Connecting Bimaximal Neutrino Mixing to a Light Sterile Neutrino

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Abstract

It is shown that if small neutrino masses owe their origin to the conventional seesaw mechanism and the MNS mixing matrix is in the exact bimaximal form, then there exist symmetries in the theory that allow one of the righthanded neutrinos to become naturally massless, making it a candidate for the sterile neutrino discussed in the literature. Departures from the exact bimaximal limit leads to tiny mass for the sterile neutrino as well as its mixing to the active neutrinos. This provides a minimal theoretical framework where a simultaneous explanation of the solar, atmospheric and LSND observations within the so-called 3+1 scenario may be possible.

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I. INTRODUCTION

At the moment, there are three classes of experiments that provide positive indications for neutrino oscillations: solar neutrino data from seven different experiments, Chlorine, Kamiokande, Super-Kamiokande, SAGE, GALLEX, GNO, SNO, all experiments giving evidence for missing electron neutrinos emitted from the solar core, which can be understood in terms of neutrino oscillations [1]; atmospheric neutrino data from Super-Kamiokande which has not only confirmed earlier indications of a muon neutrino deficit but also has provided more precise data that has established at a high confidence level that the muon neutrinos from the cosmic rays are oscillating to tau neutrinos [2]. These two evidences are on very solid footing. The third piece of evidence is from the Los Alamos LSND experiment that shows an oscillation from the muon neutrino to the electron neutrino. This experiment however has neither been confirmed nor refuted by other experiments [3]. The KARMEN [4] experiment which looked for $\nu_{\mu} - \nu_e$ oscillation did not find any evidence for it and eliminated a large fraction of the parameter space allowed by LSND. It is hoped that the Mini-BOONE experiment at FERMILAB will settle the issue in near future.

Theoretical analyses have made it clear that explanation of the observations in terms of neutrino oscillations require that the three mass differences $\Delta m_{\odot}^2$, $\Delta m_{\text{atm}}^2$, and $\Delta m_{\text{LSND}}^2$ are all of very different orders of magnitude. This poses a theoretical problem for models with only three active neutrinos since with three neutrinos, one can at most have two different mass differences. The simplest way to provide a simultaneous understanding of all the above mentioned oscillation data appears to be to postulate the existence of an additional ultralight neutrino, which in order to be consistent with the LEP data must be a sterile neutrino [5].

In the presence of a sterile neutrino, there are several ways to understand the observations. We mention only two of them, one called 2+2 scheme [5] and another called 3+1 scheme [6] in the literature. The 2+2 scheme has the $\nu_{\mu,\tau}$ neutrinos with mass around an eV and $\nu_{e,s}$ with mass near $10^{-3}$ eV, with the later explaining the solar neutrino data, the former explaining the atmospheric neutrino data and the gap between the two pairs explaining the LSND results. Recent SNO data disfavors the original version of the model where all the missing solar $\nu_e$’s are converted via a small angle MSW mechanism only to the sterile neutrinos. The situation where only a fraction of the missing $\nu_e$ convert to $\nu_s$ and the rest to active ones has been studied in several papers [7]. A particular challenge in this model would be to fit both the SNO Super-Kamiokande gap and the more or less flat neutrino energy distribution observed in both the SNO and Super-Kamiokande experiments.

In the 3+1 picture, on the other hand, it is assumed that the three active neutrinos are bunched together at a small mass value (say around $6 \times 10^{-2}$ eV or so), with the sterile neutrino at a mass near an eV. The atmospheric and solar neutrino data is explained by the oscillations among active neutrinos whereas the LSND data is explained by indirect oscillations involving the sterile neutrino [8]. The advantage of this picture in view of the SNO data is that the flat energy spectrum is explained by postulating bimaximal [9] MNS mixing pattern among the active neutrinos and using the large mixing angle MSW solution. The SNO-Super-Kamiokande gap is understood in terms of the neutral current interactions of the $\nu_\mu$’s to which the solar $\nu_e$’s convert. It must however be pointed out that 3+1 pattern works only for certain values of the $\Delta m_{\text{LSND}}^2$ as has been noted in ref. [6] and there are papers
noting that it may have problem accommodating all accelerator and reactor constraints as well as the positive signals for oscillation [10] and yet have room in its parameter space to explain the LSND results.

If experiments ultimately establish the existence of a sterile neutrino, a key theoretical challenge would be to understand its origin in the context of physics beyond the standard model, specially why its mass is so small even though it has no electroweak quantum numbers. Most existing scenarios postulate new fermions beyond the conventional seesaw framework i.e. new singlet fermions beyond the three right handed dictated by quark lepton symmetric extension of the standard model. They are usually taken to be mirror neutrinos [11], extra singlet fermions [12] or modulinos [13], $E_6$ singlets [14] or bulk neutrinos [15] as in models with large extra dimensions.

Our goal in this brief note is to point out a more economical possibility. We show that when one tries to obtain the exact bimaximal form for the MNS matrix in the framework of the seesaw [16] mechanism, there appear new symmetries of the neutrino mass matrix in special limits that predict that one of the right handed neutrinos, which normally would have been superheavy, remains massless whereas the other two are superheavy as expected in the seesaw mechanism. The massless right handed neutrino can therefore be identified with the sterile neutrino, providing not only a completely new picture for the sterile neutrino but also connecting it to the bimaximal mixing among the active neutrinos.

Slight departures from the bimaximal pattern endow this massless sterile neutrino with an ultralight mass as well as mixings with the active neutrino, so that one can now use this to understand known oscillation data. The resulting picture is the 3+1 type discussed above. Thus if the 3+1 scenario stands the test of time, the model discussed in the present paper would provide an interesting minimal theoretical scenario that would connect two apparently different phenomena without going beyond the usual quark lepton symmetric framework.

**II. SYMMETRY REASON FOR BIMAXIMAL MIXING STERILE NEUTRINO CONNECTION**

Let us start by writing down the generic bimaximal MNS mixing matrix with the definition:

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i;$$  \hspace{1cm} (1)

where $\nu_\alpha$ and $\nu_i$ are respectively the flavor and mass eigenstates.

$$U_{\text{bimax}} = \begin{pmatrix} c & -s & 0 \\ \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} & \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} & \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} \end{pmatrix}.$$  \hspace{1cm} (2)

When $c = s = \frac{1}{\sqrt{2}}$, we will refer to this as the exact bimaximal limit alluded to above. It was shown in ref. [17] that the most general mass matrix involving the active neutrinos, which leads to the strict bimaximal pattern in the basis where the charged leptons are mass eigenstates is
This mass matrix has a permutation symmetry $S_2$ operating on the $\nu_\mu$ and $\nu_\tau$. Let us call this $S_{2L}$ since it acts only on the left-handed neutrinos. In the limit of exact $SU(2)_L$ gauge symmetry of the electroweak interactions, this symmetry will also have consequences for the charged lepton masses (i.e. $\mu, \tau$ masses). The difference of the $\mu$ and the $\tau$ mass will then be attributed to its breaking. Note that the gauge interactions are invariant under $S_{2L}$.

In the matrix $M_3$, $m, a, b$ are three free parameters. In order to further relate the bimaximal mixing to symmetries of the leptonic world, we will restrict ourselves to the case when $a = b = 0$. In this case, the matrix $M_3$ has the symmetry $L_e - L_\mu - L_\tau$ symmetry [18] in addition to the $S_{2L}$ symmetry. Let us therefore consider the combination of $S_{2L} \times U(1)_{e-\mu-\tau}$ as a symmetry of the weak Leptonic Lagrangian to zeroth order.

The next step in our discussion is the seesaw [16] mechanism which provides an explanation of the small neutrino masses. In order to implement this, we will include three right handed neutrinos in the theory (to be denoted by $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$) in the standard model. This makes the theory quark lepton symmetric. If we denote the mass matrix for the right handed neutrinos as $M_R$, then the complete $6 \times 6$ mass matrix involving the Dirac mass and Majorana mass for the neutrinos can be written as

$$M_{LR} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}.$$  \hfill (4)

When $M_R$ is not a singular matrix, one can obtain the mass matrix for the light neutrinos as

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D.$$ \hfill (5)

This is the so-called type I seesaw formula (see e.g. [19]). On the other hand when $M_R$ matrix is singular, this means that it has one or more zero eigenvalues and one must “take them out” of the matrix before using the seesaw formula to obtain the light neutrino mass matrix\footnote{For earlier discussion of singular seesaw to get light sterile neutrinos, see [20]. None of these papers connected the existence of the sterile neutrino to the bimaximal mixing among neutrinos, as is done in this paper.}. In this case, one of the three righthanded neutrinos will have a mass which is far below the seesaw scale. If it has a Dirac mass with one or more of the left handed neutrinos at the tree level, its mass will be its Dirac mass, which, apriori, can be quite large (of order or less than the weak scale). Without further restriction, the light right handed neutrino in general is not light enough to qualify as the desired ultralight sterile neutrino needed for understanding the oscillation data.

On the other hand, if the theory has the symmetry dictated by the mass matrix discussed above i.e. $U(1)_{e-\mu-\tau} \times S_{2L} \times S_{2R}$ where $S_{2R}$ operates on the righthanded neutrinos, then the Dirac mass connecting the massless righthanded neutrino to the left handed neutrinos...
vanishes and the eigenvector of the righthanded neutrino mass matrix corresponding to the zero eigenvalue emerges as a viable candidate for the sterile neutrino. Its tree level mass is zero.

To study this in detail, note that the symmetries of the theory force the $M_R$ to take the form

$$ M_R = \begin{pmatrix} 0 & M & M \\ M & 0 & 0 \\ M & 0 & 0 \end{pmatrix}. $$

(6)

The Dirac mass for the neutrinos that connects the left and the right handed neutrinos takes the form

$$ M_D = \begin{pmatrix} m_{11} & 0 & 0 \\ 0 & m_0 & m_0 \\ 0 & m_0 & m_0 \end{pmatrix}. $$

(7)

Note that both the $M_D$ and $M_R$ have one zero eigenvalue each. They correspond to the linear combinations $L_\nu_{-}L_\nu$ and $R_\nu_{-}R_\nu$. The seesaw mechanism now can be applied to the remaining $2 \times 2$ matrix to yield the following light neutrino mass matrix in the original $3 \times 3$ basis.

$$ M^{(0)} = \begin{pmatrix} 0 & m & m \\ m & 0 & 0 \\ m & 0 & 0 \end{pmatrix}; $$

(8)

with $m = \frac{m_{11} m_0}{\sqrt{2} M}$. This matrix on diagonalization leads to the strict bimaximal MNS matrix given above. We thus see that in the strict bimaximal limit there are two massless neutrinos: one, $\nu_-$ is an active left-handed neutrino and the other, $\nu_s$ is a right handed neutrino, that has no weak interactions of Fermi strength. The latter can therefore play the role of a sterile neutrino.

The special case of the mass matrix in equation (8) $M^{(0)}$ predicts an inverted spectrum for neutrinos and oscillation of atmospheric muon neutrinos dictated by the $\Delta m^2_{\text{ATMOS}} = 2m^2$; it is however not realistic since it leads to $\Delta m^2_{\odot} = 0$ and therefore predicts no oscillation of the solar neutrinos. In order to make this model useful, we must add small corrections to it. Such corrections to the mass matrix can arise once the $L_e - L_\mu - L_\tau$ as well as the permutation symmetries are broken. In that case both the sterile neutrino mass as well as its mixings with the active neutrinos can arise. In the next section, we give an example of a model for this case.

To study such realistic situations, let us consider the case, when $L_e - L_\mu - L_\tau$ and $S_{2R}$ are broken softly in the Higgs sector. An example of a mass matrix with this symmetry breaking can be of the form (in the basis $(\nu_e, \nu_\tau, \nu_s)$, where $\nu_{\pm} \equiv \frac{\nu_e \pm \nu_\tau}{\sqrt{2}}$):

$$ M^{(0)} + M^{(1)} = \begin{pmatrix} 0 & \sqrt{2}m & 0 \\ \sqrt{2}m & \delta_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \delta_1 \end{pmatrix}. $$

(9)
Diagonalization of this mass matrix in the approximation \( \delta_2 \ll m \ll \delta_1 \), leads to the following form for the generalized \( 4 \times 4 \) MNS matrix \( \tilde{U} \):

\[
\tilde{U} = \begin{pmatrix}
c & s & 0 & \epsilon \\
-s & c & \frac{1}{\sqrt{2}} & \frac{\alpha}{\sqrt{2}} \\
\frac{\alpha}{\sqrt{2}} & -\frac{1}{\alpha c} & 0 & 1 \\
\end{pmatrix}
\]

(10)

where \( \alpha = \frac{\delta_2}{\delta_1} \) and \( \epsilon = \frac{\delta_2 m}{\delta_1} \). This form leads to \( \Delta m^2_{\text{ATMOS}} \approx 2m^2 \) as before; however, now we have \( \Delta m^2_{\odot} \approx \sqrt{2}(2m\delta_3 - \frac{2m\delta_3^2}{\delta_1^2}) \) making it possible to understand solar neutrino deficit in terms of the large mixing angle MSW solution. Clearly a cancellation between \( \delta_3 \) and \( \frac{\delta_3^2}{\delta_1} \) is required for this purpose. To see the extent of fine tuning, note that, we get for the LSND mixing \( \theta_{\text{LSND}} \approx \alpha \epsilon \approx \frac{\delta_2 m}{\sqrt{2} \delta_1} \). If we choose \( \delta_1 \approx 0.6 \text{ eV} \) (the LSND mass) and LSND mixing to be 0.02, then we get \( \frac{\delta_3^2}{\delta_1} \approx 0.16 \); so to get the right order of magnitude for the \( \Delta m^2_{\odot} \), we need fine tuning at the level of one per cent or so. The extent of fine tuning needed is less severe for lower values of \( \Delta m^2_{\text{LSND}} \) e.g. for \( \delta_1 \approx 0.2 \text{ eV} \), the required fine tuning is at the level of 10%. In this model \( U_{e3} = 0 \) but

\[
U_{e4} \approx -\frac{\delta_2 m}{\delta_1} \approx 0.04 - 0.06,
\]

(11)

which may be accessible to future high precision disappearance searches for \( \nu_e \)'s. This may qualify as a realistic 3+1 scenario.

### III. THEORETICAL ORIGIN OF ALMOST BIMAXIMAL FORM AND STERILE NEUTRINO MASS

In this section, we discuss theoretical schemes where by extending the standard model to include the right handed neutrinos and several extra Higgs fields, one can obtain the almost bimaximal mass matrix that leads to the MNS matrix discussed in the previous section. One may use other alternative schemes such as the use of higher dimensional terms that include Higgs fields that break both the \( S_{2L} \) and \( S_{2R} \) symmetry. Here we give an example of only the first kind.

The radiative scheme uses only the standard model gauge group with the right handed neutrinos to implement the seesaw mechanism and to get the massless sterile neutrino, as has already been discussed. The leptonic multiplets \( (L_\mu, L_\tau) \) are \( S_{2L} \) multiplets, singlets \( (\mu_R, \tau_R) \) and \( (\nu_{\mu R}, \nu_{\tau R}) \) are the \( S_{2R} \) multiplets. The extra Higgs fields included are: iso-singlet singly (positive) charged fields \( \eta_2^{+}, \eta_0^{+} \), iso-singlet doubly (positive) charged fields \( H_2^{++}, H_0^{++} \), where the superscripts correspond to the \( S_{2L} \times S_{2R} \) quantum numbers (+ means even and - means odd under permutation) and subscripts to \( L_e - L_\mu - L_\tau \) quantum numbers. We also include an extra standard model like doublet \( \phi_2^{+-} \) in addition to the usual Higgs doublet \( \phi_1 \). The standard model Lagrangian is then augmented by the inclusion of the then following terms:
\[ L' = M_1 \eta_0 \phi_1 \phi_2 + f_1 \eta_2 \ell_{+R} \nu_{-R} + f_2 \eta_0 e_{R} \nu_{-R} + h_1 H_2 (\ell_{+R} \ell_{+R} + a \ell_{-R} \ell_{-R}) \\
+ H_0 e_R \ell_{+R} + M H_2^* \eta_0 \eta_2 + M' H_0^* \eta_0 \eta_0 + \mu M H_2^* H_0 + h.c. \]  

(12)

where we have denoted \( S_{2L} \) even and odd lepton doublet combinations by \( \ell_{\pm} \), \( S_{2R} \) odd RH neutrino combination as \( \nu_{-R} \) (previously called \( \nu_s \)) and \( S_{2R} \) even and odd combinations of the charged leptons as \( \ell_{\pm R} \). We assume all the new fields to be in the TeV scale. This is an extra assumption which is at the same level as, say, the \( \mu \) problem of supersymmetry and presumably can be solved once the nature of physics beyond the standard model becomes more clear.

Using these interactions, one gets at the two loop level [21] (for a generic diagram, see Fig. 1) \( \nu_{-R} \nu_{-R} \) (or \( \nu_s \nu_s \)) and \( \nu_{-R} \nu_{+L} \) as well as \( \nu_{+L} \nu_{+L} \) contributions to the \( 4 \times 4 \) neutrino mass matrix. The magnitude of these contributions are \( \sim \frac{f^2 h_1 \eta}{16\pi^2} \) where we assume \( \mu \) to be of order of the weak scale. For \( f \simeq h \simeq 10^{-2} \), this gives the new contributions to the neutrino mass matrix of order .1 - 1 eV, which can then lead to a realistic picture for all three neutrino oscillations (solar, atmospheric and LSND). The charged lepton sector can be made diagonal using appropriate number of Higgs fields which we do not discuss here.

In conclusion, it is pointed out in this brief note that there may be an intimate connection between the exact bimaximal neutrino mixing matrix and the possible existence of a light sterile neutrino within the conventional seesaw picture for neutrino masses. This connection arises from new symmetries of the neutrino mass matrix that yield the exact bimaximal MNS matrix. The resulting scenario is a 3+1 scheme for understanding the present neutrino oscillation data, which, though highly constrained at the moment, may still be a viable possibility. A crucial prediction of these models is the inverted spectrum for neutrinos.

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Note added: It is possible to show that there is a connection between the near bimaximal neutrino mixing and the existence of a light sterile neutrino if the theory has a smaller symmetry \( U(1)^{\imath}_{L_e - L_{\mu} - L_{\tau}} \times S_{2R} \) symmetry. The \( S_{2R} \) symmetry acts only on the right handed neutrinos. As a result the charged lepton sector is as in the standard model. This is the subject of a forthcoming article.
REFERENCES


