Zero-Branes, Quantum Mechanics and the Cosmological Constant

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Abstract

We analyse some dynamical issues in a modified type IIA supergravity, recently proposed as an extension of M-theory that admits de Sitter space. In particular we find that this theory has multiple zero-brane solutions. This suggests a microscopic quantum mechanical matrix description which yields a massive deformation of the usual M(atrix) formulation of M-theory and type IIA string theory.

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1 Introduction

Any compelling theory of cosmology must resolve the cosmological horizon, flatness, and topological defect abundance problems, and must also generate a scale-invariant spectrum of density fluctuations. An elegant idea which does all of these things is the idea of inflation, which asserts that there was a period of accelerating expansion in the early universe. The beauty of inflation is that it doesn’t depend on what happened before inflation, so it has many model-independent properties. Furthermore recent observational results point towards a small but non-zero cosmological constant in our observable universe.

It is now generally accepted that most fruitful candidate for a unified description of the fundamental interactions, including quantum gravity, is the mysterious ‘M-theory’, which encompasses the perturbative superstring theories. The low energy limit of this theory is eleven-dimensional supergravity, and compactifications of this theory on circles, tori and orbifolds give rise to the perturbative string theories in ten dimensions. While the M-theory moduli space contains a huge variety of supergravity solutions, to date it has proved remarkably difficult to obtain inflation or de Sitter space directly from compactification of the low-energy limit of M-theory. In addition de Sitter space poses intriguing difficulties for quantum theory since its entropy is associated to a system with only a finite number of degrees of freedom (e.g. see [1, 2]).

In a recent paper [3], we have attempted to improve this situation. In particular we studied a supergravity, dubbed ‘MM-theory’, which may be regarded as a slight extension of eleven-dimensional supergravity which still preserves eleven-dimensional supersymmetry. This extension describes the most general form of eleven dimensional supersymmetry that is compatible with on-shell superfields. This problem was studied in [4] and involves extending the local structure group of spacetime from Spin(1,10) to a conformal spin group CSpin(1,10). The superspace constraints then assert that the conformal connection is locally trivial. In effect this procedure gauges the global scale symmetries of the M-theory equations of motion.

Moreover, as long as the eleven-dimensional space is simply connected, this theory can be obtained through a gauge transformation of the usual eleven-dimensional supergravity [5]. However if the manifold is not simply connected then it is possible to introduce a Wilson line in the conformal connection. This is analogous to the Higgs’ effect in gauge theory compactified on a torus. After compactification, the fields which are charged with respect to the Wilson line then become massive in the lower dimensional theory.

A crucial point is that it only really makes sense to physically distinguish ‘MM-theory’ from M-theory if \( \pi_1(M_{11}) \neq 0 \). Indeed, in compactified MM-theory one finds that the connection acts as a source for the Einstein tensor and the natural vacuum is ten-dimensional de Sitter space.

While this construction shows that one can embed de Sitter space into eleven-dimensional superspace and supergravity, its main drawback is that it is not clear how, if at all, to relate MM-theory to a microscopic quantum theory and in particular M-theory. The equations of MM-theory compactified to ten-dimensions are certainly rather odd. In addition they do not come from an action. However they have the advantage that they can be readily obtained from the standard equations of M-theory. Indeed MM-theory provides a continuous deformation of type IIA supergravity to include a positive cosmological constant and hence de Sitter space. In addition the graviphoton has a tachyonic form at the linearised level (although it is not clear what is the case in the full non-linear theory). This should be contrasted with the usual analysis of the de Sitter supergravities which admit actions but generally contain ghost-like vectors [6]. The appearance of tachyons, rather than ghosts, is encouraging since they are much more easily understood within quantum theory. Thus MM-theory allows us study the emergence of de Sitter space from a deformation of type IIA supergravity and M-theory in a relatively controlled way. Therefore we also hope that our work will be insightful for other recent approaches to
the study of de Sitter space from the string theory point of view [7, 8, 9, 10, 11, 12, 13, 14].

Here we wish to address some dynamical issues and attempt to obtain an underlying microscopic
description of MM-theory. Our analysis suggests that MM-theory describes a legitimate part of M-
theory, but represents an unstable phase of the theory. We will identify the existence of zero-branes
which obey a no-force condition and thereby motivate a matrix model of the underlying dynamics.

2 MM-theory on $S^1$

The Bosonic equations of MM-theory in eleven dimensions can be written as

$$
\hat{R}_{ab} - \frac{1}{2} \hat{g}_{ab} \hat{R} = -18 \hat{D}_a \hat{k}_b + 18 \hat{g}_{ab} \hat{D}^c \hat{k}_c + 36 \hat{k}_a \hat{k}_b + 144 \hat{k}^2 \hat{g}_{ab} - \frac{1}{48} \left( 4 \hat{H}_{acde} \hat{H}_b^{cde} - \frac{1}{2} \hat{g}_{ab} \hat{H}^2 \right)
$$

$$
\hat{D}^a \hat{H}_{abcd} = -12 \hat{k}^a \hat{H}_{abcd} + \frac{1}{36 \cdot 48} \epsilon_{bede...f...} \hat{H}^{e...f...},
$$

where $\hat{H}_{abcd} = 4 \partial_a \hat{B}_{bcd} + 24 k_a \hat{B}_{bcd}$ and $\partial_a \hat{k}_b = 0$. The superspace construction of these equations
 guarantees that the the full Fermionic and Bosonic system is invariant under the supersymmetry

$$
\delta \hat{e}_a^b = -i \hat{\epsilon} \hat{\Gamma}_b^a, \\
\delta \hat{B}_{abc} = -3i \hat{\epsilon} \hat{\Gamma}_{[ab} \hat{\psi}_{c]}, \\
\delta \hat{\psi}_a = \hat{D}_a \hat{\epsilon} + \frac{1}{36} \left( \hat{f}^{bcd} \hat{H}_{abcd} + \frac{1}{8} \hat{\epsilon} \hat{\Gamma}_{bcde} \hat{H}^{bcde} \right) \hat{\epsilon} - \hat{k}^b \hat{\Gamma}_{ab} \hat{\epsilon} + 2 \hat{k}_a \hat{\epsilon},
$$

where $\hat{\psi}_a$ is the eleven dimensional gravitini. Here we have written the Weyl superspace equations of
[4] in terms of the standard eleven-dimensional Levi-Civita connection and curvature. These equations
are simply those of the usual eleven-dimensional supergravity [5] but with a $\text{CSpin}(1,10)$ connection

$$
\hat{\Gamma}_a = \frac{1}{4} \left( \hat{\omega}_a^b - 4 \hat{e}_a^b \hat{\epsilon} \hat{\omega}_{c}^d \right) \hat{\Gamma}_b^c + 2 \hat{k}_a \hat{\epsilon},
$$

instead of the usual $\text{Spin}(1,10)$ connection $\hat{\omega}_a^b$.

As discussed in [15], when we perform a standard Kaluza-Klein reduction of MM-theory on the $S^1$, we arrive at a new massive IIA supergravity, which has the appealing property that it can be obtained by compactification of eleven-dimensional supergravity on a circle, with the introduction of a ‘Wilson line’ in the conformal connection.

The equations of motion for the low energy effective action of MM-theory can be obtained by compactifying the equations of motion 2.1 with a Wilson line in the Conformal connection, $\hat{k}_a = m \delta_0^a$, and assuming that the fields are independent of the eleventh-dimension $y$. These equations were first constructed in [15], although there the two-form and three-form gauge fields were set to zero. Here we follow the methods and conventions of [15], but include all the Bosonic fields. In particular the eleven-dimensional vielbein has the form

$$
\hat{e}_a^b = \begin{pmatrix} e^{-\phi/3} e_m^a & A_m e^{2\phi/3} \\ 0 & e^{2\phi/3} \end{pmatrix},
$$

It is simplest to evaluate the eleven-dimensional equations of motion 2.1 in the eleven-dimensional

tangent frame. This leads to the ten-dimensional equations

$$
R_{mn} - \frac{1}{2} g_{mn} R = -\frac{1}{2} e^{2\phi} \left( F_{mp} F_n^p - \frac{1}{4} g_{mn} F^2 \right) - \frac{1}{4} \left( H_{mpq} H_n^{pq} - \frac{1}{6} g_{mn} H^2 \right)
$$
\[-\frac{1}{12}e^{2\phi} \left( G_{mpqr} G_{n}^{pq} - \frac{1}{8} g_{mn} G^2 \right) + 2 \left( D_m D_n \phi - g_{mn} D^2 \phi + g_{mn} (D\phi)^2 \right) \]
\[+ 18 m (D_m A_n) - g_{mn} D_p A_p) + 36 m^2 (A_m A_n + 4 g_{mn} A^2) \]
\[+ 12 m A_m (\partial_n \phi) + 30 m g_{mn} A^p \partial_p \phi + 144 m^2 g_{mn} e^{-2\phi}, \]
\hspace{1cm} (2.5)

\[6 D^2 \phi - 8 (D\phi)^2 = R + \frac{3}{4} e^{2\phi} F^2 + \frac{1}{48} e^{2\phi} G^2 - \frac{1}{12} H^2 \]
\[+ 360 m^2 e^{-2\phi} + 288 m^2 A^2 + 96 m A^a \partial_n \phi - 36 m D^n A_n, \]
\hspace{1cm} (2.6)

\[D^a F_{mn} = -\frac{1}{6} G_{mnpq} H^{npq} + 18 m A^a F_{mn} + 72 m^2 e^{-2\phi} A_m - 24 m e^{-2\phi} \partial_m \phi, \]
\hspace{1cm} (2.7)

\[D^m (e^{-2\phi} H_{mnp}) = 12 m A^m H_{mnp} e^{-2\phi} + \frac{1}{2} G_{npqr} F^{qr} + \frac{1}{36 \cdot 48} \epsilon_{npq\ldots r} G^{qr\ldots} - H^{\ldots}, \]
\hspace{1cm} (2.8)

\[D^n G_{mnpq} = 12 m A^n G_{mnpq} + 12 m H_{npq} e^{-2\phi} + \frac{1}{6 \cdot 36} \epsilon_{npqr\ldots s} G^{qr\ldots} - H^{\ldots}, \]
\hspace{1cm} (2.9)

where

\[H_{mnp} = 3 \partial_{[m} B_{np]} - 6 m C_{mnp}, \]
\[G_{mnpq} = 4 \partial_{[m} C_{npq]} + 4 A_{[m} H_{npq]} , \]
\hspace{1cm} (2.10)

follow from the reduction of the eleven-dimensional four-form $\hat{H}_{abcd}$. This reduction agrees with the equations obtained in [16] through a non-compact ‘Scherk-Schwarz’ dimensional reduction of ordinary eleven-dimensional supergravity (although these authors do not use the ‘string frame’).

Note that the usual $U(1)$ symmetry of the gauge field $A_m$ is broken. Since this is a result of eleven-dimensional diffeomorphisms we expect that it is replaced by another symmetry. Indeed one can show that these equations of motion are invariant under

\[
\begin{align*}
\phi & \rightarrow \phi - 3 m \chi, \\
A_m & \rightarrow A_m - \partial_m \chi, \\
g_{mn} & \rightarrow e^{-6m\chi} g_{mn}, \\
B_{mn} & \rightarrow B_{mn} + \partial_{[m} A_{n]} + 6 m \Omega_{mn}, \\
C_{mnp} & \rightarrow C_{mnp} + 3 \partial_{[m} \Omega_{np]} \\
\end{align*}
\]
\hspace{1cm} (2.11)

From the eleven-dimensional point of view these transformations scale the metric by the Weyl factor $e^{-4m\chi}$. Thus it is possible to gauge away $\phi$ and $B_{mn}$.

By construction these equations of motion are the restriction to Bosonic fields of a system which is invariant under the supersymmetry

\[
\begin{align*}
\delta e_m^a & = -ie^{\phi/3} \Gamma_{na} \psi_m - ie^{-2\phi/3} e_m^a \Gamma_{11} \lambda , \\
\delta \phi & = -\frac{3i}{2} e^{-2\phi/3} \epsilon_{11} \lambda , \\
\end{align*}
\]

4
The naive ground state of M-theory, where the non-metric fields are set to zero, is ten-dimensional de Sitter space:

\[
\delta A_m = -i e^{-2\phi/3} \bar{\Gamma}_{11} \psi_m ,
\]

\[
\delta B_{mn} = -i e^{-2\phi/3} \bar{\Gamma}_{mn} \lambda + 2 i e \epsilon e^{\phi/3} \bar{\Gamma}_{[m} \phi_{n]} + 2 i e \epsilon e^{4\phi/3} A_{[m} \bar{\psi}_{n]} ,
\]

\[
\delta C_{mnp} = -3 i e^{-2\phi/3} \left( \bar{\Gamma}_{[mn} \phi_{p]} + \bar{\Gamma}_{[mn} A_p \lambda - 2 e^\phi A_{[m} \bar{\Gamma}_{n]} \phi_{p]} \right) ,
\]

\[
\delta \lambda = \frac{1}{3} e^\phi \partial^n \phi_{\Gamma_{11} n} \epsilon + \frac{1}{8} e^{2\phi} F_{mn} \Gamma_{mn} \epsilon - \frac{1}{36} e^\phi H_{mnp} \Gamma_{mnp} \epsilon 
+ \frac{1}{288} e^{2\phi} G_{mnpq} \Gamma_{11} \Gamma_{mnpq} \epsilon + me^\phi A^n \Gamma_{11} \Gamma_n \epsilon + 2m \epsilon ,
\]

\[
\delta \psi_m = D_m \epsilon + \frac{1}{6} \partial^n \phi \Gamma_{mn} + \frac{1}{4} F_m^n \Gamma_{11} \Gamma_n \epsilon + \frac{1}{36} e^\phi F_{mnp} G_{mnpq} \epsilon 
+ \frac{1}{12} \Gamma_{11} H_{mnp} \epsilon + \frac{1}{288} e^\phi \Gamma_{mnpq} G_{mnpq} \epsilon 
+ \frac{1}{72} \Gamma_{mnpq} \Gamma_{11} H_{mnp} \epsilon 
+ ie^{2\phi/3} \Gamma_{11} \psi_m \lambda + mA^n \Gamma_{mn} \epsilon + me^{-\phi} \Gamma_{11} \Gamma_m \epsilon - 2mA_m \epsilon ,
\]

(2.12)

where \( \lambda = \hat{\psi}_y \) and \( \psi_m = \hat{\psi}_m - A_m \lambda \) are the ten-dimensional dilatini and gravitini respectively, and \( \Gamma_{11} = \hat{\Gamma}_y \). Although somewhat complicated these supersymmetries are merely the symmetries of Weyl superspace written in a ten-dimensional form (assuming no dependence on \( y \)) and reduce to those of massless type IIA supergravity when \( m = 0 \).

### 3 Stability of de Sitter space

The naive ground state of MM-theory, where the non-metric fields are set to zero, is ten-dimensional de Sitter space: \( R_{mn} = -36m^2 g_{mn} \). The first thing one notices about this de Sitter vacuum is that the massive vector field \( A_m \) is in fact tachyonic at the linearised level. Therefore one expects that de Sitter space is a “false” vacuum of some kind. To further understand this issue let us consider the stability of de Sitter space under a small vector perturbation. To this end we expand the equations of motion to lowest order in \( A_m = \mathcal{O}(\epsilon) \), \( g_{mn} = g_{mn}(dS) + \mathcal{O}(\epsilon) \), but with all the other fields vanishing. This gives

\[
R_{mn} - \frac{1}{2} g_{mn} R - 144m^2 g_{mn} = 18m D_{(m} A_{n)} + \mathcal{O}(\epsilon^2) 
\]

\[
D^n F_{mn} = 72m^2 A_m + \mathcal{O}(\epsilon^2) .
\]

(3.13)

Note that we have used the fact that the second equation implies \( D^n A_n = \mathcal{O}(\epsilon^2) \). This in turn implies that 2.6 is satisfied to lowest order and also that the only non-vanishing term on the right hand side of 2.5 is the one given in 3.13. Taking the trace of the linearised Einstein equation shows that the background is still of constant curvature: \( R = -360m^2 + \mathcal{O}(\epsilon^2) \). The second equation of 3.13 may now just be written as

\[
D^2 A_n = -36m^2 A_n + \mathcal{O}(\epsilon^2) .
\]

(3.14)

It is not hard to check that a natural family of solutions to these equations is just to let \( A_m = \epsilon K_m \) where \( K_m \) is a Killing vector\(^1\). In this case we find \( g_{mn} = g_{mn}(dS) + \mathcal{O}(\epsilon^2) \), i.e. to this order the metric remains that of de Sitter space. However if we choose coordinates where de Sitter space has the metric

\[
ds^2 = -dt^2 + e^{4mt} dx^i dx^i ,
\]

(3.15)

\(^1\)Recall that, in general, a Killing vector in a vacuum spacetime acts as a vector potential for a Maxwell test field [17]
where $i = 1, \ldots, 9$, then a class of Killing vectors, corresponding to translations along $x^i$, have the form $K^m = \delta^m_i$. These produce the vector field $A_m = e^{4mt} \delta^m_i$. Thus a small perturbation of de Sitter space at time $t = 0$ by an electric field will in fact grow exponentially in size as space inflates until it is of order one and we can no longer neglect the non-linear terms, including the back-reaction on the metric. Thus we conclude that the de Sitter space vacuum of MM-theory is unstable.

It is amusing to note that precisely this form of the tachyonic vector field has been studied before on de Sitter space [18]. These authors showed that, in an inflationary scenario, interactions for the tachyonic vector field give rise to primordial magnetic fields of the same magnitude that are observed in galaxies today. Here we see that such peculiar interactions readily appear in MM-theory.

We can also consider small perturbations about de Sitter space by the two-form and three-form fields. In particular consider a de Sitter background and turn on a small $H_{mnp}$ and $G_{mnpq}$ of order $\epsilon$.

To order $\epsilon$ the metric remains that of de Sitter space and the remaining equations of motion are

\begin{align}
D^m G_{mnpq} &= 12m H_{npq} + O(\epsilon^2), \\
D^m H_{mnp} &= 0 + O(\epsilon^2),
\end{align}

and now $G_{mnpq} = -\frac{1}{6m} \partial_m H_{npq}$. Clearly the first equation implies the second. Therefore we simply need to consider

\begin{equation}
4D^m D[mH_{npq}] = -72m^2 H_{npq} + O(\epsilon^2).
\end{equation}

Thus the three-form also appears to be tachyonic. In the coordinates 3.15, 3.17 splits into two equations:

\begin{align}
4\partial^k \partial_{[k} H_{0ij]} &= -72m^2 e^{4mt} H_{0ij} + O(\epsilon^2), \\
4\partial_0 \partial_{[0} H_{ijk]} + 24m \partial_{[0} H_{ijk]} - 4e^{-4mt} \partial_{[0} H_{ijk]} &= 72m^2 H_{ijk} + O(\epsilon^2).
\end{align}

One simple solution to these equations is to take $H_{ijk} = e^{\lambda mt} C_{ijk}$, $H_{0ij} = 0$ where $C_{ijk}$ is a constant tensor. Inserting this ansatz into 3.18 leads to $\lambda = 6$ or $\lambda = -12$. On the other hand one can easily check that there are no solutions with $H_{ijk} = 0$. Thus, just as with the vector field above, a small but constant three-form $H_{ijk}$ with $\lambda = 6$ will eventually grow so large that its effect on the de Sitter background spacetime can no longer be ignored.

4 Zero-branes

Given that the de Sitter vacuum of MM-theory is unstable against the spontaneous creation of electric fields, it seems natural to search for static solutions of the field equations that carry charges, i.e. solutions with $F_{0i} \neq 0$. To this end we start by considering the familiar D0-brane ansatz

\begin{align}
ds^2 &= -e^{2\alpha \phi} dt^2 + e^{2\beta \phi} dx^i dx^i, \\
A_i &= 0.
\end{align}

First we examine the non-linear vector equation 2.7. Demanding that the solution is time-independent leads to the relation

\begin{equation}
A_0 = \pm \sqrt{\frac{4}{3(1 - \alpha)}} e^{(\alpha - 1)\phi}.
\end{equation}

This then leaves the equation

\begin{equation}
\partial_i \partial^i \phi + (7\beta - 1) \partial_i \phi \partial^i \phi = -\frac{72m^2}{\alpha - 1} e^{(2\beta - 2)\phi},
\end{equation}
where the ± sign chooses between zero-branes and anti-zero-branes. If we now substitute this ansatz into the Einstein equation 2.5 we find that $\alpha = -\beta = -1/3$. One can then check that 2.6 is automatically satisfied. Thus we find the solution is simply

$$
ds^2 = -H^{-\frac{1}{2}} dt^2 + H^{\frac{1}{2}} dx^i dx^i,
A_0 = \pm H^{-1},
\quad e^{\phi/3} = H.
$$

(4.22)

This is precisely the same form as D0-branes in massless type IIA string theory. The only difference is that in M-theory the function $H$ is not harmonic but instead satisfies, from 4.21,

$$
\partial_i\partial^i H = 72m^2.
$$

(4.23)

However one can check that this solution does not preserve any supersymmetries since, although from the eleven-dimensional point of view it is ‘almost’ supersymmetric: $\delta\bar{\psi}_m = 0$, $\delta\bar{\psi}_y = 3m\epsilon$.

We note that $H$ can be written as

$$
H = H_0 + 4m^2 A_{ij} x^i x^j,
$$

(4.24)

where $A_{ij}$ is a constant matrix with $\text{Tr}A = 9$ and $H_0$ is a harmonic function which we take to have the usual $p$-brane form

$$
H_0 = 1 + \sum_n Q_n \left[\frac{1}{x^i - x^i_n}\right]^7.
$$

(4.25)

Thus we can obtain solutions describing an arbitrary collection of zero-branes in a background “condensate” of electric charge. In particular the near-horizon geometry of these zero-branes will be the same as the D0-branes in the massless type IIA theory. On the other hand, as $r \to \infty$, where $r = x^i x^i$ the spacetime is not asymptotically flat. Furthermore the dilaton grows without bound, so that asymptotically the spacetime decompactifies.

The appearance of $m^2$ in $H$ is reminiscent of Reissner-Nordstrom-de Sitter solutions. However here the solution is not asymptotically de Sitter and contains a non-vanishing dilaton. We may find another class of solutions by invoking the $U(1)$ symmetry 2.11 of the equations of motion resulting from eleven-dimensional diffeomorphisms. In particular if we set $\chi = \phi/3m$ we can transform away the dilaton and find the solution

$$
ds^2 = -H^{-2} dt^2 + H^{-1} dx^i dx^i,
A_0 = \pm H^{-1},
A_i = -\frac{1}{4m} \partial_i \ln H
\phi = 0
$$

(4.26)

where $H = H_0 + 4m^2 A_{ij} x^i x^j$ is the same as before. Note that $F_{mn}$ is unchanged by this transformation, i.e. we still only have electric fields. Although we note that now it is not clear how to take a smooth limit as $m \to 0$. However a smooth limit might exist since terms the divergent terms $A_i$ are pure gauge. Indeed the smooth limit might require a transformation to the infinite momentum frame.

Let us consider the spherically symmetric case $A_{ij} = \delta_{ij}$. Far from the zero-branes $H_0 = 1$ and hence $H = 1 + 4m^2 r^2$. It is instructive to transform to a new coordinate $\tilde{r}^{-2} = r^{-2} + 4m^2$ in which case the metric in 4.26 becomes

$$
ds^2 = -\left(1 - 4m^2 \tilde{r}^2\right)^2 dt^2 + \frac{d\tilde{r}^2}{\left(1 - 4m^2 \tilde{r}^2\right)^2} + \tilde{r}^2 d\Omega_8^2.
$$

(4.27)
This is very similar to de Sitter space in static coordinates which has the metric
\[
\begin{align*}
 ds^2_{dS} &= -(1 - 4m^2r^2)dt^2 + \frac{dr^2}{(1 - 4m^2r^2)} + r^2d\Omega^2_8. 
\end{align*}
\] (4.28)

Indeed 4.27 has the same causal structure and in particular \( \dot{r}^2 = 1/4m^2 \) corresponds to a cosmological event horizon. Furthermore, at least in these coordinates, all of the curvature components are well behaved. For distances much less than the cosmological horizon the metric looks like that of de Sitter (but with twice the cosmological constant of the vacuum de Sitter solution).

Lastly we note that the entropy of the zero-brane spacetime 4.27, viewed as one quarter of the area of the cosmological horizon, is
\[
S = \frac{\text{Vol}(S^8)}{1024m^8},
\] (4.29)

where \( \text{Vol}(S^8) \) is the volume of a unit eight-sphere. This is the same as the entropy of the de Sitter vacuum. This is particularly intriguing since this spacetime can contain arbitrarily many zero-branes, each with eight bosonic degrees of freedom. Whereas the entropy is supposed to measure the number of degrees of freedom in the corresponding quantum theory.\(^2\)

We can also lift the solution to eleven-dimensions using the ansatz 2.4. This leads to the standard pp-wave spacetime only with a non-harmonic function \( H \)
\[
 ds^2 = Hdy^2 \pm 2dydt + dx^i dx^i. 
\] (4.30)

Asymptotically, where we can neglect \( H_0 \), this spacetime is known as a Cahen-Wallach space. It can be constructed as a smooth symmetric space and has also been recently studied in conventional (i.e. \( k_a = 0 \)) M-theory [21]. A crucial difference, however, is that in [21] the source for \( H \) is provided by the four-form field strength, whereas in our analysis the source is \( \hat{k} \). In particular the four-form gives rise to a source for \( H \) with \( m^2 \to -m^2 \), i.e. anti-de Sitter, rather than de Sitter, behaviour.

Thus MM-theory naturally contains zero-branes and the cosmological constant provides a uniform and constant source for them. Since zero-branes are eleven-dimensional gravitons carrying single units of momentum around the eleventh dimension, this hints that MM-theory itself is unstable against de-compactification to eleven dimensions, where it is physically equivalent to M-theory. Thus we are led to the conjecture that MM-theory is an unstable vacuum of M-theory.

### 4.1 Other Branes

It seems natural to look for other static \( p \)-brane solutions with the ansatz
\[
 ds^2 = e^{2\alpha\phi} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2\beta\phi} \delta_{ij} dx^i dx^j, 
\] (4.31)

where \( \mu, \nu = 0, \ldots, p, i, j = p + 1, \ldots, 9 \) and \( \partial_\mu \phi = 0 \). A first attempt might be to look for six-branes, the magnetic duals to the zero-branes. However if we set \( G_{mnpq} = H_{mnpq} = 0 \) and consider static solutions with \( A_0 = 0 \) then the vector equation 2.7 does not simplify in any obvious way. Indeed the usual \( D6 \)-brane solution is often (but not necessarily) constructed by dualising \( F_{mn} \) to an eight-form. However the massive deformation discussed here makes this dualisation non-local since \( A_m \), and not just \( F_{mn} \), appears in the equations of motion.

\(^2\)In [19] it was argued that there should be an upper bound on the transverse density of partons. It would be interesting to see if similar arguments may imply that there is an upper bound on the total number of zero-branes which can be ‘packed’ within the cosmological horizon. We will not pursue these ideas here.
Given that the three-form $C_{mnp}$ also has a tachyonic instability we might try to look for $D2$-branes and $D4$-branes. If we now set $B_{mn} = A_m = 0$ then 2.7 implies, for $m \neq 0$,

$$\frac{1}{6} \partial_m C_{npq} C^{npq} = e^{-2\phi} \partial_m \phi.$$  \hfill (4.32)

For a $D2$-brane we expect that $C_{012} \neq 0$. This leads to $C_{012} = \sqrt{8/(2 - 6\alpha)} e^{(3\alpha - 1)\phi}$. However the Einstein and dilaton equation can only be satisfied if $m = 0$ and $\alpha = -\beta = -1$, in which case we obtain the standard $D2$-brane of the massless theory. In the case of four-branes we encounter the same problem that we saw for six-branes, i.e. 2.7 does not simplify and it is not clear how to dualise $G_{mnpq}$ since the equations of motion involve $C_{mnp}$ without derivatives.

In particular it might seem as if there should be eight-brane solutions, similar to those found in the Romans supergravity [22]. However inserting the ansatz 4.31 with $p = 8$ into the equations of motion and setting $F_{mn} = H_{mnp} = G_{mnpq} = 0$ ones finds that there are no Poincare invariant solutions.

## 5 MM(atrix) Theory?

The existence of solutions that represent an arbitrary number of static zero-branes is analogous to the situation of D0-branes in massless type IIA string theory. In addition it is well-appreciated that in a certain limit, known as the infinite momentum frame, the dynamics of D0-branes contains the entire physics of string theory and M-theory. In particular this allows for a matrix definition of string theory and M-theory[19]. Therefore one might hope that some modification of this matrix theory could be interpreted as the dynamics of the zero-branes we found above in MM-theory. Furthermore this naturally leads to a microscopic quantum definition of MM-theory, as a modified (or massive) MM(atrix) theory.

To construct the effective action for the zero-branes we first recall the analogous construction in of the D0-brane effective action starting from M-theory. Following [23] one starts with the superspace action for a massless superparticle in eleven dimensions

$$S = - \int d\tau \frac{1}{\hat{v}} \hat{E}_r \cdot \hat{E}_r - \hat{Y},$$  \hfill (5.33)

where $\hat{E}_r^A$ is the pull back to the worldline of the superveilbien $\hat{E}_r^B$ and $\hat{v}$ is an independent world-line density. Here $Y$ is the coordinate corresponding to the eleventh dimension, and a dot denotes differentiation with respect to the worldline coordinate $\tau$.\footnote{The last term is therefore a total derivative but is needed as explained in [23].} Using the analogous decomposition for the superveilbien that we used in 2.4, and assuming that nothing depends upon $Y$, we can remove $Y$ and $\hat{v}$ using their equations of motion to obtain (setting the Fermions to zero)

$$S = - \int d\tau e^{-\phi} \sqrt{-\hat{g}} \hat{X}^m A_m$$  \hfill (5.34)

where $\hat{g} = \hat{X}^m \hat{X}^n g_{mn}$ is the pull back of the spacetime metric to the one-dimensional worldvolume, and $\hat{X}^m A_m$ is the Wess-Zumino term which describes the coupling of the zero-brane to the RR field $A_m$.

Let us now consider a superparticle in MM-theory. If we imagine that $y$ is non-compact then we may map the superparticle of M-theory to a superparticle of MM-theory by rescaling $\hat{E}_r^A \rightarrow e^{-2\hat{\theta}} \hat{E}_r^A$ where $\hat{k} = d\hat{\theta}$. However in 5.33 this transformation is simply absorbed by the worldline density $\hat{v}$. Clearly this construction is unaffected if we now compactify $Y$, assuming that $\hat{v}$ is again taken to be...
independent of $Y$. Thus we obtain precisely the same worldline action 5.34 for the zero-branes in MM-theory. As a check on this construction we note that 5.34 is invariant under the MM-theory gauge transformation 2.11.

Let us consider two backgrounds for these zero-branes. The first example is de Sitter space and we choose the coordinates 4.28. Using static gauge $\tau = t$ and keeping only the radial coordinate $r$, the zero-brane action in this background is

$$S = - \int dt \sqrt{(1 - 4m^2r^2) - \frac{j^2}{1 - 4m^2r^2}} = \int dt \frac{1}{2} \frac{j^2}{(1 - 4m^2r^2)^{3/2}} - \sqrt{1 - 4m^2r^2} + \ldots ,$$  \hspace{1cm} (5.35)

where we have kept only the lowest order term in a derivative expansion. Here we see that de Sitter space induces a potential on the worldline action of the zero-branes. Near $r = 0$ this potential appears tachyonic. Secondly we consider a zero-brane in the background of other zero-branes. Using static gauge the action becomes

$$S = - \int dt H^{-1} \sqrt{1 - H\dot{x}^i\dot{x}^i} - H^{-1} = \int dt \frac{1}{2} \dot{x}^i\dot{x}^i + \ldots ,$$  \hspace{1cm} (5.36)

where again we have only kept the lowest order terms in a derivative expansion. Here we see that at lowest order terms are free and identical to those of D0-branes in type IIA string theory.

Next we must consider the effective action for $N$ zero-branes. In M(atrix) theory one considers the light-like limit and only the lowest order terms in the effective action survive. Furthermore one knows from D-brane quantisation via open strings that for $N$ D0-branes these terms are precisely that of a maximally supersymmetric one-dimensional $U(N)$ gauge theory

$$S_{\text{M(atrix)}} = \int d\tau \text{Tr} \left[ \frac{1}{2} \dot{X}^i\dot{X}^i - \frac{1}{2} \sum_{i<j} [X^i, X^j]^2 \right].$$  \hspace{1cm} (5.37)

The free action 5.36 is recovered from 5.37 by identifying $x^i$ with the $(U(1)$ part of $X^i$. Therefore our next task is to motivate a proposed modification to the quantum mechanics if $m \neq 0$.

There is at least one crucial difference between any proposed matrix definition of MM-theory theory and M(atrix) theory. Namely in M(atrix) theory the near horizon geometry of the D0-branes can be obtained through a limit of M-theory where the compact $S^1$ becomes light-like. To see this one starts with the usual D0-brane metric of type IIA string theory (i.e. 4.22 but with $m = 0$) and sets $H = 1 + h$, where $h$ vanishes at infinity. In the massless theory we can choose the graviphoton to have the form $A_0 = H^{-1} - 1$, which differs from the zero-brane solution 4.22 by a gauge transformation. In this case one finds that the eleven-dimensional lift of this solution does not take the form 4.30 but rather

$$ds^2_{11} = -dt^2 + dy^2 + h d(y - t)^2 + dx^i dx^i .$$  \hspace{1cm} (5.38)

Reducing to ten-dimensions on the (asymptotically) light-like circle $y - t$ then leads to the near-horizon geometry of the D0-branes (i.e. one finds 4.22 but with $H$ replaced by $h$). The interpretation of this observation is that M(atrix) theory describes M-theory states that have been asymptotically boosted into the infinite momentum frame. However in MM-theory we cannot so simply choose the graviphoton to have this form since the required gauge transformation also induces a Weyl rescaling of the metric and a shift in the dilaton. Instead if we set $\chi = t$, so that $A_0 = H^{-1} - 1$, we find

$$ds^2_{11} = e^{-4\alpha t} \left[ -dt^2 + dy^2 + h d(y - t)^2 + dx^i dx^i \right].$$  \hspace{1cm} (5.39)

Compactification on $y - t$ again reproduces the near horizon geometry of the zero-branes (in the gauge transformed variables). However asymptotically $h$ now diverges so that $y - t$ is never light-like. Of
course it is not surprising that the near horizon geometry of these zero-branes cannot be related to asymptotically light-like compactified MM-theory, because MM-theory does not have asymptotically flat solutions.

Indeed, since the $U(1)$ symmetry 2.11 lifts to a diffeomorphism combined with a Weyl transformation of the eleven-dimensional metric, we see that by a choice of gauge we can arrange for an eleven-dimensional solution which of the form 5.39 but with any conformal factor $e^{-4m\chi}$. Asymptotically $H = 1 + h \sim 4m^2A_{ij}x^ix^j$ so that from the eleven-dimensional point of view the zero-branes describe states in MM-theory which are asymptotically conformally Cahen-Wallach.

This conformal symmetry implies that there is no suitable Maldacena limit. In particular, from the point of view of MM-theory it is not meaningful to consider the limit where the energy and length scale of the system is taken to zero, since this requires a choice of conformal frame (or, from the ten-dimensional point of view, a choice of gauge for the graviphoton). In effect the zero-brane dynamics are not associated with the light-like limit of MM-theory but rather they somehow describe MM-theory at all scales. As a result it seems likely that we can not restrict our attention the lowest order terms in zero-brane action 5.34 which then leads to 5.37 if there are $N$ zero-branes. We therefore imagine a non-trivial phase of matrix quantum mechanics where none of the higher order terms can be neglected. This resolves the puzzle that at lowest order in fields, the zero-brane action 5.34 does not depend on $m$ and so one can not tell the difference between MM-theory and M-theory. However this is not the case if the higher order terms are not neglected. In addition the tachyonic nature of zero-branes in a de Sitter background is stabilised through higher order terms.

It is intriguing to note that Cahen-Wallach spacetimes are symmetric spaces with a coset structure $G/H$, where $G$ is the Heisenberg algebra associated to a system with nine degrees of freedom, combined with an outer automorphism that intertwines the momentum and coordinate variables [21]. Specifically, $G$ is the algebra whose generators satisfy [21]

$$[p_i, q_j] = 4m^2A_{ij}, \quad [R, q_i] = p_i, \quad [R, p_i] = 4m^2A_{ij}q_j,$$

with all other commutators vanishing. The subalgebra $H$ is generated by the momenta $p_i$ and the corresponding symmetry is presumably that of coordinate translations. We are led to conjecture that $G/H$ is the symmetry group of the underlying quantum mechanical MM(atrix) theory in an asymptotically conformally Cahen-Wallach spacetime. Since $H$ is linearly realised this suggests that there is no potential in the Hamiltonian. It would be interesting to see if the generalised conformal mechanics of D0-branes [24] is related to a possible MM(atrix) theory.

This situation can be sharply contrasted with the D0-branes of the massless IIA string. In that case, the supergravity solution is asymptotically flat and one may choose the string coupling to be arbitrarily small so that ordinarily the physics would be strictly ten-dimensional. However, if one stacks a large number ‘$N$’ of D0-branes in the core of the spacetime, then a bubble of eleven-dimensional spacetime will open up in the region $N^{1/9}/M < r < N^{1/7}/M$ [25, 26]. For the massive zero branes that we have found this effect will persist, the key difference is that the geometry can never be asymptotically flat.

Given that the symmetry which we use to (locally) transform from M-theory to MM-theory is also a symmetry of the D0-brane effective action, there is a bit of a puzzle here: How could de Sitter space possibly be the ground state of MM-theory, given that it is definitely not a solution of the massless IIA string? Again, our main answer to this conundrum is that there should exist a phase of matrix quantum mechanics where higher order terms survive. However, it is possible to see that this theory will be sensitive to the fact that the bulk R-R vector is tachyonic, even at the linearized level. This is because on a D0-brane the world-volume quantum mechanical theory includes sixteen Fermionic operators $\theta$ [20]. The zero-modes of these Fermions generate an $SO(16)$ Clifford algebra, and consequently may be written as $2^{16/2} = 256$-dimensional gamma matrices. It follows that a D0-brane has 256 internal
degrees of freedom, or polarization states. These states are explicitly constructed in [27], and they are precisely the space of polarization states of the supergraviton in eleven dimensions. In the weak field approximation, the worldvolume fermions couple to small fluctuations of the background metric, \( g_{mn} \), NS-NS 2-form potential, \( B_{mn} \), and R-R one-form and three-form potentials, \( A_m \) and \( A_{mnp} \) [28, 27]. The precise interaction Lagrangian for the R-R vector takes the form

\[
L_{int} = -\frac{i}{8} (\nabla_m A_n) \bar{\theta} \Gamma^{mn} \theta .
\]  

(5.41)

In this sense the zero-brane internal degrees of freedom are ‘sensitive’ to an MM-theory background. Understanding this interaction properly will involve working out the full family of superpartners for the purely bosonic zero-brane which we have been discussing in this paper. In other words, while a finite \( N \) matrix formulation of DLCQ MM-theory should provide a (partial) microscopic description of MM-theory, it would also be of interest to better understand the asymptotic structure of these massive zero-branes. In particular, it would be interesting to work out the long-range supergravity fields which arise when we polarize one of these massive Bosonic zero-branes with a higher multipole moment. For example, it should be possible to polarize one of these zero-branes with a two-brane dipole moment as discussed in [29]. However, this is complicated by the observation above that it seems difficult (though not necessarily impossible) to find a two-brane solution for this theory.

6 Relaxation of the cosmological constant through domain wall nucleation?

As described above, in order to construct the new massive IIA supergravity of [15], we must introduce an exact one form \( \hat{k} = mdy \) in eleven-dimensions. As described in [30], in ten-dimensions we can introduce a ten-form field strength

\[
F_{10} = *m .
\]

(6.1)

Thus \( d \ast F_{10} = 0 \) and \( dF_{10} = 0 \) automatically. From this point of view it is natural to consider the existence of eight-brane or domain wall solutions that couple to \( F_{10} \). Of course, when we say that states of this massive IIA theory couple electrically to the 10-form \( F_{10} \), what we really mean is that there exists a nine-form potential \( A_9 \), which couples to the worldvolume of the eight-brane, and which is related to the ten-form in the usual way:

\[
F_{10} = dA_9 .
\]

(6.2)

Unfortunately, at present there is no reason why we should make this assumption. This is related to the fact that at present we still don’t have a good microscopic realization of the degrees of freedom underlying the ten-form formulation of this theory. This can be contrasted with the Romans massive IIA theory [31], where the D8-branes are of course places where F-strings can end. We have also seen that no Poincare invariant eight-brane solutions to the equations of motion exist, whereas they do exist in the Romans theory [22].

At any rate, if we could justify the statement that such domain walls exist then we could invoke the mechanism of Brown and Teitelboim ([32], [33]), who originally studied the stability of a theory in four dimensions where the effective (positive) cosmological constant is generated by a four-form flux, with fundamental membranes coupled electrically to the four-form. They proved that the effective cosmological constant will be decreased through the nucleation of membranes which couple to the four-form. This decrease of the cosmological constant through brane nucleation is generically known as the Brown-Teitelboim mechanism. Clearly, if the massive IIA theory which we have been studying is
just a higher-dimensional realization of the scenario originally studied by Brown and Teitelboim, then we may use the Brown-Teitelboim mechanism to conclude that at late times the cosmological constant will be driven to zero. Research on this possibility is currently underway.

7 Conclusions

In this paper we have addressed some dynamical issues in a modified type IIA supergravity. In particular we showed that it admits multi-zero-brane solutions and that the effective description of these zero-branes can be related to a massive deformation of the well-known M(atrix) formulations of M-theory and type IIA string theory. This suggests a matrix theory formulation of MM-theory in which the massive equations of motion arise as the the low energy effective dynamics.

The massive deformation of type IIA supergravity discussed here can almost certainly be extended to other supergravities. Indeed the mechanism is in some sense trivial. What is, in our opinion, non-trivial is that this construction is completely compatible with eleven-dimensional superspace. Indeed it is the most general solution to the constraints of eleven-dimensional supersymmetry. We find that this is a compelling reason to explore the conjecture that it also has a microscopic description related to that which underlies M-theory. We find this motivation particularly exciting, since our results imply that a cosmological constant is generated spontaneously.

Extrapolating the discussion here of M-theory to a realistic four-dimensional phenomenology, we are presented with the following inflationary scenario: at its creation the universe is in fact three-dimensional. It undergoes a period of very rapid, Planck scale, inflation where an additional fourth-dimension opens up. The size of this dimension also rapidly expands well beyond any meaningful observable scale. The eventual ground state in such a model is then four-dimensional Minkowski space (perhaps with a small remnant of the cosmological constant). The idea that our universe is, in some sense, three-dimensional has appeared before. Notably with Witten’s suggestion that the cosmological constant problem might be solved if our universe is in a strongly coupled phase of a three-dimensional supersymmetric theory [34]. It would be intriguing if the supergravity discussed here provides a cosmological realization of this mechanism.

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Appendix: Conventions

Here we give the conventions we have used for the spin connection and curvature. We follow the same conventions as in [15] where underlined indices refer to the tangent frame, $m, n = 0, ..., 9$ label ten-dimensional indices and $a, b = 0, ..., 10$ eleven-dimensional ones. We also use hats to denote eleven-dimensional fields. When discussing $p$-branes the worldvolume coordinates are labelled by $\mu, \nu = 0, ..., p$ and the transverse coordinates by $i, j, k, ....$
We define the spin connection by
\[
\partial_m e_n^p - \partial_n e_m^p + \omega_{mq}^p e_n^q - \omega_{mq}^q e_n^m = 0 ,
\]
and hence the torsion-free Christoffel components can be obtained as
\[
\Gamma^p_{mn} = e_q^p (\partial_m e_n^q + \omega_{mq}^q e_n^m) .
\]
The curvature tensor is defined as
\[
R_{mpn}^q = \partial_m \Gamma_{np}^q - \partial_n \Gamma_{mp}^q - \Gamma_{mr}^q \Gamma_{np}^r + \Gamma_{np}^r \Gamma_{rm}^q ,
\]
and the Ricci tensor is \(R_{mn} = R_{mpn}^p\). In particular one finds that \([D_m, D_n]V_p = -R_{mpn}^q V_q\).

References


