Gauge groups from brane-anti-brane systems at angles

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Abstract
We discuss a system formed by two pairs of brane-anti-brane that form an arbitrary angle in a plane. We identify the gauge groups from this system which presumably could be used to construct gauge theories.

1 Introduction
D-branes are hyperplanes in superstring background on which the corresponding open strings can end\(^1\). Some of the solitons of the low energy effective field theory of strings are also interpreted as D-branes. In general, the solitons have a rich geometrical structure\(^2\). Another way of describing the D-branes is as boundary states in the Fock space of closed strings. One feature of D-branes is that their moduli spaces include gauge fields of Chan-Paton groups of open strings as well as pull-back form fields of superstrings background. The effective actions of these world-volume fields is of the Dirac-Born-Infeld type since branes or parallel branes preserve 1/2 of the supersymmetry of the background in \(d=10\)\(^4\). This shows that D-branes are BPS-states of the non-perturbative string theories. From BPS-branes extended in various dimensions one can construct supersymmetric gauge theories in \(d = 6, 4, 3\) and 2 with the gauge groups \(SU(N), U(N)\) or \(SU(N) \times SU(M)\) and also some non-supersymmetric theories\(^5\).

In this paper we report on the gauge field content of the two brane-anti-brane pairs at angle. These are the preliminary results of an undergoing project that studies the stability of two brane-anti-brane pairs\(^6\). The brane-anti-brane states have been recently discussed in\(^7\).

2 Brane-anti-brane pairs at one angle
The system under consideration is formed by two identical Dp brane-anti-brane pairs denoted by \((p - \alpha p)\) and \((p' - \alpha p')\), respectively. The coordinates of the

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tangent space are denoted by $X^i$, where $i = 0, 1, \ldots, p$ while the coordinates of the transversal space are denoted by $Y^a$, where $a = p + 1, \ldots, 9$. The case of parallel pairs is depicted in Fig.(1).

The relevant strings are the ones between different branes. The rest of them represent just excitations of a single brane.

The pairs we consider form a single angle $\theta \pi$ in the plane $(p, p + 1)$ (see Fig.(2)). If the brane $p$ is taken as reference the relative orientations of the other branes are given by

$$p - ap = \pi, \quad p - p' = \theta \pi, \quad p - ap' = \pi(\theta + 1). \quad (1)$$

Actually, we consider a modified version of the system above in which the branes are separated in $p + 2, \ldots, 9$ directions by the distances $L^m$. The boundary conditions for the bosonic coordinates of strings between two branes rotated by $\phi \pi = (\pi, \theta \pi, (\theta + 1) \pi)$ are given by

$$Y^a = 0, \quad a = p + 1, \ldots, p$$
$$\partial_\sigma X^i = 0, \quad i = 0, 1, \ldots, p, \quad (2)$$
in \( \sigma = 0 \), and by

\[
Y^m = L^m, \quad m = p + 2, \ldots, 9
\]
\[
\partial_\sigma X^u = 0, \quad u = 0, 1, \ldots, p - 1
\]
\[
X^p \sin \phi \pi - Y^{p+1} \cos \phi \pi = 0.
\]
\[
\partial_\sigma X^p \cos \phi \pi + \partial_\sigma Y^{p+1} \sin \phi \pi = 0,
\]

(3)

in \( \sigma = \pi \). Here, \( \sigma \) and \( \tau \) are parameters of the open string world-sheet.

The corresponding boundary conditions for the fermionic coordinates can be obtained by imposing the world-sheet supersymmetry \( \delta X = \bar{\psi} \psi \).

3 Massless states and gauge groups

From the equations (2) and (3) and the corresponding fermionic relations the mass spectrum of the theory is found to be

\[
\alpha' M^2 = \frac{i^2}{(4\pi \alpha')^2} + N(\phi)
\]

(4)

where the operator \( N(\phi) \) represents the sum of the following Ramond and Neveu-Schwarz operators

\[
N_R(\phi) = \sum \alpha_{-n} \cdot \alpha_n + \sum \mu d_{-n} d_n + \sum \alpha_{-n+} \alpha_{n+} + \sum \alpha_{-n-} \alpha_{n-}
+ \sum n_{-n}^+ d_{n+}^p + \sum n_{-n-}^+ d_{n-}^p + \frac{1}{2}(a_{\phi} a_{\phi} + a_{-\phi} a_{\phi}) + \frac{1}{2} \phi (d_{\phi} d_{-\phi} + d_{-\phi} d_{\phi})
\]

(5)

\[
N_{NS}(\phi) = \sum \alpha_{-n} \cdot \alpha_n + \sum \nu b_{-n} b_n + \sum \alpha_{-n+} \alpha_{n+} + \sum \alpha_{-n-} \alpha_{n-}
+ \sum r_{-n}^+ b_{n+}^p + \sum r_{-n-}^+ b_{n-}^p + \frac{1}{2}(a_{\phi} a_{-\phi} + a_{-\phi} a_{\phi}) + \frac{1}{2} \phi (b_{\phi} b_{-\phi} + b_{-\phi} b_{\phi})
\]

(6)

Here, we have employed the notations from\(^8\), namely, \( \alpha, d \) and \( b \) indicate the usual string operators in canonical quantization, valid for all directions less \( p \) and \( p + 1 \). In the rotation plane the indices of the operators are \( n \pm \phi \) and \( r \pm \phi \), respectively, which stand for \( n \pm \phi \) and \( r \pm \phi \), respectively. \( n \) is integer and \( r \) is semi-integer. Applying the usual GSO projection we obtain the states we are interested in.
Let us analyse now the gauge fields from the massless NS spectrum of the theory. We can tell between four distinct types of sectors:

$p$-$ap$ and $p'$-$ap'$ sectors
Here $L = 0$, $\phi \pi = \pi$. The GSO projection preserves the odd states, and the tachyon survives the projection\(^7\). For type IIA/B string theory, it is possible to map this sector of one theory into the same sector of the other theory by a further $(-1)^{F_L}$ projection. Through the last operation one recovers the gauge fields in the other theory. The fields obtained through this procedure are in the vector field representation of $SO(1, p)$, $SO(9 - p)$, and $SO(32)$ (the last group is the Chan-Paton group).

$p$-$p'$ sector
Here $L \neq 0$, $\phi \pi = \theta \pi$ with $0 \leq \phi \leq 1$. The massless states are the same as of the open string theory in $0, 1, \ldots, p - 1$ and $p + 2, \ldots, 9$ directions, namely in the vector representation of $SO(1, p - 1)$, $SO(7 - p)$ and $SO(32)$.

$p$-$ap'$ sector
Here $L \neq 0$, $\phi \pi = (\theta + 1)\pi$, $0 \leq \phi \leq 1$. The massless states in $0, 1, \ldots, p - 1$ directions and in $p + 2, \ldots, 9$ directions are as in the above case. However, there is an ambiguity in defining the GSO projection which might change the field content of the theory and which will be discussed later.

$ap$-$p'$
This case is like the above one. The unique difference is that the branes are reversed, but the relative orientation of them is preserved. Thus, the massless spectrum is the same.

$ap$-$ap'$
This is the symmetric case with $p - p'$ one but all of the branes have a reversed orientation.

Some comments are in order now. The above spectrum was obtained with the usual GSO projection defined such that all the tachyons are killed. This definition should verify the modular invariance of string theory\(^8\). However, in\(^7\) it was shown that the tachyon from the brane-anti-brane sector survives, which poses the question whether this also happens in the third and fourth cases above. It is likely that a tachyon survives under the GSO projection in these sectors, making the system unstable. Indeed, if we compute the contribution of the massless modes to the amplitude of exchanging one closed string in the approximation where the GSO projection kills the tachyons from all the sectors\(^8\) we obtain the total potential

$$V(\theta \pi) = 0$$

according to the following formula

$$V(\phi \pi) = -4V_p(4\pi^2 \alpha')^{3-p}G_{8-p}(L^2) \sin(\pi \phi)$$

(7)

(8)

4
which expresses the contribution to the potential from RR and NSNS modes of any sector. Here, $V_p$ is the finite volume of brane and $G_{p-p}$ is the Green function in the directions transversal to the $(p, p+1)$ plane. This result shows that the system is stable for any angle. However, if one modifies the GSO projection such that the tachyons from brane-anti-brane sectors be allowed, this might change the above conclusion. We hope to clarify these aspects in a future paper.

4 Conclusion

The massless vector states in the present system are in vector representations of $SO(1, p-1), SO(1, p), SO(9 - p)$ and $SO(7 - p)$. There are also fields coming from the Chan-Paton factors of $SO(32)$.

To get a full picture of this system one has to study the action of the generalized GSO projection on all the sectors, the amplitude for the exchange of closed string states and the stability of the system under the tachyonic decay in $p - ap, p' - ap', p - ap'$ and $ap - p'$ sectors.

6. I. V. Vancea, C. T. Echevarria, work in progress
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