II. THE ANOMALOUS ACCELERATION

The observed anomaly is to show that the Earth’s rotation is
the possibility of the effect, it can be explained, the
precessing and precessing motion of the Earth.

Here, we assume that the Earth is a perfect sphere and
that it is rotating about its own axis.

We use the equations of motion with the measured
acceleration, which are adjusted to account for the
Earth’s rotation.

Each of these effects is accounted for, and a net force
...
More recent analyses have refined these results somewhat, though the main conclusions remain unchanged. Table I shows the most recent results from [2], which fits a constant, independent acceleration in each interval. SIGMA and CHASMP are two different trajectory modelling programs each with many possible analysis options. We show here the best Weighed Least Squares (WLS) results from each program. Accelerations are in units of $10^{-8}$ cm s$^{-2}$. The difference between the programs is a better estimate of the real uncertainties since it is far greater than the formal errors. For convenience, we show the amount of directed power, in watts, that would be needed to account for each acceleration, assuming the 241 kg estimate of spacecraft mass from [2]. So the best available data, taken at face value, shows that 57 watts are needed in 1998, and that a 3% decrease was observed between interval I and interval III.

### Table I: Summary of results from Anderson, et al. [2]

<table>
<thead>
<tr>
<th>Interval</th>
<th>SIGMA accel.</th>
<th>watts</th>
<th>CHASMP accel.</th>
<th>watts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 87 - Jul 90</td>
<td>6.93 ± 0.11</td>
<td>57.3</td>
<td>7.84 ± 0.01</td>
<td>56.7</td>
</tr>
<tr>
<td>Jul 92 - Jul 98</td>
<td>8.25 ± 0.03</td>
<td>59.9</td>
<td>7.94 ± 0.01</td>
<td>57.2</td>
</tr>
</tbody>
</table>

### III. Previous Work

For the convenience of the reader, section III A consists of direct quotes from [2], covering the relevant details of the Pioneer spacecraft, and Figure 3, from [4]. Many other papers [4] and web [5] descriptions are available. In section III B we summarize the existing literature on the hypothesis that non-isotropic radiation is responsible for the unmodelled acceleration.

#### A. General description of the Pioneer spacecraft, from [2]

The main equipment compartment is 36 cm deep. The hexagonal flat top and bottom have 71 cm long sides. Most of the scientific instruments’ electronic units and internally-mounted sensors are in an instrument bay (“squashed” hexagon) mounted on one side of the central hexagon.

At present only about 65 W of power is available to Pioneer 10 [6]. Therefore, all the instruments are no longer able to operate simultaneously. But the power subsystem continues to provide sufficient power to support the current spacecraft load: transmitter, receiver, command and data handling, and the Geiger Tube Telescope (GTT) science instrument.

The sunward side of the spacecraft is the back, and the anti-sunward side, in the direction of motion, is the front [7].
B. Non-isotropic radiation – previous work

Murphy suggests that the anomalous acceleration seen in the Pioneer 10/11 spacecraft can be, “explained, at least in part, by non-isotropic radiative cooling of the spacecraft.” [8] Anderson, et al. argue in reply [9] that over the data span in question the louver doors were already closed. They conclude “the contribution of the thermal radiation to the Pioneer anomalous acceleration should be small.” They also argue that the spacecraft power is decreasing, but the unmodelled acceleration is not. Scheffer[10] points out that the front of the spacecraft has a much higher emissivity than typical thermal blankets, and therefore the majority of the heat will radiate from the front even though the louvers are closed. Anderson, et al. [11] dispute this, based on the emissivity data in [4].

Katz[12] proposes that at least part of the acceleration is generated by radiation from the RTGs reflecting off the back of the antenna. Anderson, et al., argue that this effect must be small since the antenna is end-on to the RTGs, and hence gets very little illumination.

Slusher (as credited by Anderson) proposed that the forward and backward surfaces of the RTGs may emit non-equally. Anderson, et al., conclude there is no credible mechanism to explain the large difference in surfaces that would be required if this was to explain the whole effect.

IV. DISCUSSION

We consider asymmetrical radiation from 5 sources - the RTGs themselves, the two spacecraft compartments, RTG radiation reflected from the antenna, the radioisotope heater units (RHUs) on the spacecraft, and radiation from the feed that misses the antenna. We also consider one modelling error, a mis-estimation of the reflectivity of the antenna to solar radiation.

Consider thermal radiation from the spacecraft with the louvers closed, as they have been since 9 AU. A simple thought experiment shows that the electrical power dissipated in the spacecraft must result in thrust. From Figure 3, the simplest possible model consists of the main compartment as a 60 watt isotropic radiator, and the back of the antenna a mirror. The antenna subtends 120 degrees as seen from the instrument compartment, so if the emitted radiation is isotropic, the antenna intercepts 1/4 of the total radiation, and reflects it back to the right. Since the main compartment is centered behind the antenna, and since the sides, if anything, are worse radiators than the front, we conclude a large fraction of main compartment power must be converted to thrust.

Instrument compartment heat will also contribute to thrust, but with much less efficiency. This is because the instruments will radiate preferentially at right angles to the spin axis since that is the direction of their observation ports through the thermal blankets. Furthermore, the science compartment is much closer to the edge of the dish than the main compartment, so the dish shadows less of the thermal radiation.

A more detailed analysis shows the radiation is even more anisotropic that these arguments would suggest. Assuming a uniform internal temperature, the power emitted from each surface is proportional to the area times the effective emissivity of the surface. The front and back of the central compartment have about 1.3 m² area, and the sides about 1.5 m² total. The sides and the rear of the compartment are covered with multi-layer insulation (MLI) [4]. When calculating radiation from multi-layer insulation, the correct value to use is the “effective” emissivity, ε_{eff}, which accounts for the lower temperature of the outer layer [13]. Anderson [11] points out that the outer layer of the MLI has an emissivity of 0.70 according to [4]. This is not a contradiction because of the temperature difference between the interior of the spacecraft and the outer layer of the MLI. From [13], the multilayer insulation from on Pioneer 10 has an effective emissivity of 0.007 to 0.01 (see Figure 4). Assuming a value of 0.0085, and a 1998 internal tempera-
ture of 241 K [14], the main compartment will lose about 4 watts total through the MLI on the sides and back. Allowing a few watts for conduction losses through wires and struts, perhaps 10% of the power (about 6 watts) goes through the back, 10% through the sides, and the remaining 80% through the front. The back radiation will have a near 0 efficiency (it squirts out from between the dish and the compartment at right angles to the flight path). Radiation from the side should be about 10% efficient, assuming Lambertian radiation and a 45 degree obstruction by the dish. Radiation from the front will be about 66% efficient, again assuming Lambertian emission. The overall efficiency of main bus radiation is therefore about 54%.

A. Feed pattern of the radio beam

An ideal radio feed antenna would illuminate its dish uniformly, with no wasted energy missing the dish. However, the feed is physically small and cannot create such a sharp-edged distribution, so some radiation always spills over the edge. Since dish area is wasted if not fully illuminated, an optimum feed (for transmission) normally allows about 10% of the total power to miss the dish. This power is converted to thrust with an efficiency of 1.7 since it directly subtracts from the sun directed power and adds anti-sun power at a roughly 45 degree angle to the spin axis.

B. Radiation from the RHUs

From the diagrams in [4], 10 1-watt (in 1972) radioisotope heater units are mounted to external components (thrusters and the sun sensor) to keep them sufficiently warm. The diagram is not very specific, but the units to which they are mounted are primarily behind the main dish. If we assume these radiate isotropically into the hemisphere behind the antenna, then they contribute the equivalent of 4 watts of directed force in 1998. This component decreases with a half life of 88 years.

C. Asymmetrical radiation from the RTGs

The RTGs might contribute to the acceleration by radiating more to the front of the spacecraft than the rear. In [2] this is estimated to contribute no more than 6 watts of thrust. Another argument for a small RTG asymmetry is the constancy of spin in interval III. Barring a fortuitous cancellation with some other effect, the spin rate change in interval III corresponds to an directed radiation of about 4.3 w-meters. Since the RTGs are about 3 meters from the axis, we conclude their radiation in the spinward and anti-spinward direction must differ by less than 1.5 watts.

D. Revisiting RTG reflection

The RTGs are not on-axis as viewed from the antenna. From Figure 3, we see that the centerline of the RTGs is behind the center of the antenna. Measurements from this diagram indicate this distance is about 23.8 cm. Another figure (not included here) from [4] shows the far end
of the RTGs is 120.5 inches (or 3.06 meters) from the centerline. The near end of the RTGs will then be about 60 cm further in, or at about 2.46 meters from the center. The antenna extends 1.37 meters from the center, so the rim of the antenna is 69.8 cm off axis and 1.09 meters away radially. Thus the edge of the antenna, where the illumination is by far the brightest, views the inner RTG at an 32.6 degree angle. This is far from on-axis.

The fins of the RTGs radiate symmetrically, and all are visible from the antenna, so the center of this illumination will be 23.8 cm behind the antenna. The cylindrical center of the RTG is about 8.4 cm in radius [16] so this illumination will come from at about 15.4 cm behind the antenna. The fins have more area than the cylinder, so for this calculation we take a rough weighted average and assume a cylindrical Lambertian source 20 cm behind the antenna. We assume the inner RTG is centered 2.66 meters from the center, and the outer RTG 2.91 meters.

The area blocked by the antennas is shown in Figure 5 in spherical coordinates. Rough integration of the two areas shows about 12 watts for the near RTG and 8 watts for the far one if the total RTG power is 2000 watts.

![Antenna size in spherical coordinates](image)

**FIG. 5:** Antenna size in spherical coordinates from RTGs. Radial axis is angle from the centerline in radians; other axis is angle around this line with the magnetometer defined as zero.

This energy is turned into thrust by two effects. First, the antenna shadows radiation which would otherwise go forward. An angle in the middle of the antenna is about 17 degrees forward; this corresponds to an efficiency of 0.3 (the true efficiency is probably higher since the edge is both at a greater angle and more brightly illuminated.) Next, the energy that hits the antenna must go somewhere. Some will be absorbed and re-radiated; some will bounce into space, and some will bounce and hit the instrument compartment, and be reflected or re-radiated from there. A detailed accounting seems difficult, but an overall efficiency of 0.8 seems reasonable (0.3 for shadowing and 0.5 for reflection and re-emission).

### E. Total of all effects

Here we sum all the effects as of 1998. The total is more than enough to account for the acceleration, giving us the freedom to reduce some of the efficiencies if needed to fit the data.

<table>
<thead>
<tr>
<th>Source of effect</th>
<th>Total Power (W)</th>
<th>Thrust</th>
<th>Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rad from RHUs</td>
<td>8</td>
<td>0.5</td>
<td>4</td>
</tr>
<tr>
<td>Antenna shadow</td>
<td>20</td>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td>Antenna radials</td>
<td>20</td>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>RTG asymmetry</td>
<td>3000</td>
<td>0.053</td>
<td>6</td>
</tr>
<tr>
<td>Feed pattern</td>
<td>0.8</td>
<td>1.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Rad, main bus</td>
<td>50</td>
<td>0.54</td>
<td>-32</td>
</tr>
<tr>
<td>Rad, instr.</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>10.5</td>
<td></td>
</tr>
</tbody>
</table>

### F. Antenna solar reflectivity

The calculation of consistency in [2] at the end of section 7.1 seems to be incorrect. Pioneer 10, for example, had a fitted $K$ of 1.73. If the mass was really 241 kg instead of the assumed 251.8 kg, then the true value of $K$ must be less by 241/251.8 ($K = 1.66$, instead of the $K = 1.87$ of the paper) to account for the same acceleration. Similarly, for Pioneer 11 the fitted value was 1.83. If the mass was really 232 kg instead of 238.7 kg, then $K$ should be 1.77. At 5.2 AU the total power on the antenna is about 298 watts. If we assume that the dimensions and paints are identical for Pioneer 10 and 11, and ascribe the difference to radiated power, then Pioneer 10 had $(1.77 - 1.66) \times 298 = 33$ more watts of thrust than Pioneer 11 at this time.

The fitting procedure cannot completely separate a contribution from spacecraft radiation from the $1/r^2$ solar reflectivity, since the radiation contribution is also decreasing (the total power is decreasing, and the efficiency will decrease as well as the lenses close). Nevertheless, since the solar radiation varies over a wider range than the hypothesized thermal radiation, it might be possible to see this effect in archival data. It would show up as a decrease in the fitted value of $K$ as the spacecraft receded from the sun.

The solar constant also provides a natural explanation for the onset of the anomalous acceleration. Assume the acceleration has existed all along, and might even have been stronger closer to the sun. When Pioneer was closer to the sun, the fitting programs absorbed the extra acceleration by adjusting the value of $K$, the reflectivity of the antenna. As Pioneer receded, the power available from this source decreased, and the fitted $K$ would start to run into the limits allowed in the fit. (Physically reasonable values perhaps range from 1.5 to about 1.9; they are certainly greater than 1.0 and less than 2.0).
Once the magnitude of the solar pressure becomes small enough, then $K$ reaches the limit allowed in the fit, and the anomalous acceleration appears. This argument is not specific to radiation induced acceleration - any small radial acceleration can be compensated for by adjusting the value of $K$.

In this paper, we model the effect of any error in $K$ by introducing a fictitious force, whose value is simply the solar force on the spacecraft times the error in $K$.

V. COMPARISON WITH EXPERIMENT

How well does this explanation account for the acceleration? The explanation has 6 adjustable parameters. In theory all are separable since they decay at different rates.

- $\epsilon_{RHU}$, the proportion of RHU heat converted to thrust
- $\epsilon_{RTG}$, the proportion of RTG heat converted to thrust. Includes both direct asymmetry and reflection from the antenna.
- $\epsilon_{FEED}$, the fraction of RF power that misses the antenna
- $\epsilon_{INST}$, the fraction of instrument heat that is converted to thrust
- $\epsilon_{BUS}$, the fraction of main compartment heat that is converted to thrust
- $K_{SOLAR}$, the amount by which the solar reflection constant is underestimated.

We compute the net thrust as follows: let $d$ be the date in years. The total electrical power, in watts, is modelled as

$$E(d) = 68 + 2.6 \cdot (1998.5 - d)$$

The RHU power, in watts, is

$$RHU(d) = 10.0 \cdot 2^{-(d-1972)/88}$$

The RTG heat dissipation, in watts, is

$$RTG(d) = 2580 \cdot 2^{-(d-1972)/88} - E(d)$$

We assume the distance from the sun, measured in AU, increases linearly from 20 AU in 1980 to 78.5 AU in 2001:

$$r(d) = 20 + (d - 1980)/21 \cdot (78.5 - 20)$$

The power incident upon the antenna, in watts, is

$$SOLAR(d) = \pi(1.37 \text{ m}^2)f_0/r^2(d)$$

where $f_0 = 1367 \text{ W/m}^2$ is the “solar radiation constant” at 1 AU. We assume 8 watts goes into the radio beam. The instrument power $INST(d)$ is given in Table III.

Other equipment shut-off (PSE, DSL) does not affect the instrument heat since it simply substitutes one heat source in the main compartment for another.

The power that does not go into the instruments or the radio beam goes into the main compartment:

$$BUS(d) = E(d) - INST(d) - 8.0$$

We sum the individual sources, then convert to acceleration by dividing by $c$, the speed of light, and $m$, the spacecraft mass (here 241 kg):

$$acc(d) = \frac{1}{c \cdot m} \left[ \epsilon_{RHU} \cdot RHU(d) + \epsilon_{RTG} \cdot RTG(d) + \epsilon_{FEED} \cdot (8 \text{ w}) \cdot 1.7 + \epsilon_{INST} \cdot INST(d) + \epsilon_{BUS} \cdot BUS(d) + K_{SOLAR} \cdot SOLAR(d) \right]$$

To examine the fit, we use the plots from [2, 3], and try to fit them with our model. We make two fits, the first with minimal assumptions and the second to get the best possible fit by varying the different effects of instrument and main compartment heat.

The “minimal assumptions” fit assigns the same efficiency to main compartment heat and instrument heat. This avoids much of the need to look at spacecraft construction details and instrument history, since the acceleration only depends on the total electrical power. Even with this restriction, we get a reasonable fit over the entire data span with the following coefficients:

- $\epsilon_{RHU}$ = 0.5, the RHUs radiate like point sources behind the antenna.
- $\epsilon_{RTG}$ = 0.0108, 0.28% RTG asymmetry, 30% blockage efficiency, and 50% reflection efficiency.
- $\epsilon_{FEED}$ = 0.1. 10% of the feed power misses the antenna
- $\epsilon_{INST}$ = 0.35. Instruments same as main bus for simple fit.
- $\epsilon_{BUS}$ = 0.35. About half the main bus heat goes out the front, with Lambertian efficiency.
- $K_{SOLAR}$ = 0.3. Antenna reflection estimates are too low by 0.3

The fit to the data is shown in Figures 6 and 7. The agreement seems reasonable in both regimes. In particular, the values of the early anomalous acceleration are
sensitive to variations in the causing mechanism. This is illustrated in Figure 6, which compares the data from Figure 1, the proposed mechanism, and a hypothetical constant acceleration. The proposed model provides a significantly better fit in this region.

![Image](https://example.com/image.png)

**FIG. 6:** Data from Figure 1 (error bars), model prediction from this paper (solid line), and constant acceleration (dashed line)

The fit from 1987 to 1998 also looks OK:

![Image](https://example.com/image2.png)

**FIG. 7:** Figure from [3], with fitted data added. The dotted line is Turyshkin's empirical fit; the solid line is the model hypothesized in this paper.

Finally, we compare against the most recent results[2] that fit a constant acceleration in each interval. Using the model above, the average thrust is 57.8 watts in interval I, and 51.0 watts in interval III. We can normalize the result to get the correct overall average, or the right acceleration in interval I, but in either case we would expect to see an 11.8% decrease from interval I to III, where only a 3% decrease is observed. The two different measurements of the effect (SIGMA and CHASMP) themselves differ by 4% in interval III. If we treat this difference as a statistical result (a procedure of dubious merit, but the best we can do) then the 9% discrepancy is 2.25 standard deviations out. This makes it unlikely at about the 2% level that this hypothesis alone accounts for all the measured result.

We can get a better result at later times by assigning different efficiencies to instrument heat and main compartment heat. For example, assume $\epsilon_{\text{INS}} = 0.1$ (instruments radiate mostly to the side) and $\epsilon_{\text{BUS}} = 0.4$ (main compartment radiates mostly to the front) gives a better fit, with only a 6.5% discrepancy on the I-III decline and an equally good fit at earlier times. This is only about 1.5 standard deviations out, a considerably better fit. However, figuring out the maximal reasonable difference between instrument efficiency and main compartment efficiency is difficult[11]. One the one hand the two compartments are separate, the instrument bay is closer to the edge of the antenna, and it has side facing ports that extend through the thermal blankets. On the other hand the two compartments are radiatively and conductively coupled. So it’s very hard to tell how big the difference in efficiencies could be and still be physically plausible.

In any case, the proposed explanation, by accounting for the bulk of the effect, makes it more likely that conventional physics can account for the entire unmodelled acceleration. Conventional explanations for the remaining discrepancy include other unmodelled effects such as gas leaks, inaccuracies in the simple thermal model, or the effects of a complex fitting procedure applied to noisy data.

**VI. CONCLUSIONS AND FUTURE WORKS**

There is surely an unmodelled effect on the Pioneer spacecraft, based upon its thermal characteristics. Rough estimates show it can account for the magnitude of the unmodelled acceleration to within the errors, but overpredicts the rate of change. The antenna shadowing of main compartment radiation and the radiation from the RFGs falling on the antenna seem particularly robust sources of acceleration since they are only based on geometry. These effects alone account for more than half the acceleration. The other sources - BHU radiation, differential RFG radiation, and differential emissivity - depend more on construction details, but all seem plausible.

This explanation also explains some other puzzles: the values of acceleration of Pioneer 10 and 11 would be expected to be similar, but not identical, as observed. The acceleration would not have a strong effect on the spin; most of the radiation will generate little torque. Other spacecraft, built along the same general principles, would be expected to show a similar effect, but planets and other large bodies would not, as is observed.

More detailed modeling, using the Pioneer materials, construction details, and history, could provide a much better estimate of the magnitude of this effect. A suit-
ably detailed thermal model, measured in a cold vacuum chamber, would provide the strongest evidence for or against this hypothesis.

If Pioneer 10 remains operational, additional data may allow us to improve our understanding of the unmodelled acceleration. Longer term, other proposed experiments such as LISA[17] are designed specifically to reduce the systematics that bedevil retrospective analyses like Pioneer. (LISA is expected to be about 10^3 times better in this respect.) If the anomalous acceleration is not detected in these more precise experiments, then almost surely the unmodelled acceleration of Pioneer 10 is caused by some overlooked prosaic sources such as those proposed here.

VII. ACKNOWLEDGEMENTS

I'd like to thank Edward Murphy and Jonathan Katz for comments and suggestions on an earlier version of this document; Edward Murphy also sent copies of the documents he found while investigating the same effect. Larry Lasher and Dave Lowier of the Pioneer project were kind enough to answer questions about the probe. John Anderson suggested adding the statistical likelihood calculations.

[5] For web summaries of Pioneer, go to:
http://quest.arc.nasa.gov/pioneer10,
http://spaceprojects.arc.nasa.gov/
SpaceProjects/pioneer/Fl1home.html
[6] This is a “theoretical value,” which does not account for inverter losses, line losses, and such. It is interesting to note that at mission acceptance, the total “theoretical” power was 175 Watts.
[7] When a Pioneer antenna points toward the Earth, this defines the “near” direction on the spacecraft. The equipment compartment placed on the other side of the antenna defines the “front” direction on the spacecraft.
[14] This temperature is the average of the 4 thermistors in the main compartment, from telemetry on 29 Sept 98. This data was provided by Larry Lasher of the Pioneer project.
LISA-tech-report.pdf