Effective Neutrino Mass Operators:  
A Guide to Model Building\(^1\)

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Abstract

Effective operators relevant for generating small Majorana masses for the neutrinos in the Standard Model will be considered. These operators serve as a useful guide for building models of neutrino mass. Some of these operators are represented by familiar models in the literature, and others lead to interesting new models. The number of the relevant operators will be drastically reduced if neutrinoless double beta decays are observed in current experiments.

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I would like to report some recent work with K.S. Babu[1]. In this work, we compiled all the effective higher dimensional ($5 \leq d \leq 11$) operators (and considered renormalizable models that can produce them) which can generate small Majorana neutrino masses consistent with that indicated by neutrino oscillation experiments. As you know, data from these experiments suggest a neutrino spectrum with masses of order 1 eV or less. It is one of the challenges of the fermion mass problem to understand why the neutrinos are so much lighter than the charged leptons and quarks.

A natural way to comprehend the smallness of the neutrino mass is to assume that they are generated via some underlying new physics at an energy scale $\Lambda$ which is higher than the electroweak scale. Typically, $\Lambda$ corresponds to the scale at which the lepton number symmetry is broken. At or below the electroweak scale, the neutrino mass will then be described by higher dimensional ($d > 4$) effective operators which are suppressed by appropriate powers of $\Lambda$. One can then understand the smallness of the neutrino mass on purely dimensional grounds, without having to know exactly what the underlying new physics is.

An example of this effective–operator description is the well-known seesaw mechanism [2] in which heavy $SU(2)_L$ singlet right–handed neutrinos, $N$, are introduced which can form a Dirac mass term with the left–handed neutrinos. Upon integrating out the heavy $N$ fields, one obtains an effective theory without the right–handed neutrinos, but with the dimension 5 operators [3] (see Ref.[1] for an explanation of the notation): $O_1 = (L_a^T C L_b^I) H^k H^l \epsilon_{ik} \epsilon_{jl}$, which can generate small Majorana masses for the left–handed neutrinos. When the neutral component of $H$ develops its vacuum expectation value, $v$, $O_1$ will produce a Majorana mass matrix for the neutrinos, with eigenvalues of order $v^2/\Lambda$. In order to generate a neutrino mass of order 1 eV or less, $\Lambda$ must be greater than $10^{13}$ GeV or so.

On the other hand, the nonobservation of lepton number violating processes such as $\mu \rightarrow eee$ and $\mu \rightarrow e\gamma$ only constrains $\Lambda$ to be larger than a few TeV. If the actual lepton number breaking scale is closer to this experimental lower limit$^3$, $O_1$ will generate too large a neutrino mass and other effective neutrino mass operators will have to be considered. In view of the

$^3$Such may be the case, e.g., if lepton number is broken by quantum effects of gravity. Then $\Lambda$ will be of order the Planck scale which can be as low as a few TeV if large extra dimensions exist.
current interests in neutrino mass models, it will be useful to identify all such operators.

Since we are interested in effective operators that can lead to a Majorana mass term for the left-handed neutrino fields in the Standard Model, the operators must violate lepton number, \( L \), by two units. They must be \( SU(3)_C \times SU(2)_L \times U(1)_Y \) invariant and are also required to conserve baryon number\(^4\), \( B \). Thus, these operators violate \( (B - L) \) and may be relevant for models of leptogenesis\(^4\).

The full list of effective neutrino mass operators can be found in Ref.[1]. It is also shown there how to construct neutrino mass models systematically from these operators. We classify the effective operators according to the number of fermion fields they contain. Operators containing two fermion fields are just the \( d = 5 \) seesaw operators given by \( O_1 \), which generate neutrino masses at tree level.

Operators containing four fermion fields will generate neutrino masses radiatively at the one–loop level. Examples of this class of operators include \( O_2 = L^i L^j L^k e^c H^i \epsilon_{ij} \epsilon_{kl} \), which have a realization in the Zee model \(^5\), and \( O_3 = L^i L^j Q^k d^c H^i \epsilon_{ik} \epsilon_{jl} \), which are realized in the supersymmetric standard model with \( R \)-parity violation\(^6\). As an example of how to use the effective operators to obtain new neutrino mass models, consider the operator \( O_4 = L^i L^j Q^k u^c \bar{H}^k \epsilon_{jk} \), which can generate one–loop neutrino masses via an \( SO(10) \) grand unified model (see Ref.[1] for details).

Operators containing six fermion fields will generate neutrino masses as two–loop radiative corrections. Because of the additional suppression factors, the scale \( \Lambda \) in this class of models will be much lower than the seesaw scale, and may even be close to the electroweak scale.

We have not considered operators with eight or more fermion fields because operators with \( d \geq 12 \) will be highly suppressed and will generate neutrino masses that are too small to satisfy the atmospheric neutrino data which require at least one neutrino to have a mass of about 0.03 eV. Even with this truncation, our list contains a large number of operators. It will be helpful if one can find a way to reduce the number of relevant operators. Neutrinoless double beta \((2\beta 0\nu)\) decays may provide such a way. This is because there is a subset of operators (e.g., \( L^i L^j Q^k d^c Q^l d^c \epsilon_{ik} \epsilon_{jl} \), \( L^i d^c \bar{Q}^j \bar{u}^c \bar{e}^c \bar{H}_i \bar{H}_j \), etc.) which

\(^4\)Otherwise, the nonobservation of proton decays will limit \( \Lambda \) to be larger than \( 10^{14} \) GeV and the resulting neutrino masses will be too small to be of interest.
have the special property that they can produce $2\beta\nu$ decays directly, but can only generate neutrino masses at the two–loop level. These operators can therefore generate $2\beta\nu$ decay amplitudes which are large enough to be observable in current experiments even though the neutrino masses they induce are as small as that indicated by the solar and atmospheric neutrino data. Thus, if $2\beta\nu$ decays are observed in the current round of experiments, this subset of operators will be singled out as the most likely effective neutrino mass operators.

References


