and 3). We can then apply independent limits on $f_\star$ (0.01–0.15) to further constrain $P_{\text{cosmo}}$. Figure 4 displays one such case in the $f_\star$–n subspace for the projected constraints from MAP’s T and T+P data. Despite the error ellipses being lower bounds to those that MAP will provide (given our assumption of successful foreground removal), the entire astrophysical permitted band for $f_\star$ can still reduce the 1-$\sigma$ error for $n$. Although one may propose alternate ranges for $f_\star$, we anticipate that the main point here— that known constraints on $f_\star$ have the power to strengthen limits from

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Fig. 3.— Constraints from the reionization model and projected data from MAP in the $\tau$–$\sigma_8^2$ plane, after 2-D marginalization over the $[\tau, \sigma_8^2]$ space with all other parameters fixed at their SM values. Plot legend is the same as in Figure 2.
the CMB on $P_{\text{cosmo}}$ will still hold true.

![Diagram](image)

Fig. 4.— Constraint from the projected data from MAP in the $f_{\ast}-n$ plane after full 7-D marginalization over the $[f_{\ast}, P_{\text{cosmo}}]$ space. The dark outer and light inner ellipses correspond to the 1 $\sigma$ joint confidence regions from MAP’s temperature, and temperature plus polarization data. Solid horizontal band represents the entire allowed astrophysical range of 0.01–0.15 for $f_{\ast}$.

In summary, using a reionization model can break degeneracies in CMB parameter estimation related to $\tau$, and improve the errors from MAP data on $n$ and $\sigma_8^2$ by factors of at least 3–6 and 3–10 respectively for the case of $f_{\ast} = 0.05$. Alternatively, known astrophysical limits on $f_{\ast}$ can reduce the errors on $P_{\text{cosmo}}$ from MAP, e.g., by up to a factor of 2 for $n$ from MAP temperature data. The strongest cross-constraint in the near future may be provided by an independent measurement of $z_{\text{reion}}$, which could reduce the 1 $\sigma$ errors on parameters that are degenerate with $\tau$, such as $n$ or $\sigma_8^2$, by factors of 3–10. The non-trivial advantage of this last method is that it is independent of one’s choice of reionization model.

3.2. Using CMB Data to Constrain a Reionization Model

Given the framework of this paper, there are at least two ways that forthcoming CMB data may be used to constrain aspects of reionization. First, we can use a reionization model to extract $[f_{\ast}, P_{\text{cosmo}}]$ rather than $[\tau, P_{\text{cosmo}}]$ from CMB data. Table 1 displays the 1 $\sigma$ errors from MAP T and T+P data for full marginalization over the 6-D $[P_{\text{cosmo}}]$ and the 7-D $[\tau, P_{\text{cosmo}}]$ parameter spaces.
Table 1: Projected 1 σ errors from MAP data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without τ</th>
<th>With τ</th>
<th>Ideal MAP</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>T</td>
<td>T+P</td>
<td>T</td>
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<tr>
<td>τ [f⋆]</td>
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<td>0.193</td>
<td>0.022</td>
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<tr>
<td>σ₈²</td>
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<tr>
<td>Ω_b</td>
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<td>0.003</td>
<td>0.004</td>
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<td>h</td>
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<td>0.022</td>
<td>0.026</td>
</tr>
<tr>
<td>n</td>
<td>0.013</td>
<td>0.013</td>
<td>0.03</td>
</tr>
<tr>
<td>Ω_Λ</td>
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</tr>
<tr>
<td>Ω_M</td>
<td>0.019</td>
<td>0.018</td>
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</tr>
</tbody>
</table>

Note. — The 1 σ errors anticipated from MAP, with temperature (T), and temperature plus polarization (T+P) data. The respective columns are: Without τ: 6-D marginalization over [P_{cosmo}] space only; With τ: 7-D marginalization over the full [τ, P_{cosmo}] space; Ideal MAP: sky coverage of 50%, and data is cosmic variance limited to \( l \sim 500 \). With/without τ columns assume 65% sky coverage and include the effects of instrumental noise for MAP. Note that excluding τ from the analysis leads to deceptively small errors for \( n \) and \( σ₈² \) from the temperature data. Entries in brackets represent 1 σ errors from 7-D marginalization over [f⋆, P_{cosmo}] space, using the reionization model.

Both the 6-D and 7-D cases assume 65% sky coverage and factor in the effects of instrumental noise for MAP. Including τ in the analysis significantly worsens error bars from MAP’s T-data, particularly for \( σ₈² \) and \( n \); this can be expected from the degeneracies discussed above. Put another way, excluding τ or setting it to be zero can lead to deceptively small errors in parameters such as \( σ₈² \) and \( n \).

Using the reionization model now to relate \( f_⋆ \) and \( P_{cosmo} \) (eqns. 2 and 3), we see from Table 1 that MAP’s T and T+P data do not constrain \( f_⋆ \) very strongly. If, however, MAP can achieve being cosmic variance limited to \( l \sim 500 \) with 50% sky coverage, which we label as “Ideal MAP” in the table, it is possible to determine \( f_⋆ \) to significantly greater accuracy with T+P data than its currently allowed range. Given that we have not factored in foreground contamination of the CMB polarization signal, which particularly degrades parameter extraction on the (large) scales at which reionization has a unique signature (Tegmark et al. 2000; Baccigalupi et al. 2001), our prediction of strong limits on \( f_⋆ \) from the CMB may be somewhat optimistic.

A second possibility involves using a measurement of τ, particularly through polarization in the CMB; current data place only rough upper limits of \( τ \lesssim 0.3 \). A low net value of τ would imply
that star formation cannot be widespread, or that it has to be fairly inefficient. The majority of the theoretical models to date imply that reionization takes place between \( z \sim 8-20 \). In our SM, we allow star formation only in halos of virial temperature \( \gtrsim 10^4 \) K. If we replace this mass threshold scale in our Press-Schechter evolution with the Jeans mass scale at each redshift, then, for the SM cosmological parameters, we obtain \( \tau \sim 0.078 (0.11) \), and \( z_{\text{reion}} \sim 11.25 (14.2) \) with (without) clumping. Thus, as an example, if \( \tau \) were measured in the future to be \( \lesssim 0.05 \), it would imply that early star formation has to be relatively rare, i.e., occurs in high-mass rather than in low-mass halos at high redshifts, or that it is relatively inefficient (values of \( f_* \) significantly less than in the SM). While this statement relies on our assumptions and adopted reionization model in this work, it is a potential constraint in the near future.

In summary, forthcoming CMB data may be able to constrain the fraction of baryons that participated in early star formation, and, more speculatively, the sites of such stellar activity as well, if reionization were caused by stars.

### 4. Conclusions

We have extended previous work on the mutual constraints that are possible between a reionization model and parameter estimation from CMB data to a more general parameter set in a \( \Lambda \)CDM cosmology, and for the data anticipated from the MAP satellite. A reionization model provides valuable complementary information for cosmological parameter extraction from the CMB. In particular, the well-known \( \tau - \sigma_8^2 \) and \( \tau - n \) degeneracies, which continue to be present in the most recent data from the DASI, MAXIMA-1 and Boomerang experiments, can be broken (see Figures 2 and 3), even when allowing for the effects of the astrophysical uncertainty in the reionization model. Furthermore, using the reionization model in this work improved the projected errors on \( n \) and \( \sigma_8^2 \) from MAP data by respective factors of about 3–6 and 3–10.

Alternatively, we may use the reionization model to relate the astrophysics of reionization to cosmology: independent theoretical limits on \( f_* \) can reduce the forecasted errors on \( P_{\text{cosmo}} \) from MAP, e.g., by up to a factor of 2 for \( n \) (Figure 4). Applying reionization models to CMB data provides the only way, in the absence of an alternate determination of \( z_{\text{reion}} \), to utilize the strong sensitivity of \( \tau \) through \( z_{\text{reion}} \) to parameters such as \( n \) and \( \sigma_8^2 \), which are important inputs to models of inflation and the evolution of structure. The specific dependence of \( z_{\text{reion}} \) on \( n \) through the reionization model can be seen in Figure 2: for the \( f_* = 0.05 \) case, \( z_{\text{reion}} \) increases from 7.75 to 8.2, with respective values of \( \tau \) from \( \sim 0.046 \) to \( \sim 0.05 \), as \( n \) varies from 0.98 to 1.02.

Forthcoming CMB data also have the potential to constrain the sites of early star formation, as well as the fraction of baryons that participate in it, if reionization were caused by stellar activity at high redshifts (§3.2). In particular, if MAP can achieve 50% sky coverage and is cosmic variance limited to \( l \sim 500 \), the \( 1 \sigma \) error for \( f_* \) could be significantly smaller than the current uncertainty in its value (Table 1, “Ideal MAP” column), although it requires a detection of polarization in
the CMB at large angular scales. This signal is, however, of sufficiently small magnitude for late reionization (Figure 1) that it will prove extremely challenging to detect experimentally, especially when foregrounds are included, which we have assumed here can be effectively subtracted. Thus, the utility of CMB data in constraining reionization models, besides being model-dependent, is optimistic at best.

While the anticipated errors from MAP in Table 1 are dependent on the size of our chosen parameter space, any analysis of CMB data cannot include very many fewer parameters than we have considered here. Larger parameter spaces and the inclusion of foregrounds will only increase the projected errors in this work, thereby enhancing the importance of techniques to break parameter degeneracies, including the three presented here— the use of a reionization model, applying known astrophysical limits, and an independent measurement of the reionization epoch. The last method appears particularly promising from the recent detection of a GP trough in the spectra of a quasar at $z \sim 6.3$ (Becker et al. 2001), which could well represent the last stages of non-uniform reionization. We anticipate that this may provide the strongest cross-constraint in the near future, which we have shown ($\S$3.1) could reduce the 1 $\sigma$ errors on parameters that are degenerate with $\tau$, such as $n$ or $\sigma_8^2$, by factors of 3–10 for data from MAP. The great advantage of using a detection of $z_{reion}$ to break such degeneracies is that it is not subject to the details of “gastrophysics” that partly determine the optical depth to reionization. A measurement of $z_{reion}$ cannot necessarily be translated to a unique prior on $\tau$ in multidimensional analyses, as the latter is also determined by cosmological parameters. Thus, an independent determination of $z_{reion}$ is best utilized in the specific parameter spaces that are degenerate with $\tau$ (Figures 2 and 3).

In conclusion, this is a special time for cosmology (and for those employed in its study!), when observational efforts to detect the epoch of hydrogen reionization are rapidly narrowing the bracketed range of possible redshifts— from the lower end, through spectroscopic studies of the highest-redshift objects, and from the upper end, with data from past and ongoing CMB experiments. This has provided a unique opportunity to jointly test theoretical models of the CMB and of the growth of structure, in order to understand the nature and birth sites of the first luminous objects. We can look forward to the next few years of data from such endeavors, which are likely to settle important frontiers in cosmology including the epoch when the universe returned to a fully ionized state.

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