Do Hadronic Charge Exchange Reactions Measure Electroweak $L = 1$ Strength?

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Abstract

An eikonal model has been used to assess the relationship between calculated strengths for first forbidden $\beta$ decay and calculated cross sections for $(p,n)$ charge exchange reactions. It is found that these are proportional for strong transitions, suggesting that hadronic charge exchange reactions may be useful in determining the spin-dipole matrix elements for astrophysically interesting leptonic transitions.

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I. INTRODUCTION

Charge exchange reactions \((A_Z, A_{Z\pm 1})\) induced by hadronic projectiles are a powerful tool for probing spin-isospin degrees of freedom in nuclei [1–4]. The spin-isospin parts of the operators that mediate charge exchange reactions such as \((p, n)\) are the same as those involved in the corresponding processes induced by electromagnetic and weak interactions. As a result, the matrix elements that describe hadronic charge exchange reactions are closely related to those that describe the rates of \(\beta\) decay or the cross sections of reactions induced by neutrinos. It would be fortunate if this relationship were quantitatively accurate, since it is often difficult to study the leptonic processes directly. For example, the range of excitation energy kinematically accessible in a \(\beta\) decay transition does not encompass the majority of the allowed (Gamow-Teller) strength and the experimental study of neutrino induced reactions is difficult.

A promising direction of future activity is to determine leptonic strengths for otherwise inaccessible nuclides by studying charge exchange reactions using radioactive (secondary) beams in inverse kinematics [5]. This would provide nuclear properties important for problems of nuclear physics, particle physics, astrophysics and cosmology [4,6,7]. One could clarify the relationship between the spatial properties of nuclear halo systems and the nature of soft multipole modes [8]. One could also determine the strength of neutrino-nucleus interactions needed to describe the chemical evolution of the Universe, especially the abundances of the light elements [9] and the products of \(r\)-process nucleosynthesis [10], and to calibrate terrestrial detectors of supernova neutrinos [11].

However, it is not obvious a priori that the correlation of charge exchange and leptonic matrix elements is sufficiently close for this purpose. In contrast to leptonic processes, hadronic reactions involve operators that have an additional radial dependence and are subject to distortion by complex nuclear potentials; the medium renormalization of effective operators and the contributions of multi-step processes introduce additional uncertainties. Therefore it was an important advance to establish that there is an approximate proportion-
ality between the cross section of charge exchange reactions at very forward angles leading to the Gamow-Teller (GT) excitations (with transferred $T = 1, L = 0, J = S = 1$) and the transition strength $B$(GT) determined by intrinsic nuclear matrix elements [8,12–16]. The proportionality has been confirmed for strong transitions in a variety of nucleon and nucleus induced charge exchange reactions; a more detailed analysis is needed for weak GT processes [17]. The model-independent character of the relation between the $L = 0$ cross section for reactions induced by $^{12}$C projectiles and the GT strength was clarified by a theoretical analysis [18] based on a sensitivity function which identified the important part of the target transition density in momentum space.

In contrast to GT transitions, the first forbidden matrix elements of weak processes explicitly include orbital degrees of freedom. The corresponding nuclear response in the $L = 1$ channel is associated with the states forming the spin-dipole and giant-dipole charge-exchange resonances (SDR and GDR). This excitation was discovered [19] and studied on different targets [20] mostly using $(p, n)$ reactions. Recently the energy splitting of the $L = 1$ charge exchange resonances (GDR and SDR) was determined [21]. There is very little information about the quantitative relationship of charge exchange cross sections and leptonic strength for these excitations. Here we take a first step in providing this information by studying the relationship between first forbidden strength and $(p, n)$ cross sections, both calculated from the same wave functions. We show that, at the same level of accuracy as for the GT case, for strong transitions one can expect an approximate proportionality between the observed cross sections of charge exchange reactions populating spin-dipole states and the corresponding nuclear transition probabilities. We follow the general approach that was successfully applied to GT excitations by Osterfeld et al. [18], extended to describe $L = 1$ transitions and the effects of the real part of the optical potential.
In order to understand the relationship between the charge exchange cross section and the nuclear response strength, we need to examine the effects specific to the excitation of the SDR. For this purpose we consider the influence of distortion in a simple eikonal approximation (EA). In many cases, even at rather low energy, the EA gives a good qualitative description of the reaction cross section in the SDR region. We obtain the relevant wave functions and transition densities from the shell model.

The strength $B_J$ of the SDR in the long wavelength limit and the transition form-factors $F_J(q)$ are calculated in terms of the elements of the single-particle density matrix of the given transition $i \rightarrow f$ from the ground state, $\rho_{fi}(\nu, \nu') = \langle f| a^\dagger_\nu a_{\nu'} |i \rangle$,

$$B_J(i \rightarrow f) = \left| \sum_{\nu \nu'} \rho_{fi}(\nu, \nu')(\nu'||r O_J||\nu') \right|^2,$$

(1)

$$F_J(q) = \sum_{\nu \nu'} \rho_{fi}(\nu, \nu')(\nu'||j_1(q r) O_J||\nu'),$$

(2)

where $(\nu'||j_1(q r) O_J||\nu)$ is the product of the radial matrix element of the spherical Bessel function and the reduced matrix element of the charge exchange spin-dipole operator $O_{JM} = \{ \sigma \otimes Y_1(\hat{r}) \}_{JM} \tau^\pm$. In the limit of low momentum transfer, $q R \ll 1$, the squared transition form-factor (2) is related directly to the strength of the SDR,

$$F_J^2(q) \rightarrow \frac{q^2}{9} B_J(i \rightarrow f).$$

(3)

In the case of the GT resonance a direct proportionality between the experimentally measured cross sections of charge exchange reactions at very forward angles and nuclear matrix elements was confirmed by a number of studies [8,12–16] at a level of accuracy of 10-15%. From the viewpoint of the underlying physics, this important result is based on a single-step mechanism for the process, a simple bare operator which does not include orbital degrees of freedom, and the dominance of the central spin-isospin interaction $V_{\sigma \tau}$ over a broad range of energies. It is a priori unclear whether these features pertain to the SDR.
At the maximum of the differential cross section for the SDR, $q R \sim 1$. The $q$-dependence of the hadronic operator and the effects of tensor forces may show up at the larger $q$. As a result, there may not be a simple relationship between the cross section and the nuclear response strength.

In the SDR case, the direct part of the $(p, n)$ reaction amplitude (the exchange part will be discussed later) is

$$T_{fi}^{dir} = \int d^3 r \chi_f^{(-)\ast}(k_f, r) \int \frac{d^3 q'}{(2\pi)^3} F_J(q') V_{JM}(q') \exp(-iq' \cdot r) \chi_i^{(+)}(k_i, r),$$

(4)

where the effective operators for the channels with angular momenta $J^\pi = 0^-, 1^-$ and $2^-$ contain contributions from central and tensor forces,

$$V_{JM}(q') = V_{JM}^c(q') + V_{JM}^t(q'),$$

(5)

$$V_{JM}^c(q') = 4\pi i \sqrt{\frac{2}{2J+1}} t_{\sigma\tau}^c(q') \{\sigma \otimes Y_1^*(\hat{q}')\}_{JM},$$

(6)

$$V_{JM}^t(q') = 4\pi i \sqrt{\frac{2}{2J+1}} t_{\tau}^t(q') \{\sigma \otimes Y_1^*(\hat{q}')\}_{JM} - 3(\sigma \cdot \hat{q}') \{\hat{q}' \otimes Y_1^*(\hat{q}')\}_{JM}.$$  

(7)

In Eqs. (6) and (7), $t_{\sigma\tau}^c(q')$ and $t_{\tau}^t(q')$ are, respectively, the central and tensor components of the nucleon-nucleon $t$-matrix, see Franey and Love [22], that are responsible for spin-isospin transfer. The excitation of different $J$-components of the SDR proceeds via different combinations of the amplitudes of the nucleon-nucleon effective interaction. For the $0^-$ part, the tensor interaction can be combined with the central one using the relations

$$\{\hat{q}' \otimes Y_1^*(\hat{q}')\}_{00} = -\frac{1}{\sqrt{4\pi}}, \quad \{\sigma \otimes Y_1^*(\hat{q}')\}_{00} = -\frac{(\sigma \cdot \hat{q}')}{\sqrt{4\pi}}.$$

(8)

These identities produce the combination $t^t(q) = t_{\sigma\tau}^c(q) - 2t_{\tau}^t(q)$ which is just the spin-longitudinal component of the nucleon-nucleon $t$-matrix. For the $1^-$ part, the operator $\{\hat{q}' \otimes Y_1^*(\hat{q}')\}_{1M} = 0$, and the amplitude in Eq. (4) becomes proportional to $t^t(q) = t_{\sigma\tau}^c(q) + t_{\tau}^t(q)$, which is the spin-transverse component of the nucleon-nucleon $t$-matrix. For the $2^-$ part, both components contribute, and the amplitude in Eq. (4) has a more complicated form.
The functions $\chi^(-)$ and $\chi^($+ in the amplitude (4) are optical-model wave functions describing the motion of the initial proton and final neutron in the optical potential of the target nucleus. To disentangle the nuclear transition form-factor from the observed cross section, one needs to unravel the intrinsic nuclear dynamics masked by the distorted waves $\chi^{(\pm)}$.

III. DISTORTION FACTOR

The effective operators (6) and (7) in the reaction amplitude (4) are evaluated at the value $q'$ of the local momentum transfer that corresponds to the charge exchange event. However, because of the distortion by the optical potential, $q'$ does not coincide with the asymptotic momentum transfer $q = k_i - k_f$. In the absence of distortion (the plane wave approximation) we would have

$$\left[\chi_f^(-)(k_f, r)\chi_i^(+)(k_i, r)\right]_{PW} = \exp(iq \cdot r)$$

so that the integration over $r$ could be performed explicitly resulting in $\delta(q - q')$. For distorted waves this is no longer true.

In the EA, the product of two optical-model wave functions in Eq. (9) can be estimated by

$$\chi_f^(-)(k_f, r)\chi_i^(+)(k_i, r) = \exp(iq \cdot r)D(r_\perp),$$

where the distortion factor $D(r_\perp)$ is defined by

$$D(r_\perp) = \exp\left[-\frac{i}{\hbar}\int_{-\infty}^{\infty} dz U_{opt}(z, r_\perp)\right].$$

In the spirit of the eikonal approximation, the longitudinal momentum is still preserved, while the distortion is effective in the plane perpendicular to the trajectory. In Eq. (11), the optical potential is different in the initial and final channels. In first order we account for this difference by assuming a fast single-step process which leads to
\[
\frac{U_{opt}}{v} = \frac{1}{2} \left( \frac{U_{opt}^i}{v_i} + \frac{U_{opt}^f}{v_f} \right).
\]

For a square well potential of depth \(U_0\) with a sharp boundary at \(r = R\) the distortion factor can be calculated analytically:

\[
D(r) = \exp \left( - \frac{2U_0}{\hbar v} \sqrt{R^2 - r^2} \right),
\]

and \(D(r) = 1\) for \(r > R\). This approximation might be insufficient in the region of minimum of the cross section where the details of the potential shape are essential. However, near the maximum, which is our region of interest, the cross section is insensitive to the diffuseness of the optical-model potential. We have checked this point by varying the diffuseness in a DWIA calculation.

The exchange part of the reaction amplitude can be estimated with the aid of the standard “fixed \(Q\)” approximation \([23,24]\). In the laboratory frame the exchange momentum \(Q\) coincides in this approximation \([23]\) with the initial momentum \(k_i\). For the exchange amplitude we obtain

\[
T_{fi}^{ex} = \sqrt{2} \left[ \left( \tilde{t}_{\sigma\tau}^c(k_i) + \tilde{t}_{\tau}^t(k_i) \right) \sigma \cdot \tilde{k} \right] \cdot \langle f | O^{ex}(q) | i \rangle,
\]

where the central \(\tilde{t}_{\sigma\tau}^c\), and tensor, \(\tilde{t}_{\tau}^t\), interactions are defined by Franey and Love \([22]\). In Eq. (14) the effective exchange operator

\[
O^{ex}(q) = \sum_j \exp (iq \cdot r_j) D(r_{\perp j}) \sigma_j \tau_j^{-}
\]

includes the distortion factors \(D(r)\) specific for each nucleon inside the nuclear matrix element.

**IV. SENSITIVITY FUNCTION**

In Eq. (4) the integration over \(r\) is not well defined at large distances. It is convenient to single out a no-distortion contribution proportional to \(\delta(q - q')\) by using the decomposition \(D(r) \to 1 + [D(r) - 1]\). The first term describes the plane wave contribution and the second
the effects of distortion. Since $|D(r)| \leq 1$, the distortion term reduces the plane wave contribution. A convenient form of Eq. (4) can be obtained for transitions to $0^-$ and $1^-$ states by writing it as

$$T_{fi}^{\text{dir}} = \frac{\{\sigma \otimes T^{(J)}\}_{JM}}{\sqrt{2J+1}}.$$  \hspace{1cm} (16)

where

$$T^{(J)}_m = T^{(J)}_m(PW) + \int_0^\infty dq' S^J_m(q, q') F_J(q')$$ \hspace{1cm} (17)

is the amplitude describing the excitation of the SDR with the longitudinal, $m = 0$, or transverse, $m = \pm 1$, relative proton-neutron spatial oscillations. In Eq. (17), $T^{(J)}_m(PW)$ is the plane wave contribution, and we have introduced the sensitivity function [18]

$$S^J_m(q, q') = \frac{2\sqrt{2}}{\pi q'^2} \int d^3r \exp (i\mathbf{q} \cdot \mathbf{r})(D(r) - 1)Y_{lm}(\hat{\mathbf{r}})j_1(q'r)$$

$$\times \begin{cases} t^l(q') \text{ for } J = 0, \\ t^r(q') \text{ for } J = 1. \end{cases} \hspace{1cm} (18)$$

The sensitivity function in Eq. (18) characterizes the range of $q'$ which contribute importantly to the charge exchange cross section for a given asymptotic momentum transfer $q$.

**V. EXAMPLE: $^{12}$C($P,N)^{12}$N REACTION**

As an example of application of the method we performed numerical calculations for the $^{12}$C($p,n)^{12}$N reaction to compare with experimental data [25] for the excitation of spin-dipole states at a proton energy of 135 MeV.

For $^{12}$C, with the optical-model potential of Ref. [26], $D(r)$ varies smoothly inside the nucleus. Near the surface it changes rapidly from its value at the center $D(0) \approx 0.5$, to the value of 1. It is then a good approximation to write the exchange matrix element in Eq. (14) as

$$\langle f | O^{ex}(q) | i \rangle \approx D(r_0) \langle f | \sum_j \exp (i\mathbf{q} \cdot \mathbf{r}_j) \sigma_j \tau_j^- | i \rangle.$$  \hspace{1cm} (19)
The result is not very sensitive to a particular choice of the reference point $r_0$; we used $r_0 = 0$. This approximation underestimates the exchange part. Near the maximum of the cross section its contribution is not significant. It becomes important at large angles where the difference of distortion along different trajectories is noticeable; however, the cross section at large angles is small.

In our calculations, the wave functions and transition densities for the spin-dipole states were obtained using a harmonic oscillator basis including the orbitals of $p$, $sd$ and $pf$ shells that form the $3\hbar\omega$ model space necessary for the description of the $L = 1$ excitations. The calculations were performed with the WBN residual interaction and a harmonic oscillator parameter of $1.64(A/A - 1)^{1/2}$ fm [27]. The cross sections for the $^{12}$C$(p,n)^{12}$N reaction leading to the $1^-$ state at $E_x = 1.8$ MeV and the $2^-$ state at $E_x = 4.3$ MeV were calculated as the sum of the direct and exchange amplitudes, Eqs. (7) and (14). The results are shown in Fig. 1 together with data from Ref. [25]. For comparison, a calculation with the DW81 code is also presented. The calculations give similar cross section shapes near the maximum and significantly overestimate the magnitude of the cross section. The results are very similar for other excited states. They are also similar to the distorted wave results obtained for the same transition in Ref. [25] using a $1\hbar\omega$ model space and the MK interaction.

The systematics of the cross sections at their maximum divided by the calculated $\beta$ decay strengths are shown in Fig. 2 for $0^-$, $1^-$ and $2^-$ states. As seen from Fig. 2, there is an approximate proportionality between the cross section at the maximum and the spin-dipole strength, accurate to within 10-15%, for states with strength $B_J > 0.1$ fm$^2$. This is the same level of proportionality as for GT ($L = 0$) excitations at very forward angles. One may ask whether the validity of this conclusion is affected by the poor agreement in the magnitude of the cross section for the 1.8 MeV $1^-$ state. We would argue that this is not the case: since the wave functions are sufficiently complex, they provide a reasonable sample of possible behavior with respect to the operators involved. Furthermore, one might expect proportionality to fail for such weak transitions.
VI. DISCUSSION

It is not clear a priori that the high degree of proportionality shown in Fig. 2 should occur. The cross section involves an integral of the transition form factor over a range of \( q' \) while the value of \( B_J \) is determined by evaluating the form factor at \( q' \approx 0 \). To examine what leads to the observed proportionality, we return to Eq. (17). Two factors determine the \((p,n)\) cross section: the transition form-factor \( F_J(q) \) and the sensitivity function \( S^I_m(q,q') \).

In Fig. 3 we show the transition form-factors for different \( 1^- \) states normalized to the same maximum value in order to compare their shapes. The shapes are very similar near the maximum but differ at higher momentum transfer \( q \). If the region of high \( q' \) does not contribute significantly in the integration over \( q' \) in Eq. (17), the integrals for different form-factors will be proportional.

Samples of the imaginary parts of the sensitivity functions are shown in Figs. 4 and 5; the real parts have very similar shapes and are typically a factor of two smaller in magnitude. A general remark should be made about the \( q' \)-dependence at small \( q' \). Since \( D(r_\perp) \) does not depend on the longitudinal coordinate \( z \), the \( z \)-component of the local momentum transfer \( q' \) must coincide with the \( z \)-component of the asymptotic momentum transfer \( q \). When the absolute value of \( q' \) is smaller than \( q_z \), this condition cannot be fulfilled at any angles of \( q' \), and the sensitivity function must be equal to zero.

As noted above, the sensitivity function for \( J = 0 \) is proportional to the spin-longitudinal component of the effective interaction, and that for \( J = 1 \) to the spin-transverse one. We, therefore, expect a different \( q' \)-dependence reflecting the different behavior of \( t^I(q') \) and \( t^{tr}(q') \). For \( 0^- \) states the projection \( m = 0 \) dominates, corresponding to the spin-longitudinal behavior of the reaction amplitude for \( J = 0 \); the sensitivity function for \( m = 1 \) is smaller by an order of magnitude. At the small scattering angle corresponding to the peak of the cross section, \( \theta = 4.3^\circ \), the momentum transfer \( q \) is almost parallel to the initial proton momentum \( k_i \), thus enhancing the \( m = 0 \) component. For \( J = 1 \) the picture is different, as is seen in Fig. 5. Projections \( m = 0 \) and \( m = \pm 1 \) give comparable contributions.
Given the nature of the sensitivity functions it is clear why the cross sections and $B_J$ are closely proportional. For both $J = 0$ and $J = 1$ the main contribution comes from the peak region where the transition form factors have the same shape, leading to the observed proportionality.

In summary, our results imply that there will be an approximate proportionality of the observed cross section at the maximum of the charge exchange reaction exciting spin-dipole modes and the leptonic strength. This supports the possibility of using such reactions for extracting leptonic strengths of astrophysical interest. Having established here the basic apparatus to examine this issue, it will next be important to examine transition densities for heavier nuclides, so as to determine whether their shapes are similar enough that cross sections and $B_J$ strengths will be proportional. It will also be important to examine the nature of the sensitivity functions for heavier nuclei, to ascertain whether they remain concentrated in a relatively small range of $q'$ where the transition densities are similar.

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FIG. 1. Cross sections for the reaction $^{12}\text{C}(p,n)^{12}\text{N}$ leading to the $1^{-}$ state at $E_x = 1.8$ MeV and the $2^{-}$ state at $E_x = 4.3$ MeV. The cross sections shown as solid lines are the results of the eikonal approximation calculations described here; a DWIA calculation done with the same parameters is shown by solid dots. The data shown as solid squares are from Ref. [25], as are the DWIA calculations shown as open circles. All the theoretical calculations have been multiplied by the factor shown in the Figure.

FIG. 2. Ratios of the cross sections for excitation of the spin-dipole states, taken at their maximums, to the corresponding spin-dipole strengths $B_J$ for states with different $B_J$. The upper panel is for $0^{-}$ states; the middle panel for $1^{-}$ states; and the lower panel for $2^{-}$ states.

FIG. 3. Transition form-factors for the $1^{-}$ states normalized to their maximum values. The state with the anomalous shape corresponds to the high point near $B_1 = 0.45\text{fm}^2$ in Fig. 2, middle panel.

FIG. 4. The imaginary part of the sensitivity functions $S_m(q,q')$ for $J = 0$. The small size of $S_1$ reflects the spin-longitudinal origin of the reaction amplitude.

FIG. 5. The imaginary part of the sensitivity functions $S_m(q,q')$ for $J = 1$. $S_1$ and $S_0$ are comparable for this spin-transverse dominated reaction amplitude.
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