Gluonic mechanism for $B \rightarrow D \eta'$

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We present a calculation of the process $\bar{B}^0 \rightarrow D^0 \eta'$ within a heavy-light chiral quark model. We assume that the η' has a gluonic component, described via an effective glue-gluon- η' vertex. The main contribution comes from the nonfactorizable part of the effective weak Lagrangian at the quark level. Within our model-dependent assumptions, we find from the gluonic mechanism alone a branching ratio $\text{BR}(\bar{B}^0 \rightarrow D^0 \eta') = (1.9\text{--}3.3) \times 10^{-4}$, somewhat below the experimental upper bound of 9.4×10^{-4} .

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I. INTRODUCTION

After the CLEO [1] results reporting a very large branching ratio for the inclusive production of η' in $B \rightarrow \eta' X_s$ decays, Atwood and Soni [2] proposed an interesting explanation of the data. The suggested mechanism is based on the subprocess $b \rightarrow s g^* \rightarrow s \eta' g$, where the virtual gluon g^* emerging from the standard model penguin diagram couples to η' via an effective $g g^* \eta'$ vertex related to the gluonic triangle anomaly. The structure of this vertex was reexamined in [3], where the running of the effective coupling of η' to gluons, assumed to be constant in [2], is also taken into account. The possibility that the $g g^* \eta'$ vertex could dangerously be affected by noncontrollable nonperturbative effects was soon after discussed in [4]. Some further criticism can be found in [5].

Interestingly, the production of η' by thermal gluon fusion at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC) has recently been discussed in [6] again using the gluon- η' vertex idea. The authors of [7] have also described a gluon fusion process to study the inclusive $B \rightarrow \eta' X_s$ and the exclusive $B \rightarrow K^{(*)} \eta'$ decays: the gluon g of the $g g^* \eta'$ vertex is supposed to be emitted by the light quark inside the B meson, while the g^* comes from the $b \rightarrow s$ penguin diagram. The $B \rightarrow K \eta'$ decay has been further investigated in the context of perturbative QCD in [8]. To assess the reliability of such mechanisms, a study of the gluon content of the η' is in order. The estimate found in [9] results in a gluon component of 26%. This seems again to favor the possibility that the η' can be coupled reliably through gluons in hadronic processes.

A different scenario proposed to explain the abundant production of η' in $B \rightarrow K \eta'$ data was based on the idea of a strong intrinsic charm component in the η' [10]. Although appealing, the $b \rightarrow c \bar{c} s$ Cabibbo-favored process generating

an η' via its intrinsic $c \bar{c}$ requires too large a charm component to explain the data. Recently the charm content of the η' has been thoroughly investigated [11] and it is estimated to be too small to motivate the $b \rightarrow c \bar{c} s$ mechanism.

The nonleptonic decays including an η' in the final state, such as $\bar{B}^0 \rightarrow D^0 \eta'$ and $\bar{B}^0 \rightarrow D^{*0} \eta'$, are also quite interesting on both theoretical and experimental grounds. Measuring the branching ratios of these processes may shed light on the nature of the η - η' mixing and of the decay constants f_η , $f_{\eta'}$. From the experimental side, at present the only known limits on these decays are from CLEO [12].

The simplest possible mechanism to consider for $B \rightarrow D \eta'$ decay is naive factorization of quark currents as in [13]. The quark models considered there do not involve gluons. In contrast, in the present paper we consider the possibility that a gluon mechanism could be at work even in a process like $\bar{B}^0 \rightarrow D^0 \eta'$, where there is no gluon arising from a penguin diagram (which is the case in $B \rightarrow K \eta'$). We will assume that the main contribution to the decay $\bar{B}^0 \rightarrow D^0 \eta'$ comes from the gluonic mechanism, as illustrated in Fig. 1. The η' couples to the gluons as shown in Fig. 2.

In general, nonleptonic decays are described by a quark level effective Lagrangian at some chosen scale. This is built up by coefficients (containing all short-distance effects above the chosen scale) times quark operators, typically given as

FIG. 1. Bosonization of the effective Hamiltonian, $\Gamma \equiv \gamma^\mu (1 - \gamma_5)$.

FIG. 2. $\eta' g^* g$ vertex.

the product of two currents. In order to get an effective Lagrangian describing the physical degrees of freedom, the mesons, a “bosonization procedure” is required. This typically amounts to transforming (see e.g. [14]) the original Lagrangian by extracting collective composite fields with boson quantum numbers.

Historically, bosonizing by factorization of the currents, the so-called vacuum saturation approximation (VSA) has been a starting point for bosonizing the quark operators. The VSA turns out to be badly broken in K -meson decays. However, in some classes of heavy meson (B) decays, factorization holds up to order $1/m_b$ [15], but not for the process we consider.

A priori, it would be preferable to keep the description as model independent as possible. Unfortunately, some model dependence has to be introduced within our framework. Still the amplitude for $\bar{B}^0 \rightarrow D^0 \eta'$ presented here will be less model dependent than the amplitude for the process $B \rightarrow K \eta'$ within the same framework.

One should keep in mind that, when the four quark operators responsible for nonleptonic B decays are Fierz transformed, they will contain products of colored currents, such that a virtual gluon might be emitted from a (light) quark in the B meson due to such a current. Furthermore, this virtual gluon might (via the effective $\eta' g g^*$ vertex) be transformed into an η' and a soft gluon. In this paper, this soft gluon is assumed to make a gluon condensate with another soft gluon emitted from the light quark within the D meson.

Inclusion of such gluon condensates gave, within chiral quark models (χ QM) [16,17], significant effects for $K \rightarrow 2\pi$ amplitudes, and the chiral quark model works well for K decays in general [18–20]. More specifically, it has been shown that the inclusion of gluon condensates within the χ QM works well in order to understand the $\Delta I = 1/2$ rule and the CP -violating quantity ε'/ε [19]. (It should also be noted that the value used for the gluon condensate in [19] is also in fair agreement with the result obtained within generalized factorization [21].) This gives a strong motivation for using models for mesons containing a light and a heavy quark [22–24], and with soft gluon effects in terms of gluon condensates added [25]. This was already used for the process $D \rightarrow K^0 \bar{K}^0$ in [26].

II. FRAMEWORK FOR THE PROCESS $\bar{B}^0 \rightarrow D^0 \eta'$

Nonleptonic decays with $\Delta B = 1, \Delta C = \pm 1$ are described by an effective Lagrangian at quark level [27]:

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [C_1(\mu) Q_1 + C_2(\mu) Q_2], \quad (1)$$

where $Q_{1,2}$ are operators given in terms of the quark fields b, c, u, d :

$$Q_1 = \bar{d} \gamma^\mu (1 - \gamma_5) b \bar{c} \gamma_\mu (1 - \gamma_5) u,$$

$$Q_2 = \bar{c} \gamma^\mu (1 - \gamma_5) b \bar{d} \gamma_\mu (1 - \gamma_5) u. \quad (2)$$

The Wilson coefficients C_1 and C_2 contain all short-distance effects down to the scale $\mu = \mathcal{O}(m_b)$. As our quark model calculation of hadronic matrix elements will be based on the heavy quark effective theory (HQET), we should in principle also calculate the short-distance effects from the scale $\mathcal{O}(m_b)$ down to a scale $\mathcal{O}(\Lambda_\chi \sim 1 \text{ GeV})$ and evaluate our matrix elements there [28] (also some new operators should then appear in the QCD mixing). However, in this work these complications will be neglected (more on this in Sec. V).

The operators defined in Eq. (2) may be Fierz transformed. Using the identity

$$\delta_{\alpha\delta} \delta_{\gamma\beta} = 2t_{\alpha\beta}^a t_{\gamma\delta}^a + \frac{1}{N_c} \delta_{\alpha\beta} \delta_{\gamma\delta} \quad (3)$$

for the color matrices t^a , the operator Q_2 takes the form

$$Q_2 = 2\bar{c} t^a \gamma^\mu (1 - \gamma_5) u \bar{d} t^a \gamma_\mu (1 - \gamma_5) b + \frac{1}{N_c} Q_1. \quad (4)$$

Fierz transforming the operator Q_1 gives a similar expression that describes just the process $B \rightarrow D \pi$.

The first part of Q_2 contains a product of two colored currents. As in Refs. [18,19,29], this contribution will be calculated in terms of the lowest order gluon condensate. In the heavy-light chiral quark model (HL χ QM) considered here, there is also a link between the matrix element of the chromomagnetic operator of the heavy quark Lagrangian and the mass difference of B and B^* [30]. The matrix element of the chromomagnetic operator also depends on gluon condensates and will be truncated by the lowest order gluon condensate (with dimension 4), which fixes the value of this gluon condensate for a given value of the light constituent quark mass.

The idea is now that the decay $\bar{B}^0 \rightarrow D^0 \eta'$ goes in two steps. We calculate the amplitude for $\bar{B}^0 \rightarrow D^0 g^* g$, where g^* is the hard virtual gluon and g is a soft gluon. Then $g^* \rightarrow \eta' g'$, where g' is another soft gluon. The soft gluons g and g' are then assumed to form a gluon condensate. For $\bar{B}^0 \rightarrow D^0 g^* g$ we may use a generalized version [18,19,26] of the VSA, and obtain [see Eqs. (1), (2), and (4)]

$$\begin{aligned}
\langle D^0 g^* g | \mathcal{L}_{eff} | \bar{B}^0 \rangle = & \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} \left\{ \left(C_1 + \frac{C_2}{N_c} \right) [\langle D^0 g^* g | \bar{c} \gamma_\mu (1 - \gamma_5) u | 0 \rangle \langle 0 | \bar{d} \gamma^\mu (1 - \gamma_5) b | \bar{B}^0 \rangle \right. \\
& + \langle D^0 | \bar{c} \gamma_\mu (1 - \gamma_5) u | 0 \rangle \langle g^* g | \bar{d} \gamma^\mu (1 - \gamma_5) b | \bar{B}^0 \rangle] + 2C_2 [\langle g D^0 | \bar{c} t^a \gamma_\mu (1 - \gamma_5) u | 0 \rangle \\
& \times \langle g^* | \bar{d} t^a \gamma^\mu (1 - \gamma_5) b | \bar{B}^0 \rangle + \langle D^0 g^* | \bar{c} t^a \gamma_\mu (1 - \gamma_5) u | 0 \rangle \langle g | \bar{d} t^a \gamma^\mu (1 - \gamma_5) b | \bar{B}^0 \rangle] \Big\}. \quad (5)
\end{aligned}$$

Within our model, the four terms in Eq. (5) give rise to four diagrams. Two of them are shown in Fig. 1, where we have also included the $\eta' g^* g$ vertex. The other two diagrams (which are suppressed and will be dropped in the end) are similar, except for the fact that the hard gluon is emitted from the D^0 meson instead of the $\bar{B}^0$ meson. The first diagram in Fig. 1 corresponds to the factorizable part of the Lagrangian (proportional to $C_1 + C_2/N_c$) and the second diagram to the nonfactorizable part (proportional to $2C_2$). The dominant contribution comes from the nonfactorizable part because of the small ratio of Wilson coefficients ($C_1 + C_2/N_c$)/($2C_2$), which is $\simeq 1/12$ at $\mu = m_b$. It should also be noted that the Wilson coefficient $2C_2$ in front of the dominant nonfactorizable contribution is rather stable with respect to variations in the renormalization scale μ .

Instead of $C_{1,2}$ one often uses the scale and scheme independent phenomenological coefficients $a_{1,2}$, which also should include nonfactorizable effects [31]. However, in this paper we *calculate* nonfactorizable effects in terms of the gluon condensate, and it would therefore be inconsistent to use the a_i 's within our framework.

The form factor $F_{\eta' g^* g}$ has been examined by many groups following [2,3] (see also [32]), and takes the form given in Fig. 2.

In the Fock-Schwinger gauge one replaces $\varepsilon_\nu^{*b} \rightarrow -(i/2)(\partial/\partial q_{1\rho})G_{\rho\nu}^b(0)$ [33], so that the $\eta' g^* g$ vertex now takes the form

$$-\frac{1}{2} F_{\eta' g^* g} \delta^{ab} \varepsilon^{\mu\nu\rho\sigma} \varepsilon_\mu^{*a} G_{\nu\rho}^b(0) q_\sigma, \quad (6)$$

where q is the momentum of the virtual gluon. We will use the parametrization of Hou and Tseng [3] where the form factor is

$$F_{\eta' g^* g} = \sqrt{\frac{3}{2}} \frac{\alpha_s(q^2)}{\pi f_\pi}. \quad (7)$$

This is not the only possible parametrization. In [32] several parametrizations are considered and compared. The parametrization in Eq. (7) should be reasonable for q^2 of order 1–2 GeV², which is relevant in our case.

III. ELEMENTS OF THE HEAVY-LIGHT CHIRAL QUARK MODEL

Some of the matrix elements in Eq. (5) are known (e.g. the ones given by the decay constants f_D and f_B), and some

are model dependent within our approach. Our calculation will, to some extent, be based on what we will call a heavy light chiral quark model [25] (see also [26]). This is a quark loop model supposed to describe interactions between quarks with momenta below a scale of order the chiral symmetry breaking scale $\Lambda_\chi \sim 1$ GeV. By assumption, the model should fill the gap between perturbative QCD and chiral perturbation theory.

The model is similar to the ones that have been used by several groups [22–24], but it contains in addition soft gluons which may form gluon condensates, similar to one version of the chiral quark model in the light sector [17–19]. The key ingredient in this kind of model is a phenomenological term in the Lagrangian, which couples the quark fields directly to the meson fields. This enables one to integrate out the quark degrees of freedom and obtain a Lagrangian containing only meson fields [34]. A drawback with these models is that they do not incorporate any confining mechanism for quarks. However, such models, like the related Nambu–Jona-Lasinio models [22–24,35], describe chiral symmetry breaking reasonably well, and that is important for our purpose. As for the light sector [16,18,19,35], the coefficients of various pieces of chiral Lagrangians can be calculated.

The Lagrangian of the HL χ QM is

$$\mathcal{L} = \mathcal{L}_{HQET} + \mathcal{L}_{\chi QM} + \mathcal{L}_{Int} + \dots, \quad (8)$$

where

$$\mathcal{L}_{HQET} = \bar{Q}_v i v \cdot D Q_v + \mathcal{O}(m_Q^{-1}) \quad (9)$$

is the Lagrangian for heavy quark effective field theory. The heavy quark field Q_v annihilates a heavy quark with velocity v and mass m_Q , and D_μ is a covariant derivative containing the gluon field and the photon field.

In the pure light sector, the Lagrangian of the chiral quark model can be written as

$$\mathcal{L}_{\chi QM} = \bar{\chi} [\gamma^\mu (i D_\mu + \mathcal{V}_\mu + \gamma_5 \mathcal{A}_\mu) - m] \chi, \quad (10)$$

where m is the constituent mass (200–300 MeV) of the light quarks. The field χ transforms under the unbroken $SU(3)_V$ symmetry, so that the quark fields transforming under $SU(3)_L \times SU(3)_R$ are

$$q_L = \xi^\dagger \chi_L \quad \text{and} \quad q_R = \xi \chi_R, \quad (11)$$

where ξ is a 3×3 matrix containing the Nambu-Goldstone bosons of QCD (π, K, η):

$$\xi = e^{-i\Pi/f},$$

where

$$\Pi = \frac{\lambda^a}{2} \phi^a(x)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 \end{bmatrix}. \quad (12)$$

Moreover, $f \approx f_\pi = 93.2$ MeV and $\lambda^a, a=1, \dots, 8$, are the Gell-Mann matrices. The vector and axial vector fields are

$$\mathcal{V}_\mu \equiv \frac{i}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi) \quad \text{and} \quad \mathcal{A}_\mu \equiv \frac{i}{2} (\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi). \quad (13)$$

The meson-quark interaction for the heavy-light system is [22–25]

$$\mathcal{L}_{int} = -G_H [\bar{\chi}_f \overline{H}_{vf} Q_v + \overline{Q}_v H_{vf} \chi_f], \quad (14)$$

where G_H is a coupling constant (in [22–24] it corresponds to a renormalization constant), and H_f is a field containing both the singlet (0^-) and triplet (1^-) meson fields:

$$H_f \equiv P_+ (P_{\mu f} \gamma^\mu - i P_{5f} \gamma_5),$$

$$\overline{H}_f \equiv \gamma^0 (H_f)^\dagger \gamma^0 = [(P_{\mu f})^\dagger \gamma^\mu - i (P_{5f})^\dagger \gamma_5] P_+, \quad (15)$$

where

$$P_+ = (1 + \not{v})/2 \quad (16)$$

is a projection operator. The index f runs over the light quark flavors u, d, s . The field P_5 (P^μ) annihilates a heavy meson with spin parity 0^- (1^-), with velocity v .

The way of regularizing divergent loop integrals has to be taken as a part of the definition of the model. One might use some kind of cutoff regularization (sharp cutoff as in [22] and cutoff in the proper time formalism as in [23,24]), cutting the momenta in the loop between $\Lambda_\chi \sim 1$ GeV and some infrared cutoff μ_{IR} . The infrared cutoff could be zero as in [23] or $\sim m$ as in [22,24]. Still, dimensional regularization may be used if there are no mass scales bigger than Λ_χ entering the loop. Within our model we will relate all divergent loop integrals to some physical parameter (as for instance f_π), as was done in [17–19]. This means in particular that we will treat quadratic, linear and logarithmic divergences as different. The type of divergence can easily be seen by inspection.

FIG. 3. Feynman rule for a quark coupling to a soft gluon.

The calculation of soft gluon effects in terms of gluon condensates is carried out in the Fock–Schwinger gauge. In this gauge one can expand the gluon field as

$$A_\mu^a(k) = -\frac{i(2\pi)^4}{2} G_{\rho\mu}(0) \frac{\partial}{\partial k_\rho} \delta^{(4)}(k) + \dots, \quad (17)$$

where k is the gluon momentum which is put to zero after the derivation is performed. The fact that the soft gluon tensor appears explicitly in this expression makes it a powerful tool to derive gluon condensate contributions to many processes, as demonstrated in [32].

Since each vertex in a Feynman diagram is followed by an integration over loop momentum, we get the Feynman rule for interactions between light quarks and soft gluons shown in Fig. 3. From this Feynman rule it immediately follows that the coupling of a soft gluon to a heavy quark is suppressed by $1/m_Q$, since the vertex to this order is proportional to $v_\mu v_\nu G^{a\mu\nu} = 0$, v_μ being the heavy quark velocity.

The diagrams contributing to the kinetic term of heavy-light mesons are shown in Fig. 4. By including also diagrams with the insertion of the $\mathcal{A}_\mu$ ($\mathcal{V}_\mu$) fields on light quark loop lines, one obtains the low energy effective Lagrangian for heavy mesons coupling to the light mesons $\pi, K, \eta, \dots$ in the usual form [35,36]

$$\mathcal{L}_{Str} = -\text{Tr}[\overline{H}_a (i v \cdot D - \Delta) H_a] + \text{Tr}[\overline{H}_a H_b v^\mu \mathcal{V}_{\mu ba}] - g_A \text{Tr}[\overline{H}_a H_b \gamma^\mu \gamma_5 \mathcal{A}_{\mu ba}], \quad (18)$$

where Δ is the mass difference between the heavy meson(s) and the corresponding heavy quark. The trace is taken only over the gamma matrix indices. Note that the $\mathcal{V}_\mu$ field and the covariant derivative D_μ (containing the photon field) can be combined into a total covariant derivative $\mathcal{D}_{\mu ba} \equiv D_\mu \delta_{ba} + i \mathcal{V}_{ba}$. The heavy-light meson fields, P_{5f} and $P_{\mu f}$, have been rescaled by factors of $\sqrt{M_P}$ and $\sqrt{M_{P^*}}$. The mass dimension of the heavy-light meson fields is now $3/2$. Typical values of the axial coupling g_A found in the literature lie between $1/4$ and $2/3$.

In order to obtain Eq. (18), we have to identify the square of the coupling G_H in Eq. (14), multiplied by some loop integrals with the coefficients $g_V \equiv 1$ for the kinetic term (including the vector term) and g_A for the axial vector term. Such loop integrals contain logarithmic and linear diver-

FIG. 4. Self-energy for a heavy-light meson.

gences plus finite terms (gluon condensate terms are finite in this case). Details will be given in [25].

The logarithmic divergent integral appearing here is the same as the one appearing in the pure light sector [17–19], and is related to f_π . The quantity $\delta g_A \equiv (1 - g_A)$ is related to a linear divergent integral plus finite terms (including gluon condensates). Eliminating the linearly divergent integral (appearing as finite in dimensional regularization) and identifying the logarithmic divergent integral with f_π as in the pure light case, we obtain a relation for G_H in terms of f_π , the constituent light quark mass m , δg_A , and the gluon condensate [25]:

$$G_H^2 = \frac{2m \left(1 - \frac{3}{4} \delta g_A \right)}{f_\pi^2 + N_c m^2 / 8\pi - (\kappa / 2m^2) \langle (\alpha_s / \pi) G^2 \rangle},$$

where

$$\kappa \equiv \frac{96 - 3\pi}{768}. \quad (19)$$

Thus, in the formal limit where only the logarithmic divergent integral is kept, one obtains the simple expression [26]

$$G_H \simeq \frac{\sqrt{2m}}{f_\pi}. \quad (20)$$

The gluon condensate and the constituent quark mass can be linked to the mass difference between the 0^- and 1^- states via the matrix element of the color magnetic operator between heavy states [30]. Calculating this matrix element in our model [25] gives a gluon condensate range $(314 \text{ MeV})^4 < \langle (\alpha_s / \pi) G^2 \rangle < (344 \text{ MeV})^4$ for a constituent light quark mass between 230 and 270 MeV. Then we find that numerically also the simple expression (20) is close to Eq. (19) because the second term ($\sim N_c m^2$) and third term $[\sim \langle (\alpha_s / \pi) G^2 \rangle]$ in the denominator of Eq. (19) tend to cancel.

When calculating the diagram for $\bar{B}^0 \rightarrow g^*$, the virtual gluon is not soft. (The light quark emerging from the weak vertex carries a momentum of order 2 GeV.) In order to avoid mass scales bigger than Λ_χ in the loop integral, as explained earlier, one has to split off this high momentum in terms of a large energy effective theory (LEET) [38–40] for the quark propagating between the weak vertex and the η' vertex (indicated by two arrows in Fig. 1). This is analogous to splitting off the heavy quark mass within the HQET. The momentum of the quark is written as $p = E_{\eta'} n + k$, where $E_{\eta'}$ is the energy of the η' and $n^\mu = (1; 0, 0, 1)$ is parallel to the four momenta of the η' . Furthermore, k is the residual momentum which has to be less than $\Lambda_\chi \sim 1 \text{ GeV}$.

The LEET tells us that the effect of a soft gluon that couples to a fast light quark is suppressed by $1/E_{\eta'}$, since the leading order coupling is proportional to $n_\mu n_\nu G^{a\mu\nu} = 0$. Neglecting $1/m_Q$ and $1/E_{\eta'}$ corrections, soft gluons can couple only to the spectator quark in the B and in the D meson.

Still, for the $B \rightarrow g^*$ transition involved in the dominant contribution, we will not rely on the details of the LEET, but relate the $B \rightarrow g^*$ transition to $B \rightarrow \rho$ which is known from the literature.

IV. THE AMPLITUDE FOR $\bar{B}^0 \rightarrow D^0 \eta'$

The first part of the diagram to the right in Fig. 1 describes the matrix element for the transition $\bar{B}^0 \rightarrow g^*$, where the gluon is off shell and carries a momentum q , with $q^2 = m_{\eta'}^2$. Within our model, light vector mesons $V = (\rho, \omega, \phi)$ might couple to light quarks like the field $\mathcal{V}_\mu$ in Eq. (10) by letting $\mathcal{V}_\mu \rightarrow (\mathcal{V}_\mu - \rho_\mu)$ [37]. Thus, since the gluon is a vector particle, the transition $\bar{B}^0 \rightarrow g^*$ is, within our model, related to the transition $\bar{B}^0 \rightarrow V = (\rho, \omega, \phi)$. In other words, writing down the loop integral for $\bar{B}^0 \rightarrow g^*$ and for $\bar{B}^0 \rightarrow V$, the integrals are the same, only the couplings are different. Therefore we can avoid going into details about the loop integral because the relevant form factor for $\bar{B}^0 \rightarrow \rho$ is known from lattice and light cone sum rules [41]. Using this fact will reduce the model dependence of our calculation. Considering a vector meson dominance induced coupling of the ρ to the light quarks, $\gamma^\mu m_\rho^2 / f_\rho$ (see [42] for more details), we have in the LEET approximation

$$\begin{aligned} J^{\mu a}(\bar{B}^0 \rightarrow g^{*d}) &\equiv \langle g^{*d} | \bar{d} t^a \gamma^\mu (1 - \gamma_5) b | \bar{B}^0 \rangle \\ &\simeq \frac{\delta^{ad}}{2} \frac{g_s f_\rho}{m_\rho^2} \langle \rho | \bar{d} \gamma^\mu (1 - \gamma_5) b | \bar{B}^0 \rangle, \end{aligned} \quad (21)$$

where m_ρ is the ρ mass and $f_\rho = 0.152 \text{ GeV}^2$. Furthermore, a and d are color indices from the quark-gluon vertex and the factor $\delta^{ad}/2$ comes from the trace over the $SU(3)_c$ matrices. This current can now be parametrized by the form factors from the LEET result for $\bar{B}^0 \rightarrow \rho$ [38]:

$$\begin{aligned} J^{\mu a}(\bar{B}^0 \rightarrow g^{*d}) &= \frac{\delta^{ad}}{2} \frac{g_s f_\rho}{m_\rho^2} 2E_{\eta'} \zeta_\perp [i \varepsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \varepsilon_\sigma^* \\ &\quad - (n \cdot v) \varepsilon^{*\mu} + (\varepsilon^* \cdot v) n^\mu]. \end{aligned} \quad (22)$$

A relation corresponding to Eq. (21) cannot be used for the process $B \rightarrow K \eta'$. The other part of the diagram, with a light quark moving in the background field of soft gluons, contains a logarithmic divergence, which can be related to f_π ; the result of the calculation is [25, 26]

$$\begin{aligned} \langle g D^0 | \bar{c} t^a \gamma_\mu (1 - \gamma_5) u | 0 \rangle &= - \frac{G_H g_s \sqrt{M_D}}{8m^2 N_c} G_{\gamma\delta}^a \\ &\quad \times \left[\frac{m^2 N_c}{4\pi} (v'^\delta g^{\mu\gamma} - v'^\gamma g^{\mu\delta}) \right. \\ &\quad \left. - i \varepsilon^{\mu\alpha\gamma\delta} v'_\alpha \left(f_\pi^2 + \frac{m^2 N_c}{4\pi} \right) \right], \end{aligned} \quad (23)$$

where v' is the velocity of the D meson. Combining Eqs. (6), (22), and (23), we find for the nonfactorizable contribution to $\bar{B}^0 \rightarrow D^0 \eta'$

$$A^{NF} \equiv C_2 \pi G_H \sqrt{M_D} \frac{E_{\eta'}^2 F_{\eta' g^* g} \zeta_{\perp} f_{\rho}}{6 m_{\eta'}^2 m_{\rho}^2} (n \cdot v)(n \cdot v') \times \left(\frac{2 \pi f_{\pi}^2}{m^2 N_c} + 1 \right) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle. \quad (24)$$

In the calculation we have used $q^2 = m_{\eta'}^2$, in the gluon propagator and the $\eta' g^* g$ vertex. In the other diagram not shown in Fig. 1, where the virtual gluon g^* (and thereby the η') is emerging from the loop with the charm quark, we have an effective annihilation “Vac” $\rightarrow g^* D$ vertex, where there is a large momentum squared M_B^2 floating through the weak, colored vertex. This amplitude [corresponding to the last term of Eq. (5)] should therefore be form factor suppressed, and will be neglected.

The factorizable contributions are described by the diagram to the left in Fig. 1. The $B \rightarrow D g g^*$ part of the diagram (connected to the η' vertex) is finite. The second part is the current $\langle D^0 | \bar{c} \gamma_{\mu} (1 - \gamma_5) u | 0 \rangle$, which is by definition given by the decay constant f_D . The finite diagrams have to be calculated using LEET, and the result of the calculation is (again, the corresponding diagram not shown in Fig. 1 is form factor suppressed and neglected)

$$A^F \equiv - \left(C_1 + \frac{C_2}{N_c} \right) \pi G_H \sqrt{M_B M_D} \frac{E_{\eta'} F_{\eta' g^* g}}{288 m m_{\eta'}^2} \times f_D \sqrt{M_D} n \cdot v' \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle. \quad (25)$$

As we will discuss in the next section, this amplitude is uncertain because the Wilson coefficient $(C_1 + C_2/N_c)$ is very sensitive due to the choice of renormalization scale and scheme. It is, however, relatively small compared to the nonfactorizable amplitude in Eq. (24).

V. NUMERICAL RESULTS

The total amplitude is

$$\langle D^0 \eta' | \mathcal{L}_{eff} | \bar{B}^0 \rangle = - \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} (A_{q\bar{q}} + A^F + A^{NF}), \quad (26)$$

where A^F and A^{NF} are given in Eqs. (25) and (24) and $A_{q\bar{q}}$ is the $B \rightarrow D \eta'$ amplitude considered in Ref. [13], which is based on the quark-antiquark picture $B \rightarrow D \eta_8$ amplitude. In the LEET limit the kinematics is

$$n \cdot v = 1, \quad n \cdot v' = M_B / M_D, \quad E_{\eta'} = (M_B^2 - M_D^2) / (2 M_B). \quad (27)$$

We will use $M_B = 5.280$ GeV, $M_D = 1.870$ GeV, $m_{\eta'} = 0.958$ GeV, $G_F = 1.166 \times 10^{-5}$ GeV $^{-2}$, $V_{cb}^* V_{ud} = 0.037$, and $f_{\pi} = 93.2$ MeV, which can be found in the Particle Data Group (PDG) books [43]. The value of α_s has to be taken at the point $q^2 = m_{\eta'}^2$, where $\alpha_s(m_{\eta'}^2) \approx 0.56$ [44]. The Wilson coefficients, C_1 and C_2 , have been calculated [27] in next leading order (NLO) to be $C_1(m_b) = -0.184$ and $C_2(m_b) = 1.078$ [where the native dimensional reduction (NDR) scheme and $\Lambda_{\overline{MS}}^{(5)} = 225$ MeV have been used].

Extrapolating $C_{1,2}$ naively down to $\mu = \Lambda_{\chi}$, we find that C_2 does not change much, $C_2(\Lambda_{\chi}) \approx 1.17$, while C_1 is more sensitive, $C_1(\Lambda_{\chi}) \approx -0.37$. This gives $(C_1 + C_2/N_c)/(2C_2) \approx 10^{-2}$ at Λ_{χ} . However, this number should not be taken too seriously. Using HQEFT between the scales m_b and Λ_{χ} , there are also more operators entering as in [28]. In general, taking μ as low as Λ_{χ} , which is intrinsic in our model, the precision of perturbative QCD is of course poorer. However, the important thing for our calculation is that the coefficient C_2 of the dominating operator Q_2 is rather stable with respect to variations of the scale.

Our estimate for the branching ratio will be based exclusively on A^{NF} corresponding to the diagram in Fig. 1, right. This does not necessarily mean that $A_{q\bar{q}}$ and A^F can be neglected. For instance, using the naive numbers for $(C_1 + C_2/N_c)$ at Λ_{χ} , the amplitude A^F will lower the branching ratio by an amount of order 10%. The amplitude $A_{q\bar{q}}$ is hard to estimate within our framework. It might be relatively small due to the small mixing of η_8 into η' . Still it might interfere significantly with A^{NF} and change our estimate.

We extract the numerical value for the LEET form factor $\zeta_{\perp}$ from lattice and light-cone sum-rule results [41], from which we obtain the value $\zeta_{\perp} \approx 0.32$. The only model-dependent parameters entering Eq. (26) are the gluon condensate and the constituent light quark mass m . As mentioned above, the range $230 < m < 270$ MeV for the constituent quark mass gives the range $(314 \text{ MeV})^4 < \langle (\alpha_s/\pi) G^2 \rangle < (344 \text{ MeV})^4$ for the gluon condensate.

Inserting all these values in Eq. (26), and including the uncertainty in C_2 , we obtain the decay rate

$$\Gamma(\bar{B}^0 \rightarrow D^0 \eta') = \frac{M_B^2 - M_D^2}{16 \pi M_B^3} |\langle D^0 \eta' | \mathcal{L}_{eff} | \bar{B}^0 \rangle|^2 = (0.8 - 1.4) \times 10^{-16} \text{ GeV}, \quad (28)$$

which gives a branching ratio

$$\text{BR}(\bar{B}^0 \rightarrow D^0 \eta') = (1.9 - 3.3) \times 10^{-4}. \quad (29)$$

The ranges in Eq. (28) and (29) come from the variation of the constituent light quark mass m in the range 230–270 MeV, and the uncertainty in C_2 at $\mu = \Lambda_{\chi}$, i.e. the uncertainty within the model. In addition, there is the uncertainty from applying this model.

IV. CONCLUSIONS

In this paper we have computed the $\bar{B}^0 \rightarrow D^0 \eta'$ exclusive process coupling the η' via its gluonic component and exploiting the idea of a glue-glue- η' effective vertex, connected to the gluonic triangle anomaly. The calculation is performed using a heavy-light chiral quark model containing soft gluons which may form gluon condensates. Such a model is an extension of constituent quark models of the type obtained by bosonizing an underlying Nambu-Jona-Lasinio interaction involving heavy and light quarks.

The results found, when compared with those in [13], where a branching ratio $\text{BR}(\bar{B}^0 \rightarrow D^0 \eta') \approx 0.30 \times 10^{-4}$ is obtained, show, remarkably, that the gluon mechanism could be more important than the one obtained from the generalized factorization framework [31]. This estimate was based on the value $a_2 = 0.29$, while a recent phenomenological analysis [48] shows that the effective value of a_2 might be almost twice this in some processes. This means that nonfactorizable contributions proportional to C_2 might dominate the $\bar{B}^0 \rightarrow D^0 \eta_8$ amplitude and alter the quark-mechanism prediction for $\bar{B}^0 \rightarrow D^0 \eta'$. In our opinion, looking at $\bar{B}^0 \rightarrow D^0 \eta'$ from another perspective, as we have done in this paper, should therefore be welcome.

It would be very interesting to check this directly, comparing with new data on exclusive B decays to η' mesons. At the moment only an upper bound is known for this exclusive branching ratio, amounting to $\text{BR}(\bar{B}^0 \rightarrow D^0 \eta') < 9.4 \times 10^{-4}$ [43].

The relevance of the gluonic cloud in the η' , here considered from a phenomenological standpoint, is a crucial question within the theory. The discrepancy in mass between the η' state and the octet of (π, K, η) Goldstone modes can be attributed directly to the gluonic content of η' [45]. Moreover, the Witten-Veneziano mass formula for the η' [46], elaborated in the $1/N$ language, has very recently been derived on the lattice [47], confirming in this setup the profound relation between $m_{\eta'}$ and the breaking of the $U(1)_A$ symmetry due to the gluon anomaly.

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- [1] CLEO Collaboration, S.J. Richichi *et al.* Phys. Rev. Lett. **85**, 520 (2000).
 - [2] D. Atwood and A. Soni, Phys. Lett. B **405**, 150 (1997).
 - [3] W. Hou and B. Tseng, Phys. Rev. Lett. **80**, 434 (1998).
 - [4] D. Atwood and A. Soni, Phys. Rev. Lett. **79**, 5206 (1997); H. Fritzsch, Phys. Lett. B **415**, 83 (1997).
 - [5] A. Kagan and A. Petrov, hep-ph/9707354.
 - [6] S. Jeon, Phys. Rev. C **65**, 024903 (2002).
 - [7] M.R. Ahmady, E. Kou, and A. Sugamoto, Phys. Rev. D **58**, 014015 (1998); see also M.R. Ahmadi and E. Kou, *ibid.* **59**, 054014 (1999).
 - [8] E. Kou and A. Sanda, hep-ph/0106159.
 - [9] E. Kou, Phys. Rev. D **63**, 054027 (2001).
 - [10] I. Halperin and A.R. Zhitnitsky, Phys. Rev. D **56**, 7247 (1997); Phys. Rev. Lett. **80**, 438 (1998); E.V. Shuryak and A.R. Zhitnitsky, Phys. Rev. D **57**, 2001 (1998).
 - [11] A. Petrov, Phys. Rev. D **58**, 054004 (1998); M. Franz, P.V. Pobylitsa, M.V. Polyakov, and K. Goeke, Phys. Lett. B **454**, 335 (1999); Phys. Rev. D **62**, 074024 (2000).
 - [12] CLEO Collaboration, B. Nemati *et al.* Phys. Rev. D **57**, 5363 (1998).
 - [13] A. Deandrea and A.D. Polosa, hep-ph/0107234.
 - [14] R. Alkofer and H. Reinhardt, *Chiral Quark Dynamics* (Springer, Berlin, 1995).
 - [15] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); A. Khodjamirian, Nucl. Phys. **B605**, 558 (2001).
 - [16] J.A. Cronin, Phys. Rev. **161**, 1483 (1967); S. Weinberg, Physica A **96**, 327 (1979); A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); J. Bijnens, H. Sonoda, and M.B. Wise, Can. J. Phys. **64**, 1 (1986); D.I. Diakonov, V.Yu. Petrov, and P.V. Pobylitsa, Nucl. Phys. **B306**, 809 (1988).
 - [17] D. Espriu, E. de Rafael, and J. Taron, Nucl. Phys. **B345**, 22 (1990). See also Ref. [30].
 - [18] A. Pich and E. de Rafael, Nucl. Phys. **B358**, 311 (1991).
 - [19] S. Bertolini, J.O. Eeg, and M. Fabbrichesi, Nucl. Phys. **B449**, 197 (1995); V. Antonelli, S. Bertolini, J.O. Eeg, M. Fabbrichesi, and E.I. Lashin, *ibid.* **B469**, 143 (1996); S. Bertolini, J.O. Eeg, M. Fabbrichesi, and E.I. Lashin, *ibid.* **B514**, 63 (1998); **B514**, 93 (1998); S. Bertolini, J.O. Eeg, and M. Fabbrichesi, Rev. Mod. Phys. **72**, 65 (2000); Phys. Rev. D **63**, 056009 (2001).
 - [20] J.O. Eeg and I. Picek, Phys. Lett. B **301**, 423 (1993); **323**, 193 (1994); A.E. Bergan and J.O. Eeg, *ibid.* **390**, 420 (1997).
 - [21] H.-Y. Cheng, Chin. J. Phys. (Taipei) **38**, 1044 (2000).
 - [22] W.A. Bardeen and C.T. Hill, Phys. Rev. D **49**, 409 (1994).
 - [23] D. Ebert, T. Feldmann, R. Friedrich, and H. Reinhardt, Nucl. Phys. **B434**, 619 (1995); D. Ebert, T. Feldmann, and H. Reinhardt, Phys. Lett. B **388**, 154 (1996).
 - [24] A. Deandrea, N. Di Bartolomeo, R. Gatto, G. Nardulli, and A.D. Polosa, Phys. Rev. D **58**, 034004 (1998); A.D. Polosa, Riv. Nuovo Cimento **23N11**, 1 (2000).
 - [25] A. Hiorth and J. O. Eeg (in progress).
 - [26] J.O. Eeg, S. Fajfer, and J. Zupan, Phys. Rev. D **64**, 034010 (2001).
 - [27] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996); A. Buras, in *Probing the Standard Model of Particle Interactions*, edited by F. David and R. Gupta (Elsevier Science, Amsterdam, 1998).
 - [28] M. Ciuchini, E. Franco, and V. Giménes, Phys. Lett. B **388**, 167 (1996).
 - [29] D. Melikhov and N. Nikitin, Phys. Lett. B **494**, 229 (2000).

- [30] See, for example, M.B. Wise, hep-ph/9805468; M. Neubert, hep-ph/9801269.
- [31] See, for example, M. Neubert, and B. Stech, in *Heavy Flavours*, edited by A.J. Buras and M. Lindner, 2nd ed. (World Scientific, Singapore, 1998).
- [32] A. Ali and A.Ya. Parkhomenko, Phys. Rev. D (to be published), hep-ph/0012212; T. Muta and M. Yang, Phys. Rev. D **61**, 054007 (2000); M. Yang and Y. Yang, Nucl. Phys. **B609**, 469 (2001).
- [33] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Fortschr. Phys. **32**, 585 (1984).
- [34] D. Ebert, hep-ph/9710511.
- [35] J. Bijnens, C. Bruno, and E. de Rafael, Nucl. Phys. **B390**, 501 (1992); J. Bijnens, Phys. Rep. **265**, 369 (1996).
- [36] M.B. Wise, Phys. Rev. D **45**, 2188 (1992).
- [37] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
- [38] J. Charles, A. Le Yaouanc, L. Oliver, O. Pène, and J.C. Raynal, Phys. Rev. D **60**, 014001 (1999).
- [39] U. Aglietti and G. Corbo, Phys. Lett. B **431**, 166 (1998).
- [40] C.W. Bauer, S. Fleming, D. Pirjol, and I.W. Stewart, Phys. Rev. D **63**, 114020 (2001).
- [41] L. Del Debbio, J.M. Flynn, L. Lellouch, and J. Nieves, Phys. Lett. B **416**, 392 (1998); P. Ball and V.M. Braun, Phys. Rev. D **58**, 094016 (1998).
- [42] A. Deandrea, R. Gatto, G. Nardulli, and A.D. Polosa, Phys. Rev. D **59**, 074012 (1999).
- [43] Particle Data Group, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
- [44] <http://www-theory.lbl.gov/~ianh/alpha/alpha.html>
- [45] A. De Rujula, H. Georgi, and S. Glashow, Phys. Rev. D **12**, 147 (1975).
- [46] E. Witten, Nucl. Phys. **B156**, 269 (1979); G. Veneziano, *ibid.* **B159**, 213 (1979).
- [47] L. Giusti, G.C. Rossi, M. Testa, and G. Veneziano, hep-lat/0108009.
- [48] H.-Y. Cheng, hep-ph/0108096.