Controlling quantum coherence is one of the most fundamental issues in modern information processing [1]. The most popular solution in the field of quantum information are quantum error correction codes [2] and error avoiding codes [3], both based on encoding the state into carefully selected subspaces of a larger Hilbert space involving ancillary systems. The main limitation of these strategies for combatting decoherence is the large amount of extra space resources required [4]; in particular, if fault tolerant error correction is also considered, the number of ancillary qubits enormously increases.

For this reason, other alternative approaches which do not require any ancillary resources have been pursued, and which may be divided into two main categories, according to the form of interaction with the system under study [5]. If the interaction is one way, so that the controller acts on the system without obtaining any information about its state, then the controller is called “open loop” [6]. By contrast, if the controller acts on the system on the basis of information that it obtains about the state of the system, then it is called “closed loop” [5,7]. In standard open loop techniques, control of quantum dynamics is achieved through the application of suitably tailored, time-dependent and deterministic, driving forces. Here we want to extend open loop control of suitably tailored, time-dependent and deterministic, quantum dynamics is achieved through the application of ancillary systems. The main limitation of these techniques is that quantum coherence can be significantly suppressed if an appropriately tailored stochastic modulation of a system parameter is used. This fact is illustrated in this letter by considering the dynamics of a single radiation mode in a lossy cavity. In this open system, decoherence has a dissipative origin since it is due to photons’ leakage out of the cavity, and the stochastic control strategy will be implemented by modulating the cavity length. This modulation is responsible for non-dissipative phase-decoherence effects and we shall see that the interplay of these two kinds of decoherence may produce competitive effects yielding an effective decoherence suppression (see also [9]).

Let us consider a single radiation mode with annihilation operator $a$ within a lossy cavity, whose characteristic frequency is $\omega = n\pi c/L$, with $n$ an integer number, $c$ the speed of light and $L$ the cavity length. If photons’ leakage occurs through a partially transmitting mirror, the decay rate will be given by $\gamma = cT/2L$, with $T$ the mirror’s transmittivity.

In the case of optical frequencies, thermal excitation from the environment of the continuum of modes outside the cavity is negligible and the dynamics is well described by the master equation [10]

$$\dot{\rho} \equiv \mathcal{L}\rho = -i[\omega a^\dagger a, \rho] + \gamma \mathcal{D}[a]\rho,$$

(1)

where $\mathcal{D}[A]B \equiv ABA^\dagger - \{A^\dagger A, B\}/2$ is the Lindblad superoperator [11] describing photon decay into the vacuum. This decay is also responsible for the rapid decay of any eventual quantum coherence generated within the cavity [12].

Let us now try to preserve the quantum coherence of the radiation mode using an appropriate stochastic control strategy. In particular, we randomly modulate the cavity length, that is, $L \rightarrow L(t) = L_0[1 - \xi(t)]$. This is equivalent to a simultaneous random modulation of both the frequency and the decay rate of the cavity, that is, $\omega \approx (n\pi c/L_0)[1 + \xi] = \omega_0[1 + \xi]$ and $\gamma \approx (cT/2L_0)[1 + \xi] = \gamma_0[1 + \xi]$, in case of small noise. This random modulation of the cavity length moreover yields a dynamics which is indistinguishable from that driven by the constant, unmodulated, Liouvillian superoperator $\mathcal{L}_0 = -i\omega_0 [a^\dagger a, \ldots] + \gamma_0 D[a, \ldots]$, where the parameters $\omega_0$ and $\gamma_0$ are fixed, in the presence of a random evolution time $t'$. In fact, with the stochastic modulation of the cavity length, one has

$$\rho(t) = T \exp \left\{ \int_0^t ds L(s) \right\} \rho(0) = \exp \{ \mathcal{L}_0 t'(t) \} \rho(0),$$

(2)

where $T$ denotes time ordering, and we have defined the stochastic evolution time $t'(t) = t + \int_0^t ds \xi(s) \equiv t + W(t)$. This observation allows us to establish a connection between the present problem and the recently proposed model-independent approach to decoherence in quantum mechanics [13] in which the evolution time is regarded as a random variable.
The connection between the randomized time evolution of Ref. [13] and the model of a cavity mode with a stochastically modulated cavity length is established when we assume that the statistical properties of the cavity length modulation \( \Delta L(t)/L_0 = \xi(t) \) are determined just by the probability distribution \( P(t, t') \) of Ref. [13]. Using the equivalence between cavity length modulation and random evolution time, it means imposing that the time integrated, zero-mean, stochastic variable \( W(t) = \int_0^t ds \xi(s) \) defined above, is described by the probability distribution \([14]\)

\[
P(t, W) = \theta(W + t) e^{-\frac{(W + t)}{\tau}} \frac{\Gamma(t/\tau)}{\Gamma(t/\tau)} \frac{\Gamma((W + t)/\tau + 1)}{\Gamma((W + t)/\tau + 1)},
\]

(\( \theta(x) \) is the Heavyside step function), which is nothing but the distribution of Refs. [13] shifted by \( t \). The \( P(t, W) \) is a Gamma probability distribution \([15]\), depending on the parameter \( \tau \) which quantifies the strength of the fluctuations, i.e., \( (W(t)^2) = \tau \). Choosing this probability distribution for the stochastic modulation variable \( W(t) \) means choosing a specific, uncommon way of modulating the cavity length. In fact it is possible to see that the stochastic modulation assumes Gaussian properties \( (P(t, W) \simeq \exp[-W^2/(2\tau)]/\sqrt{2\pi t\tau}) \) only in the case \( t/\tau \gg 1 \), while it is strongly non-Gaussian in the opposite regime \( t \leq \tau \). The unusual properties of the stochastic modulation chosen can be better grasped if we consider the correlation functions of the stochastic process \( \xi(t) \), which can be derived from the explicit expression of the moments \( \langle W(t)^n \rangle \) and from the definition

\[
W(t) = \int_0^t ds \xi(s),
\]

\[
\langle \xi(t_1) \ldots \xi(t_n) \rangle = \sum_{j=0}^{n} \binom{n}{j} \sum_{r=0}^{j} C_{j,r} (-1)^{n-j} \tau^{j-r} \prod_{l=0}^{r} \delta(t_l - t_{l+1})
\]

(see \([14]\)). This shows that the cavity length modulation \( \xi(t) \) is a white non-Gaussian noise. In fact it is easy to check that, for instance, \( \langle \xi(t_1) \xi(t_2) \rangle = \tau \delta(t_1 - t_2) \), \( \langle \xi(t_1) \xi(t_2) \xi(t_3) \rangle = 2\tau^2 \delta(t_1 - t_2) \delta(t_2 - t_3) \), and so on.

The time evolution of the dissipative radiation mode in the presence of the stochastic modulation of the cavity length can be simply accounted for by first evaluating the physical quantity of interest in the absence of modulation and then averaging it over the distribution \( (3) \). A first interesting quantity is the time evolution of the cavity field which is essentially expressed by the average \( \langle a(t) \rangle \), where the bar means averaging with respect to \( (3) \). In the absence of any stochastic modulation one has

\[
\langle a(t) \rangle = \langle a(0) \rangle \exp(-i\omega_0 t - \gamma_0 t/2),
\]

showing the field decay due to photon leakage through the partially transmitting mirror. Since in this model quantum decoherence is due just to this leakage, we expect that any control exerted on the decay rate will reflect itself into a control of quantum decoherence. In the presence of the cavity length modulation one instead has

\[
\langle a(t) \rangle = \int dW P(t, W) \langle a(t + W) \rangle = \langle a(0) \rangle e^{-i\omega t - \gamma t/2}
\]

with the new effective decay rate \( \gamma \) and the effective oscillation frequency \( \omega \) respectively given by

\[
\gamma = \tau^{-1} \log \left[ (1 + \gamma_0 \tau/2)^2 + \omega_0^2 \tau^2 \right]
\]

\[
\omega = \tau^{-1} \arctan \left[ \omega_0 \tau/(1 + \gamma_0 \tau/2) \right].
\]

The dependence of these two parameters, renormalized by the effect of the stochastic modulation as a function of the modulation strength parameter \( \gamma_0 \), is shown in Fig. 1, where the upper curve refers to the ratio \( \gamma/\gamma_0 \) and the lower curve to the ratio \( \omega/\omega_0 \). The most interesting one is the upper curve, showing an initial increase of the effective cavity decay rate for increasing modulation amplitude \( \tau \). This means that for not too large \( \tau \), the modulation of the cavity length increases the decay rate, i.e., the dissipation. This decay acceleration reaches a maximum at approximately \( \gamma_0 \tau \approx 1 \) and then starts to decrease for increasing \( \tau \). What is rather unexpected is that the ratio \( \gamma/\gamma_0 \) becomes less than one and even tends to zero for larger \( \tau \), that is, when \( \gamma_0 \tau > \omega_0/\gamma_0 = Q \) (cavity quality factor). This means that the cavity field decay can be even completely inhibited by the cavity length modulation, provided that the stochastic modulation has the non-Gaussian statistical properties determined by Eq. (3) with a sufficiently large \( \tau \) parameter. The threshold value \( \tau_{\text{th}} \) for decay inhibition, \( \gamma < \gamma_0 \), depends in a transcendental way on the cavity quality factor (it is \( \gamma_0 \tau_{\text{th}} = 14.57 \) for the parameters of Fig. 1). The behavior of the renormalized frequency \( \omega \) shows instead a monotonic decrease for increasing modulation strength.

The corresponding expressions for the effective oscillation frequency and cavity decay rate in the case of a Gaussian stochastic modulation of the cavity length are simply obtained by extrapolating for all values of \( \tau \) the expansion of Eq. (5) at first order in \( \tau \), that is,

\[
\gamma_{\text{Gauss}} = \gamma_0 + (\omega_0^2 - \gamma_0^2/4) \tau \quad \omega_{\text{Gauss}} = \omega_0 (1 - \gamma_0 \tau/2).
\]

These expressions describe an accelerated decay rate (it is always \( \omega_0 > \gamma_0 \) in optical cavities) and a decreasing oscillation frequency for any modulation strength \( \tau \) as it can be easily extrapolated from Fig. 1.
The study of the behavior of $\langle a(t) \rangle$ has shown how it is possible to inhibit cavity decay and dissipation through an appropriate parameter modulation. Let us now directly address the decoherence control issue. We consider as initial state of the cavity field a linear superposition state. In order to control the coherence in the continuous variable case we shall consider the well known Schrödinger cat state, a superposition of two coherent states of the form $|\alpha\rangle + |-\alpha\rangle$ and see what happens by employing the above stochastically modulated dynamics. The same could eventually be done for a superposition of Fock states.

The time evolution of the the Schrödinger cat state in the absence of any modulation is determined by the usual Liouvillian and it can be described in the following way [12] $\rho(t) = N^2 \{[|\alpha(t)\rangle\langle\alpha(t)| + |\alpha(t)\rangle\langle-\alpha(t)|] + \exp[-2|\alpha|^2(1 - \eta(t))]|\alpha(t)\rangle\langle\alpha(t)| + |\alpha(t)\rangle\langle-\alpha(t)|\langle\alpha(t)|\}$, where we have introduced $\alpha(t) = \alpha \exp[-(i\omega_0 + \gamma_0/2)t]$ and $\eta(t) = e^{-\gamma_0 t}$. A good characterization of the time development of the quantum coherence of the state of the cavity mode is provided by the visibility with respect to an observable [12]. For the quadrature observable $X = (a + a^\dagger)/\sqrt{2}$, the quantum visibility is given by [12]

$$\mathcal{V} = \frac{|e^{-2|\alpha|^2(1 - \eta(t))}\langle X|\alpha(t)\rangle\langle-\alpha(t)|X\rangle|}{\sqrt{\langle X|\alpha(t)\rangle\langle\alpha(t)|X\rangle\langle X|\alpha(t)\rangle\langle-\alpha(t)|X\rangle}}.$$  

(7)

where $\langle X|\alpha\rangle = (\frac{1}{4})^{1/4} \exp\left[-\frac{|\alpha|^2}{2} - \frac{X^2}{2} + \sqrt{2}X\alpha\right]$. Equation (7) leads to the simple result $\mathcal{V} = \exp\{ -2|\alpha|^2(1 - \eta(t)) \}$. This is well known [12], and it shows that (dissipative) decoherence effect depends on the damping rate as well as on the separation of the coherent states, i.e. the macroscopicity.

If we now apply the stochastic modulation of the cavity length in order to achieve a stochastic control of decoherence, the corresponding visibility can be evaluated by performing an appropriate average of the dynamical quantities over the probability distribution $P(t,W)$. In particular, we have to consider the following replacements in Eq. (7)

$$e^{-2|\alpha|^2(1 - \eta(t))}\langle X|\alpha(t)\rangle\langle-\alpha(t)|X\rangle \to e^{-2|\alpha|^2(1 - \eta(t))}\langle X|\alpha(t)\rangle\langle-\alpha(t)|X\rangle,$$

$$\langle X|\alpha(t)\rangle\langle\alpha(t)|X\rangle \to \langle X|\alpha(t)\rangle\langle\alpha(t)|X\rangle,$$

and

$$\langle X|\alpha(t)\rangle\langle-\alpha(t)|X\rangle \to \langle X|\alpha(t)\rangle\langle-\alpha(t)|X\rangle,$$

for different values of the modulation strength parameter $\tau$. The relevant result is that the visibility, i.e., the quantum coherence properties of the system, behaves in the same way as the decay rate. In particular we see either an acceleration, or, more importantly, even a deceleration of decoherence according to the value of the parameter $\tau$. The usual decay of the visibility in the absence of modulation ($\tau = 0$) is shown with a dashed curve. As soon as $\gamma_0\tau$ is nonzero we observe an acceleration of the decay of the visibility (lower curve) when the modulation strength $\tau$ is not too large ($\gamma_0\tau = 1.5$ in the figure) or a slowing down of the decay (upper curves) when $\tau$ becomes sufficiently large ($\gamma_0\tau = 20, 100$ in Fig. 2). The threshold value between the two behaviors coincides with that for decay inhibition $\gamma_0\tau$.

Figs. 1 and 2 show that both cavity dissipation and decoherence can be, rather unexpectedly, inhibited if an appropriate random modulation of the cavity length is applied. This provides the first example of stochastic control of quantum coherence. Therefore, all the dynamics, and not only decoherence or dissipation, is inhibited in the limit of large $\tau$. This is confirmed by the behavior of the renormalized oscillation frequency (see Eq. (5) and Fig. 1) which also tends to zero in the large $\gamma_0\tau$ limit.

In conclusion, we have studied the possibility of a stochastic control of (dissipative) decoherence by tailoring suitable random modulations of a system parameter. Against the widespread opinion that “noise” is detrimental for quantum effects, we have shown that if the statistical properties of the modulation are appropriately chosen, this stochastic control strategy could be used in
principle to control decoherence. Here we have considered the specific model of a single cavity mode with a randomly modulated cavity length. We have seen that, when the modulation is stochastic, with strongly non-Gaussian properties, decoherence and dissipation can be inhibited. Even though the experimental implementation of this unusual random modulation is actually nontrivial, in our opinion this result is important because it shows the first example of stochastic control of decoherence. By modulating the cavity length, one gets the same modulation for the frequency $\omega$ and the decay rate $\gamma$. However, it is possible to see that one has analogous results by only modulating $\gamma$ through the mirror transmittivity [14].

Although we have considered a specific model, our results can be generalized to a generic dissipative system by considering that the usual derivation of the dissipative master equation in the Born-Markov approximation implies $\gamma_0 = G(\omega_0)$, where the function $G$ describes the spectral density of the bath modes [10]. If $G(\omega)$ has a linear dependence on frequency in an interval around $\omega_0$, then the damping rate and the frequency have the same fluctuations. That allows us to recast the the above described treatment.

Finally, our approach shares some similarities with the inhibition of atomic decay through random ac-Stark shift discussed in Ref. [16]. However our proposal is different since it strongly depends on the statistical properties of the random modulation and it is especially suited to the control of quantum decoherence. Another analogy occurs with the use of kicks to prevent the decay of a system [6]. In this latter case, dephasing introduced by kicks were deterministic processes well defined in time. Instead, the present approach is merely probabilistic, so it would be more manageable.

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