PHENOMENOLOGY OF PARTICLE PRODUCTION AND PROPAGATION IN STRING-MOTIVATED CANONICAL NONCOMMUTATIVE SPACETIME

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We outline a phenomenological programme for the search of effects induced by (string-motivated) canonical noncommutative spacetime. The tests we propose are based, in analogy with a corresponding programme developed over the last few years for the study of Lie-algebra noncommutative spacetimes, on the role of the noncommutativity parameters in the $E(p)$ dispersion relation. We focus on the role of deformed dispersion relations in particle-production collision processes, where the noncommutativity parameters would affect the threshold equation, and in the dispersion of gamma rays observed from distant astrophysical sources. We emphasize that the studies we propose have the advantage of involving particles of relatively high energies, and may therefore be useful also in scenarios in which field theories in canonical noncommutative spacetime, in light of their infrared problems, are only considered as effective theories valid in an energy range $\mu^* < E < \Lambda$.

We also notice that the relevant deformation of the dispersion relations could be responsible for the experimentally-observed violations of the GZK cutoff for cosmic rays and of an analogous cutoff for observations of hard photons from distant astrophysical sources, and we argue that this should provide motivation for future experimental searches, and theoretical studies, focusing on the range $10^{-42} \text{cm}^2 < \theta < 10^{-36} \text{cm}^2$, i.e. $(10^{4} \text{TeV})^{-2} < \theta < (10^{7} \text{TeV})^{-2}$, of the noncommutativity parameters.

I. INTRODUCTION AND SUMMARY

Most approaches to the unification of general relativity and quantum mechanics lead to the emergence, in one or another way, of noncommutative geometry. While one can in principle consider a relatively wide class [1,2] of flat noncommutative spacetimes ("quantum Minkowski"), most studies focus on the two simplest examples; “canonical noncommutative spacetimes” ($\mu, \nu, \beta = 0, 1, 2, 3$)

$$[x_\mu, x_\nu] = i\theta_{\mu,\nu}$$  \hspace{1cm} (1)

and “Lie-algebra noncommutative spacetimes”

$$[x_\mu, x_\nu] = iC^\beta_{\mu,\nu}x_\beta .$$  \hspace{1cm} (2)

The canonical type (1) was originally proposed [3] in the context of attempts to develop a new fundamental picture of spacetime\(^1\). More recently, (1) is proving useful in the description of string theory in presence of certain backgrounds (see, e.g., Refs. [4–10]). String theory in these backgrounds admits description (in the sense of effective theories) in terms of a field theory in the noncommutative spacetimes (1), with the tensor $\theta_{\mu,\nu}$ reflecting the properties of the specific background. Among Lie-algebra, type (2), noncommutative versions of flat (Minkowski) spacetime, research has mostly focused on $\kappa$-Minkowski [11–13] spacetime ($l, m = 1, 2, 3$)

$$[x_m, t] = i\lambda x_m , \hspace{1cm} [x_m, x_l] = 0 .$$  \hspace{1cm} (3)

\(^1\)But in order to play a role in a fundamental picture of spacetime the $\theta_{\mu,\nu}$ cannot be constants (e.g. they should themselves be elements of a enlarged algebra, together with the coordinates [3]).
The strategies followed on the two sides, limits on the $\theta_{\mu,\nu}$ of canonical spacetime and limits on the $\lambda$ of $\kappa$-Minkowski Lie-algebra spacetime, have been rather different and this is partly understandable in light of the differences between the two scenarios, particularly

- The fact that (3) could provide a fundamental picture of spacetime encourages the assumption that $\lambda$ be of the order of the Planck length $L_p \simeq 1.6 \times 10^{-33}$ cm, so the search of experimental tests is naturally aiming for corresponding sensitivities. Instead, in its popular string-theory application, the $\theta_{\mu,\nu}$ of (1) reflect the properties of a background field of the corresponding string-theory context, and therefore there is no natural estimate for the $\theta_{\mu,\nu}$ (they will depend on the strength of the background field).

- The new effects predicted by field theories in canonical spacetime (1) could be observably large also in the low-energy regime. Instead field theories in the Lie-algebra spacetime (3) predict noncommutativity effects that are vanishingly small for low-energy particles, but become significant in the high-energy regime (where the particle/probes have wavelength short enough to be affected by the small, $\lambda$-suppressed, noncommutative effects).

- As one easily realizes based on the fact that the $\theta_{\mu,\nu}$ of (1) reflect the properties of a background in the corresponding string-theory context, the $\theta_{\mu,\nu}$ parameters cannot be treated as observer-independent (the string-theory background takes different form/value in different inertial frames and the $\theta_{\mu,\nu}$ transform accordingly). This must be taken into account in the analysis of the experimental contexts that could set bounds on the $\theta_{\mu,\nu}$, and in particular it has important implications for the task of combining the limits obtained by different experiments\(^2\). Instead, as clarified in Refs. [14,15], noncommutativity of type (3) does not identify a preferred inertial frame ($\lambda$ is observer-independent), but rather reflects a deformed action of the boost generators. All limits on $\lambda$ can therefore be combined straightforwardly.

In spite of the fact that indeed these differences between the two noncommutativity scenarios are rather significant, in this paper we show that some of the phenomenological ideas that are being pursued to set bounds on the parameter $\lambda$ of (3) can also be used to set stringent bounds on the $\theta_{\mu,\nu}$ parameters of (1). The key ingredient for the experimental tests we propose is a deformed dispersion relation. Deformed dispersion relations arise naturally in noncommutative spacetimes. The conventional special-relativistic dispersion relation $E^2 = p^2 + m^2$ reflects the classical Lorentz symmetries of classical Minkowski spacetime. Noncommutative versions of Minkowski spacetime do not enjoy the same classical symmetries, and they therefore naturally lead to deformed dispersion relations. The $\kappa$-Minkowski spacetime (3) is known to be invariant [14] under a group of deformed Lorentz transformations; there is no loss of symmetries (infinitesimal Lorentz transformations still correspond to 6 generators, which were already well understood in the mathematical-physics studies of deformed Poincaré algebras [12,29]), but the nature of these symmetries is different from the classical case and there is a corresponding deformation of the dispersion relation. Canonical noncommutative spacetimes (1) are clearly less symmetric than classical Minkowski spacetime (again, this is easily understood considering the presence of a background field in the corresponding string-theory picture). The loss of symmetries of course is reflected in a deformation of the dispersion relation\(^3\).

Our proposal of testing the deformed dispersion relations of the canonical case (1) in the same high-energy contexts previously considered for tests of the deformed dispersion relations of $\kappa$-Minkowski might at first appear surprising. Whereas the significance of the dispersion-relation deformations encountered in $\kappa$-Minkowski increases with energy, the most significant dispersion-relation-deformation effects in canonical noncommutative spacetimes are found at low energies. One could therefore be tempted to test the latter in low-energy contexts. However, we are here concerned

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\(^2\) Limits on the $\theta_{\mu,\nu}$ obtained using data analyses that assume different inertial frames cannot be directly combined/compared; one must first transform all analyses into a single inertial frame (e.g. the one naturally identified by the cosmic microwave background). This is unfortunately not taken into account by some authors.

\(^3\) Besides the fact that in $\kappa$-Minkowski Lie-algebra spacetime there is no symmetry loss (only symmetry deformation), while in canonical noncommutative spacetimes there is a net loss of symmetries, another important difference is the fact that in field theories on $\kappa$-Minkowski the deformation of the dispersion relation is already encountered at tree level, while field theories on canonical noncommutative spacetimes have an unmodified (i.e. special-relativistic) tree-level propagator and the dispersion-relation deformation emerges only through loop corrections.
with the possibility that the low-energy predictions of field theories in canonical noncommutative spacetimes might be unreliable because of a well-known infrared problem. We adopt in this paper a strictly phenomenological viewpoint, but we are concerned with the rather preliminary status of the understanding of the infrared problems of field theories in canonical noncommutative spacetimes. Limits on $\theta_{\mu,\nu}$ are being set in some studies based on low-energy considerations, while the conceptual understanding of the relevant regime of the theory is still in progress (for example, new issues related with the infrared structure were considered recently in Ref. [22]). High-energy tests are usually only important as ways to test theories in different regimes, providing information complementary to that obtained in low-energy studies, but in the case of canonical noncommutative spacetimes high-energy tests might be even more significant, at least if one adopts the conservative viewpoint of questioning the limits obtained in the low-energy regime in light of our present limited understanding of relevant theoretical issues.

In order to give a more explicit description of our concerns, let us consider an example in which the canonical-noncommutativity scale $\theta$ (a characteristic scale of the $\theta_{\mu,\nu}$ parameters) is of order $\theta \sim 10^{-50}cm^2 \sim (10^8 TeV)^{-2}$ and the description of spacetime based exclusively on the noncommutativity is only applicable up to a UV cutoff $\Lambda \sim 10^{16} TeV$ (for energies above the UV cutoff one might expect spacetime to acquire additional quantum features, even beyond noncommutativity). We fear that the presence of such a UV cutoff would also limit the applicability of field theories in canonical noncommutative spacetimes in the infrared. In fact, a noncommutativity relation of the type $[x, y] = i\theta$ (and the associated uncertainty relation $\delta x \delta y \geq \theta$) might naturally introduce a connection between the ultraviolet and the infrared regimes. In order to uncover the most significant noncommutative features of the spacetime ($\delta x \delta y \sim \theta$) it might be necessary to combine, e.g., high $p_x$ and small $p_y$, $p_y \sim 1/(\theta p_x)$, but the cutoff $\Lambda$ would set a maximum value for $p_x$ (and $p_y$) and correspondingly there might be a loss of information on the spacetime structure corresponding to the combinations $p_x > \Lambda$ (or $p_y > \Lambda$) and $p_y < 1/(\theta \Lambda)$ (or $p_x < 1/(\theta \Lambda)$). The presence of a UV cutoff $\Lambda \sim 10^{16} TeV$ in a theory with $\theta \sim (10^8 TeV)^{-2}$ might therefore give rise to concerns for the reliability of the description of low momenta $p < 1/(\theta \Lambda) \sim 17 TeV$. This illustrative example is particularly significant, since it shows that it is conceivable that a proper description of canonical noncommutativity might require the introduction of an IR cutoff $\mu^*$, and that this cutoff could even be higher than the energy scales we have probed in laboratory experiments. In general one might take the conservative standpoint of testing theories with noncommutativity scale $\theta$ and UV cutoff $\Lambda$ using only data involving particles with momenta in the range $1/(\theta \Lambda) < p < \Lambda$. This line of reasoning is evidently far from rigorous and far from conclusive, but some of the analyses of field theories in canonical noncommutative spacetimes recently reported in the literature (see, e.g., Refs. [8,22]) do admit interpretation as possible encouragement for our concerns. At the risk of being overly conservative, we shall therefore take the working assumption that low-energy constraints on canonical noncommutativity might eventually be understood as inapplicable, and we will focus on experimental tests in contexts involving relatively high energies, indeed some contexts that, as mentioned, had already been considered for the phenomenology of Lie-algebra $\kappa$-Minkowski spacetime.

In the next Section we discuss (relying in part on results which had already appeared in the literature) the emergence of deformed dispersion relations in field theories (mostly QED) on canonical noncommutative spacetimes. We emphasize that this effect is particularly significant for uncharged particles; in fact, in canonical noncommutative spacetimes it is possible to describe the particles that we observe as neutral with respect to the electromagnetic interactions (in experimental contexts in which however there is insufficient sensitivity to quantum properties of spacetime) as particles that do interact with the photon (as a result of the structure of the “star product” [8]) when the $\theta_{\mu,\nu}$ parameters are nonvanishing. Loop corrections, and in particular photon dressing, of neutral-particle propagators in canonical noncommutative spacetimes generally induce corrections to the dispersion relation, and these corrections diverge in the infrared (one of the ways in which the infrared problems of these field theories emerge).

In Section 3, in preparation for the later discussion of experimental tests, we analyze the energy scales at which it appears plausible to use canonical noncommutative spacetimes as effective theories. We start by describing the role of infrared and ultraviolet cutoff scales in our phenomenological programme, inspired by the considerations made above. In particular, we assume that the phenomenology of field theories in canonical noncommutative spacetimes be applicable only to processes above the scale $\mu^*$ (here treated itself as a phenomenological scale, to be determined experimentally). We also emphasize a crucial role possibly played by supersymmetry, especially for what concerns experiments searching for the anomalous properties of photons in canonical noncommutative spacetimes. In these spacetimes photons can acquire rather novel physical properties, including a polarization dependence of the dispersion relation, but these dramatic new properties disappear in the supersymmetric limit. If there is indeed a supersymmetry-restoration scale $M_{\text{susy}}$ in Nature, we would expect photons to enjoy peculiar dispersion-relation properties only in a bounded energy-scale range: above $\mu^*$ and below $M_{\text{susy}}$. This might provide an indirect way to establish the value of $M_{\text{susy}}$; if indeed $\theta_{\mu,\nu}$-dependent deformations of the dispersion relation for photons were discovered experimentally, one could then attempt to repeat the same experiment at higher energies hoping to find a threshold scale were the anomalous effect disappears. That threshold scale would be naturally interpreted as the supersymmetry-restoration scale. (This raises the intriguing possibility that we might discover at once both canonical noncommutativity and the supersymmetry-restoration scale!)
In Section 4 we outline a phenomenological programme which can set bounds on (or discover effects of) the $\theta_{\mu,\nu}$ parameters by studying the implications of the deformed dispersion relations discussed in Section 2. We focus on the implications of a deformed dispersion relation $E^2 \neq p^2 + m^2$ for: (i) the threshold condition for particle production in certain collision processes, and (ii) dispersion of signals observed from distant astrophysical sources. Since our objective here is the one of outlining a relatively wide phenomenological programme, postponing to future studies a detailed analysis of each of the proposals, we just list a few experimental tests and provide simple estimates of the sensitivity levels that appear to be within reach of these tests. Concerning the implications of the deformed dispersion relations for the dispersion of signals observed from distant astrophysical sources, we focus on the case of gamma-ray bursts [32], whose observation provides powerful constraints on dispersion for photons, and on the case of 1987a-type supernovae, which can be used to constrain possible deformations of the dispersion relation for neutrinos. Concerning threshold conditions, we argue that high sensitivity to nonvanishing values of the $\theta_{\mu,\nu}$ parameters can be achieved by analyzing the threshold for electron-positron pair production from photon-photon collisions at energy scales relevant for the expected cutoff on the energies of hard photons observed from distant astrophysical sources. Similarly, deformed thresholds associated with dispersion-relation deformations can significantly affect the GZK cutoff for the observation of cosmic rays. Ultra-high-energy cosmic-ray protons, with energies in excess of the GZK cutoff, should not be detected by our observatories because they should lose energy (thereby complying with the GZK cutoff) through photo-pion production off cosmic-microwave-background photons. If the present (commutative spacetime) estimates of the GZK hard-proton cutoff and of the analogous hard-photon cutoff were confirmed experimentally, we could obtain very stringent limits on the $\theta_{\mu,\nu}$ parameters. Interestingly, observations of hard photons emitted by the Markarian-501 blazar and of cosmic rays presently appear [30,31,27] to be in conflict with the cutoff estimates that assume the conventional dispersion relation (and the associated classical picture of spacetime). We argue that, with more refined data, and careful analyses of these kinematical cutoffs in canonical noncommutative spacetime, we might end up being confronted with a “discovery” of the $\theta_{\mu,\nu}$ parameters (rather than simply setting bounds on their values) and we also suggest that $10^{-2}<\theta<10^{-3}\text{cm}^2$ (i.e. $(10^2\text{TeV})^{-2}<\theta<(10\text{TeV})^{-2}$) is a promising range of canonical-noncommutativity scales for this type of research programme.

Finally, Section 5 is devoted to some closing remarks, particularly concerning the outlook of the phenomenological programme here outlined.

II. DEFORMED DISPERSION RELATIONS IN CANONICAL NONCOMMUTATIVE SPACETIME

The subject of field theory in canonical noncommutative spacetimes has been extensively studied, and there is a wide literature where the reader can find rather pedagogical introductions (see, e.g., Refs. [8,33–40]). We here just mention some well-established features of these field theories, and focus on a few loop corrections that are relevant for the analysis of the emergence of deformed dispersion relations. The tree-level propagators are unmodified by the noncommutativity. Loop corrections to the two-point functions introduce terms that correspond to deformations of the dispersion relations and reflect the loss of symmetry associated with nonvanishing $\theta_{\mu,\nu}$ parameters. Particularly rich are the structures of the dressed photon propagator [8] and of the photon corrections to the propagators of particles that are neutral in the commutative-spacetime limit. Certain gauge-invariant actions in canonical noncommutative spacetime describe particles that are coupled to the gauge field only for nonvanishing $\theta_{\mu,\nu}$, i.e. these are particles that in the $\theta_{\mu,\nu} \rightarrow 0$ limit no longer interact with the gauge field. These particles are natural candidates to describe the neutral particles that we observe: the idea would be that we have not yet observed their interactions with the photon for the same reasons why we have not yet observed any other signature of the $\theta_{\mu,\nu}$ parameters (if indeed Nature hosts nonvanishing $\theta_{\mu,\nu}$ parameters, their effects evidently were negligible in the experiments we have been able to perform). In the remainder of this section we discuss the $\theta_{\mu,\nu}$ deformation of the dispersion relations for photons, neutral spin-1/2 fermions, and neutral spin-0 bosons. Here and in the following we are describing as (QED-)“neutral” the type of particles described above (no interactions with the photon in the $\theta_{\mu,\nu} \rightarrow 0$ limit, but some $\theta_{\mu,\nu}$-dependent

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$^4$The loop corrections we are interested in concern the photon self-energy at one loop, and the one-loop contributions involving the photon to the self-energies of “neutral” spin-1/2 fermions and spin-0 bosons. Our results on the photon self-energy reproduce the ones of the previous analysis reported in Ref. [8]. We did not find in the literature any analogous analysis of the self-energy of neutral spin-1/2 fermions. For the self-energy of neutral spin-0 bosons a related analysis has been reported in Ref. [40]; however, that study adopted a different gauge choice and it is not clear to us (it was not explicitly stated) whether it concerned “neutral” or “charged” spin-0 bosons (if it was meant to consider neutral particles, we would be in disagreement with some of the results, even taking into account the different choice of gauge).
interactions with the photon in the noncommutative spacetime). We work in Feynman gauge, we adopt the standard notation
\[ \tilde{p}_\mu \equiv p^\alpha \theta_{\mu,\alpha} , \] (4)
and we also assume throughout that \( \theta_{\mu,0} = 0 = \theta_{0,\mu} \), i.e. we consider the case in which only (some of) the space components \( \theta_{i,j} \) are nonvanishing\(^5\). Although we are planning, as discussed in Section 1, to develop a phenomenology for field theory in canonical noncommutative spacetime with ultraviolet, \( \Lambda \), and infrared, \( \mu^* \), momentum cutoffs, related by the condition \( \Lambda \mu^* \sim 1/\theta \) in this section we will mostly omit all remarks concerning these cutoffs. This is justified by the fact that the phenomenological programme discussed in Sections 3 and 4 is only sensitive to the leading \( \theta \)-dependent deformations of the dispersion relations, and this allows us to analyze self-energies with the implicit assumption that the external momenta are greater than \( \mu^* \) and that, since we are also assuming \( \Lambda \sim 1/(\theta \mu^*) \), terms of order \( \Lambda^{-2} \) can be neglected with respect to terms of order \( p^2 \theta^2 \), when \( p \) is an external momentum (\( p > \mu^* \sim 1/(\theta \Lambda) \)).

### A. Deformed dispersion relation for photons

In order to see the emergence of a deformed dispersion relation for photons in canonical noncommutative spacetimes it is sufficient to consider one-loop contributions to the photon propagator from diagrams involving as virtual particles either photons themselves or other “neutral” particles. We shall often use the notation \( \gamma \) for photons, \( \nu \) for neutral spin-1/2 fermions, and \( \Phi \) for neutral spin-0 bosons. Some relevant interaction vertices of the field theory in canonical noncommutative spacetime are:

- the four-\( \gamma \) vertex

\[
-4ig^2 \left( (g^{\alpha \gamma} g^{\beta \delta} - g^{\alpha \delta} g^{\beta \gamma}) \sin \frac{\tilde{p}_1 p_2}{2} \sin \frac{\tilde{p}_3 p_4}{2} + 
+ (g^{\alpha \delta} g^{\beta \gamma} - g^{\alpha \gamma} g^{\beta \delta}) \sin \frac{\tilde{p}_3 p_1}{2} \sin \frac{\tilde{p}_2 p_4}{2} + 
+ (g^{\alpha \beta} g^{\gamma \delta} - g^{\alpha \delta} g^{\gamma \beta}) \sin \frac{\tilde{p}_1 p_4}{2} \sin \frac{\tilde{p}_2 p_3}{2} \right) 
\] (5)

with every \( p_i \) exiting the vertex;

- the three-\( \gamma \) vertex

\[
-2g \sin \frac{\tilde{p}_1 p_2}{2} \left( g^{\alpha \beta} (p_1 - p_2) \gamma + g^{\alpha \gamma} (p_3 - p_1) \beta + g^{\beta \gamma} (p_2 - p_3) \alpha \right) 
\] (6)

with every \( p_i \) exiting the vertex;

- the \( \gamma-\nu-\nu \) vertex

\[
2g^{\gamma \mu} \sin \frac{\tilde{p}_1 p_2}{2} 
\] (7)

where \( p_1 \) (\( p_2 \)) is the momentum of the incoming (outgoing) \( \nu \);

- the \( \gamma-\gamma-\Phi-\Phi \) vertex

\[
4ig^2 g^{\mu \nu} \left( \sin \frac{\tilde{p}_2 p_4}{2} \sin \frac{\tilde{p}_1 p_3}{2} + \sin \frac{\tilde{p}_2 p_3}{2} \sin \frac{\tilde{p}_1 p_4}{2} \right) 
\] (8)

where \( p_1, p_2 \) are the photons’ momenta and \( p_3, p_4 \) the scalars’ momenta, and they all exit the vertex;

\(^5\)When also some \( \theta_{\mu,0} = -\theta_{0,\mu} \neq 0 \) it is expected [41] that the field theories in canonical noncommutative spacetimes loose unitarity (open string states cannot be neglected).
• and the $\gamma$-$\Phi$-$\Phi$ vertex

$$2g (p_1 + p_2)^\mu \sin \frac{\hat{p} \cdot p_2}{2}$$

(9)

where $p_1$ ($p_2$) is the momentum of the incoming (outgoing) scalar.

• In addition we will also need the $\gamma$-ghost-ghost vertex:

$$2gp_2^\mu \sin \frac{\hat{p} \cdot p_2}{2}$$

(10)

where $p_1$ ($p_2$) is the momentum of the incoming (outgoing) ghost.

(It is well known that abelian gauge theories in canonical noncommutative spacetime, unlike their commutative counterparts, do require the introduction of ghosts. This and other properties render abelian gauge theories in canonical noncommutative spacetime somewhat analogous to nonabelian gauge theories in commutative spacetime.)

There are three nontrivial pure-gauge one-loop contributions to the photon self-energy. For external photons with momentum $p$ and polarizations $\mu$ and $\nu$ one finds

• a tadpole-type diagram, using the vertex (5),

$$i \Sigma_{\gamma;1}^{\mu \nu} = -2ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{-ig_{\rho \sigma}}{k^2} (2g^{\rho \sigma} g^{\mu \nu} - g^{\mu \sigma} g^{\nu \rho} - g^{\mu \rho} g^{\nu \sigma}) \sin^2 \frac{\tilde{p} \cdot k}{2}$$

(11)

• a photon-loop diagram, using the vertex (6),

$$i \Sigma_{\gamma;2}^{\mu \nu} = 2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{-ig_{\rho \sigma}}{k^2} (g^{\mu \sigma} (p - k)^\rho + g^{\mu \rho} (-2p - k)^\sigma + g^{\rho \sigma} (2k + p)^\nu)
\frac{-ig_{\sigma \beta}}{(p - k)^2} (g^{\nu \beta} (-p + k)^\alpha + g^{\nu \alpha} (2p + k)^\beta + g^{\alpha \beta} (-p - 2k)^\nu) \sin^2 \frac{\tilde{p} \cdot k}{2}$$

(12)

• and a ghost-loop diagram, using the vertex (10),

$$i \Sigma_{\gamma;\text{ghost}}^{\mu \nu} = 4g^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} (p + k)^\mu \frac{i}{(p - k)^2} k^\nu \sin^2 \frac{\tilde{p} \cdot k}{2}$$

(13)

With straightforward calculations one finds [8] that the dominant $\theta$-dependent\(^6\) contribution from these diagrams are

$$i \Sigma_{\gamma;1}^{\mu \nu} = \frac{3ig^2 g^{\mu \nu}}{2\pi^2 |\tilde{p}|^2}$$

(14)

$$i \Sigma_{\gamma;2}^{\mu \nu} = \frac{-ig^2}{4\pi^2} \left( \frac{g^{\mu \nu}}{|\tilde{p}|^2} - 10 \frac{\tilde{p}^\mu \tilde{p}^\nu}{|\tilde{p}|^4} \right),$$

(15)

and

$$i \Sigma_{\gamma;\text{ghost}}^{\mu \nu} = \frac{ig^2}{4\pi^2} \left( -2 \frac{\tilde{p}^\mu \tilde{p}^\nu}{|\tilde{p}|^2} + \frac{g^{\mu \nu}}{|\tilde{p}|^2} \right).$$

(16)

\(^6\)It is easy to identify the origin of these most important $\theta$-dependent contributions. They usually come from the part of the loop integration that involves large loop momenta.
Next let us consider the one-loop contribution to the photon self-energy that involves virtual neutral fermions, using the vertex (7):

\[ i\Sigma^{\mu\nu}_{\gamma;\pi} = 4g^2 \int \frac{d^4k}{(2\pi)^4} \sin^2 \frac{\hat{p}k}{2} \left( \gamma^\mu \frac{i}{\gamma^\rho (p+k)_\rho - m \gamma^\nu} \frac{i}{\gamma^\sigma k_\sigma - m} \right). \]  

(17)

for which we extract again the dominant \( \theta \)-dependent contribution

\[ i\Sigma^{\mu\nu}_{\gamma;\pi} = -\frac{4ig^2 \hat{p}^\mu \hat{p}^\nu}{\pi^2 |\hat{p}|^4}. \]  

(18)

Finally we consider the two nontrivial one-loop contributions to the photon self-energy that involve virtual neutral scalars: a tadpole-type diagram, using the vertex (8),

\[ i\Sigma^{\mu\nu}_{\gamma;\Phi;1} = 4ig^2 g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \sin^2 \frac{\hat{p}k}{2}, \]  

(19)

and a scalar-loop diagram, using the vertex (9),

\[ i\Sigma^{\mu\nu}_{\gamma;\Phi;2} = -2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \frac{i}{(p+k)^2 - m^2} (p+2k)^\mu (p+2k)^\nu \sin^2 \frac{\hat{p}k}{2}, \]  

(20)

for which we extract again the dominant \( \theta \)-dependent contributions

\[ i\Sigma^{\mu\nu}_{\gamma;\Phi;1} = \frac{ig^2 g^{\mu\nu}}{2\pi^2 \rho^2}, \]  

(21)

and

\[ i\Sigma^{\mu\nu}_{\gamma;\Phi;2} = -\frac{ig^2}{2\pi^2} \left( -2 \frac{\hat{p}^\mu \hat{p}^\nu}{|\hat{p}|^4} + \frac{g^{\mu\nu}}{|\hat{p}|^2} \right). \]  

(22)

Combining these results one obtains [8] the total contribution to the photon self-energy:

\[ i\Sigma^{\mu\nu} = \frac{ig^2}{\pi^2} (N_s + 2 - 2N_f) \frac{\hat{p}^\mu \hat{p}^\nu}{|\hat{p}|^4}. \]  

(23)

where \( N_s \) and \( N_f \) denote the number of neutral scalar fields and the number of neutral fermion fields in the theory. It is important to notice [8] that in a supersymmetric field theory in canonical noncommutative spacetime, which would accommodate an equal number of bosonic and fermionic degrees of freedom, this correction term vanishes. However, while we do want to emphasize the implications that apply in particular to supersymmetric theories, we are here interested in general in the predictions of field theories in canonical noncommutative spacetime. There is at present no direct experimental evidence of supersymmetry, and accordingly in our phenomenological analysis we will assume that either there is no supersymmetry at all or that supersymmetry is broken at low energies (processes below the supersymmetry-restoration scale \( \mathcal{M}_{\text{sup}} \), here treated as a phenomenological parameter). It is therefore important from our perspective to explore the physical content of (23) when supersymmetry is absent (or not yet restored).

It is convenient to consider the simple case in which \( \theta_{j,j} \) is only nontrivial in the (1,2)-plane: \( \theta_{1,2} = -\theta_{2,1} \equiv \theta \neq 0, \theta_{1,3} = \theta_{2,3} = 0 \). Relevant observations have already been reported in Ref. [8]: according to (23) the two, tranversely polarized, physical degrees of freedom of the photon satisfy different dispersion relations, reflecting the loss\(^7\) of Lorentz invariance introduced by \( \theta \). If, for example, \( \hat{p} \) is in the 1-direction, in the case we are considering, \( \theta_{1,3} = \theta_{2,3} = \theta_{\mu,0} = 0, \)

\(^7\)On this issue of the breaking of Lorentz invariance there is sometimes some confusion. At the fundamental level these theories in canonical noncommutative spacetimes are still special-relativistic in the ordinary sense. However, \( \theta \) has the role of a background (as well understood in the corresponding string-theory picture) and of course effective field theories in presence of some background do not enjoy Lorentz symmetry. The Lorentz invariance of the theory becomes manifest only in formalisms that take into account both the transformations of the quantum fields and of the background. The effective field theory does have
one finds that the degree of freedom polarized in the direction orthogonal to \( \tilde{p} \) satisfies the ordinary special-relativistic dispersion relation
\[
p_0^2 = \tilde{p}^0,
\]
while the degree of freedom polarized in the direction parallel to \( \tilde{p} \) satisfies a deformed dispersion relation of the type
\[
p_0^2 = \tilde{p}^0 + \frac{\zeta}{\tilde{p}^2},
\]
where \( \zeta \) is a number that depends, according to (23), on the coupling constant and on the number of bosonic and fermionic degrees of freedom present in the theory. Besides the polarization dependence, these dispersion relations are strongly characterized by the infrared singularity of the term \( \zeta / \tilde{p}^2 \), which reflects the mentioned infrared problems of field theories in canonical noncommutative spacetimes. This will encourage us to consider such theories only as effective high-energy theories, in the sense discussed in greater detail in Section 3.

B. Deformed dispersion relation for neutral spin-0 bosons

We now analyze in the same way the self-energy of scalars (neutral spin-0 bosons) in canonical noncommutative spacetime. We focus on the loop contributions involving virtual photons and on the loop contributions involving virtual scalars (self-interactions). There are two nontrivial one-loop contributions to the scalar self-energy that involve virtual photons (for a scalar of external momentum \( p \)):

- a tadpole-type diagram, using the vertex (8),
\[
i\Sigma_{\Phi;\gamma;1} = 4ig^2g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{-ig_{\mu\nu}}{k^2} \sin^2 \frac{\tilde{p}k}{2},
\]

- and a sunset-type diagram, using the vertex (9),
\[
i\Sigma_{\Phi;\gamma;2} = -4g^2 \int \frac{d^4k}{(2\pi)^4} (p+k)^\mu \frac{-ig_{\mu\nu}}{(p-k)^2} (p+k)^\nu \frac{i}{k^2 - m^2} \sin^2 \frac{\tilde{p}k}{2}.
\]

Again, the dominant \( \theta \)-dependent contribution\(^8\) from these diagrams can be established with straightforward calculations, finding
\[
i\Sigma_{\Phi;\gamma;1} = -\frac{2ig^2}{\pi^2 |\tilde{p}|^2},
\]
and
\[
i\Sigma_{\Phi;\gamma;2} = \frac{ig^2}{2\pi^2 |\tilde{p}|^2}.
\]

\( ^8\)Since these diagrams give rise to strong, power-law, \( \theta \) dependence, we drop a logarithmic dependence on \( \theta \) (proportional to the square of the mass and therefore further suppressed in the analysis of the high-energy experimental contexts we intend to consider).
The one-loop (tadpole) contribution to the scalar self-energy due to its self-interactions is a well-known prototype result [8,38] of field theory in canonical noncommutative spacetime. One finds that the most important effect of the noncommutativity in this self-interaction tadpole contribution is again of the type \( \frac{1}{\tilde{p}^2} \). Therefore combining photon dressing and self dressing of the propagator one finds a deformed dispersion relation for neutral spin-0 bosons of the type

\[
p^2_0 = \tilde{p}^2 + m^2 + \frac{\zeta_{\Phi}}{\tilde{p}^2},
\]

where again \( \zeta_{\Phi} \) is a number that depends on the couplings of the theory and on the number and type of fields involved.

C. Deformed dispersion relation for neutral spin-1/2 fermions

We close this Section by performing an analogous dispersion-relation analysis for neutral spin-1/2 fermions. We only consider the self-energy contribution that is due to interactions with the photon. The relevant sunset-type diagram, using the vertex (7), corresponds to the integral

\[
i \Sigma_{\nu\gamma} = -4g^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \gamma^\nu \left( \frac{-ig_{\mu\nu}}{(p-k)^2} \right) \sin^2 \frac{\tilde{p}k}{2};
\]

which can be evaluated exactly, obtaining:

\[
i \Sigma_{\nu\gamma} = \frac{2g^2}{(4\pi)^2} \left( -\gamma^\mu p_\mu + 4m \right) \ln \frac{\Lambda^2}{\Lambda_{eff}^2},
\]

where \( \Lambda \) is a conventional high-momentum cutoff and

\[
\frac{1}{\Lambda_{eff}^2} = \frac{1}{\Lambda^2} + \frac{\tilde{p}^2}{4}.
\]

The physical implications are analogous to the ones found for photons and neutral spin-0 bosons. For neutral spin-1/2 fermions one has again a deformation of the dispersion relation which (if the fermion has mass) is singular in the infrared; however, the singularity is softer, only logarithmic, and accordingly the magnitude of the effects to be expected (whether or not the fermion has mass) at relatively small momenta are not as significant as, e.g., for the photon. We will therefore not express high expectations for the bounds on \( \theta \) that can be placed using observations of neutral spin-1/2 fermions, but still we will comment on some types of experiments/observations that are sensitive to the dispersion-relation deformation experienced by these particles in canonical noncommutative spacetime.

III. ENERGY SCALES FOR \( \theta_{\mu,\nu} \) SEARCHES

For our programme of experimental searches of the effects predicted by field theories in canonical noncommutative spacetime a central issue is the one of the identification of the scales \( \mu^* \) and \( M_{susy} \). As mentioned in Section 1, we are considering the possibility that these theories, because of their infrared problems, might have to be considered as effective theories with a range of applicability to physical processes that is bounded both from above and from below \( \mu^* < p < \Lambda \sim 1/(\theta \mu^*) \), and we intend to treat \( \mu^* \) as an unknown to be determined experimentally. Since some of the effects predicted by field theories in canonical noncommutative spacetime depend very strongly on whether or not supersymmetry is present (think in particular of the structure of the photon full propagator), another important unknown for this phenomenology is the scale \( M_{susy} \) of supersymmetry restoration (in particular the case \( M_{susy} = \infty \) corresponds to the case in which there is no supersymmetry restoration in Nature). The experimental \( \theta_{\mu,\nu} \) searches we propose can be done at different energy scales. When an observation at a given energy scale \( E \) gives negative

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9The role of \( \Lambda_{eff} \) in Eq. (32) and in other characteristic formulas of field theory in canonical noncommutative spacetime, obtained in previous studies, reflects the well-established “IR/UV connection” present in these theories. If one takes \( \Lambda \to \infty \) then \( \Lambda_{eff} \to 4/\tilde{p}^2 \) and \( \ln \Lambda_{eff} \) is singular in the infrared.
results it can be used to set a conditional limit on the $\theta_{\mu,\nu}$, in order to fully establish the limit one should also establish the scales $\mu^*$ and $M_{\text{susy}}$ and check that the energy scale $E$ at which the limit was derived falls within the range of applicability of field theory in canonical noncommutative spacetime. For example, if searches of anomalies in photon propagation give negative results (within the available experimental sensitivity) for photons of energy $E$ one can only conclude that one of three possibilities is realized: (i) the $\theta_{\mu,\nu}$ take values such that their effects would not be observable within the sensitivity of the experiment (limits on the $\theta_{\mu,\nu}$), (ii) the energy scale $E$ is below $\mu^*$ (and therefore the experiment sets no limit on the $\theta_{\mu,\nu}$, but rather establishes that at the scale $E$ field theories in canonical noncommutative spacetime are still not applicable), (iii) the energy scale $E$ is above $M_{\text{susy}}$ (and therefore the negative result of the experiment searching for anomalous photon-propagation properties would not set a limit on the $\theta_{\mu,\nu}$; it would rather establish that at the scale $E$ supersymmetry is already restored). In principle the experimental programme we propose could eventually, through the combined analysis of several experimental results, determine the values of the $\theta_{\mu,\nu}$ parameters and the values of the scales $\mu^*$ and $M_{\text{susy}}$. It is rather amusing that, if Nature does make use of field theories in canonical noncommutative spacetime, we would have an indirect way to establish the scale of supersymmetry restoration. Using again photons as our basic example, if photon propagation turned out to be anomalous when and only when the photon energy is within a certain range $E_1 < E < E_2$ it would then be natural to assume that $E_1 = \mu^*$ and $E_2 = M_{\text{susy}}$. The delicate role played by the scales $\mu^*$ and $M_{\text{susy}}$ in this phenomenology would actually require a corresponding level of complexity. In the present preliminary study we will set aside most of this complexity. For example we will assume that the photon propagator is exactly conventional below the scale $\mu^*$ and above the scale $M_{\text{susy}}$. This is not completely unplausible, but clearly more refined analyses should consider the (more likely) possibility that the “transitions” that occur at the scales $\mu^*$ and $M_{\text{susy}}$ might be smoother than here assumed.

IV. EXPERIMENTAL PROGRAMME SEEKING LIMITS ON THE $\theta_{\mu,\nu}$

As announced in Section 1, we are proposing to test the predictions of canonical noncommutative spacetime using the same techniques previously developed to search for the effects possibly induced by the parameter $\lambda$ of Lie-algebra noncommutative spacetime (3). In fact, both classes on noncommutative spacetimes are strongly characterized by deformations of the special-relativistic dispersion relation, and can therefore both be tested using experiments with good sensitivity to such deformations. We focus here on the role of deformed dispersion relations for the evaluation of the threshold condition for particle production in certain collision processes, and for the dispersion of signals observed from distant astrophysical sources. Since our objective here is the one of outlining a relatively wide phenomenological programme, postponing to future studies a detailed analysis of each of the proposals, we just list a few experimental tests and provide simple estimates of the sensitivity levels that appear to be within reach of these tests.

A. Gamma-ray astrophysics

It is a rather general feature [25] of “quantized” (discretized, noncommutative...) spacetimes to induce anomalous particle-propagation properties. In some pictures [42–44] of “spacetime foam” (not involving noncommutative geometry) one describes foam as a sort of spacetime medium that, like other media, induces dispersion. As discussed in the preceding Sections, in quantum pictures of spacetime based on canonical or Lie-algebra noncommutative geometry one also automatically finds deformations of the dispersion relation. The observation that gamma-ray astrophysics could be used to search for the effects of such deformed dispersion relations was put forward in Ref. [42], focusing on foam-induced dispersion. The use of the same gamma-ray astrophysics for tests of the predictions of $\kappa$-Minkowski Lie-algebra noncommutative spacetimes was then discussed in Refs. [25,45,14]. Here we observe that gamma-ray astrophysics can also be used to set stringent bounds on the $\theta_{\mu,\nu}$ parameters of canonical noncommutative spacetimes. Bounds on deformations of the dispersion relation can be set by analyzing time-of-arrival versus energy correlations of gamma ray bursts that reach our detectors coming from far away galaxies. In presence of a deformed dispersion relation (which implies that photons of different energies travel at different speeds) photons emitted in a relatively short time (a burst) should reach our detectors with a larger time spread, and the spreading should depend on energy difference. Gamma-ray bursts [32] travel over large distances, $\sim 10^{10}$ light years, and are observed to maintain time-of-arrival correlation over very short time scales, $\sim 10^{-3}$ s. For photons in the bursts that have energies in the 100KeV-1MeV range, as the ones observed by the BATSE detector [32], this has of course allowed to establish [42,46] that in vacuo dispersion (if at all present) is small enough to induce relative time-of-arrival delays between photons with energy differences of a few hundred KeV that are below the $10^{-3}$ s level. This turns out to set a stringent limit on the $\lambda$ parameter of the $\kappa$-Minkowski Lie-algebra spacetime, $\lambda < 10^{-30}$ cm, and experiments now in preparation
will allow to probe even smaller, subPlanckian, deformation scales \([42,46]\). Limits of comparable significance can be obtained analyzing the bursts of photons emitted by blazars \([47]\). An analysis of data on the Markarian 421 blazar allowed to establish \([47]\) that photons with energies of a few TeV (and comparable energy differences within the burst) acquire relative time-of-arrival delays that are below the \(10^5\) s level for travel over distances of order 100 Mpc. This again sets a limit on \(\theta\) that is of order \(\lambda < 10^{-30}\text{cm}\). With respect to the analysis of these limits on the \(\lambda\) of \(\kappa\)-Minkowski Lie-algebra spacetime, the analysis of corresponding limits on the \(\theta_{\mu,\nu}\) of canonical noncommutative spacetimes is complicated by the polarization dependence of the deformation of the dispersion relation. However, it appears safe to assume that the emissions by gamma-ray-bursts and blazars are largely unpolarized, and therefore canonical noncommutative spacetimes would imply that at least a portion of the bursts (also depending on the position of the source, which of course fixes the direction of propagation of the photons that reach us from the source) would manifest a dispersion-induced effect (a short-duration light burst travelling in a birefringent medium increases its time spread over time). This allows to sets constraints on the \(\theta_{\mu,\nu}\) parameters.

As announced we just want to estimate this type of sensitivities and we want to focus on data that are obtained as far from the infrared as possible. Considering the TeV photons of the mentioned blazar and the nominal \([47]\) \(\delta T/T \sim 10^{-12}\) accuracy (with \(\delta T\) the observed level of time-of-arrival simultaneity and \(T\) the overall time of flight), the \((p\theta)^{-2}\) behaviour of the deformation of the dispersion relation\(^{10}\) could lead to the bound \((p^2\theta)^{-2} \sim (\text{TeV}^2\theta)^{-2} < 10^{-12}\). This would allow to set a limit \(\theta > \theta_{\text{min}}\), with \(\theta_{\text{min}}\) somewhere in the neighborhood of \(10^{-40}\text{cm}^2\). Notice that this would be a lower limit on \(\theta\), while of course in the case of the \(\lambda\) parameter one obtains an upper limit. This is another manifestation of the unusual infrared structure of field theories in canonical noncommutative spacetimes, which at the level of the dispersion relation is reflected in the \((p\theta)^{-2}\) behaviour. Larger values of the \(\theta_{\mu,\nu}\) parameters correspond, somewhat counter-intuitively, to softer deformations of the dispersion relation. Here we see how phenomenological analyses are affected by the fact that the \(\theta \to 0\) limit of these theories is not analytic. In spite of the fact that the experimental bound takes the form of a lower limit, of course, even if more refined analyses of the limits imposed by blazar data should confirm our preliminary estimate, one should not conclude that nonvanishing \(\theta_{\mu,\nu}\) values are being discovered. The conclusion would be that if spacetime is described by a canonical noncommutative geometry then (some of) the \(\theta_{\mu,\nu}\) parameters should be larger than a certain minimum experimentally-allowed value. In addition one should keep in mind the issue of supersymmetry, so, actually, limits on relative time-of-arrival delays for TeV photons could be described, within canonical noncommutative spacetimes, in three ways: (i) \(\theta = 0\), (ii) \(\theta \neq 0\) and supersymmetry is already restored at the TeV scale, (iii) supersymmetry is not restored at the TeV scale and \(\theta > \theta_{\text{min}}\).

### B. Markarian hard-photon threshold

Both in study of spacetime-foam models \([42–44]\) and in the study of \(\kappa\)-Minkowski Lie-algebra noncommutative spacetimes the emerging deformed dispersion relations have also been analyzed \([45,14,27,30,31,48]\) for what concerns their implications for the determination of the threshold conditions for particle-production in collision processes. The interested reader can find these previous results in the literature \([45,14,27,30,31,48]\); here we apply the same line of analysis to the deformed dispersion relations that emerge in canonical noncommutative spacetimes. An interesting process for this type of threshold analyses is electron-positron pair production in photon-photon collisions: \(\gamma + \gamma \rightarrow e^- + e^-\). This is a significant process for the physics of blazars. Certain blazars are known to emit photons up to very high energies; however, standard relativistic astrophysics predicts that only photons with energies below 10 TeV should be able to reach our detectors. Photons of higher energies should collide with soft photons of the Far Infrared Background Radiation (FIBR) and should disappear in an electron-positron pair.

This hard-photon-absorption limit is obtained in an analysis that, besides adopting the standard special-relativistic dispersion relation and energy-momentum conservation rules, takes into account the available data on the energy/density properties of the FIBR, but for our purposes we can schematically describe the FIBR as a dense background of photons with typical energy of \(\sim 0.003\) eV. Denoting with \(E\) the energy of the hard photon emitted by the blazar, with \(\epsilon\) the energy of the FIBR photon, and with \(m_e\) the electron mass one finds indeed that pair production requires \(E > m_e^2/\epsilon \simeq 10\) TeV. This result is of course modified \([45,14,27,30,31,48]\) in theories that predict a deformed dispersion relation. Such modifications are rather attractive phenomenologically since preliminary anal-

\(^{10}\)In our estimates we take the numerical factors \(\zeta\) introduced in Section 2 to be of order 1. They are probably smaller than 1 (they involve the QED coupling constant \(\alpha\)) but at this preliminary stage we are only trying to establish a rough picture of the \(\theta\)-sensitivities of some relevant experiments, and we would not be too concerned even with inaccuracies of 1 or 2 orders of magnitude.
In the case of canonical noncommutative spacetimes it is important to take into account the polarization dependence of the deformation of the photon dispersion relation. This would lead to different threshold conditions for different polarizations. If the threshold is increased only for photons with a certain polarization, then one would expect that at the conventional special-relativistic threshold there would be a suppression of the flux, but only partial. Another delicate issue is the one of the very soft (FIBR) photon involved in the process. Clearly, at least within the conservative phenomenological approach we are adopting, these photons are too soft for field theory in canonical noncommutative spacetime, with its infrared problems, to be applicable (at those low energies we have a huge amount of data confirming the special-relativistic behaviour of photons). We shall therefore assume that only the hard Markarian-501 photon is affected by the deformed dispersion relation, while the FIBR photon involved in the process obeys the ordinary special-relativistic dispersion relation. Again, postponing a more detailed analysis to future studies, we estimate here the range of the $\theta_{\mu,\nu}$ parameters that could significantly affect the Markarian-501 absorption threshold. Since the deformation of the dispersion relation is governed by correction terms of the type $(p\theta)^{-2}$, it is easy to verify that the Markarian-501 absorption threshold could be significantly affected only if $(p\theta)^{-2} \gtrsim m^2_e$. In the case of interest $p \sim 10\text{TeV}$, and therefore the threshold can be significantly affected only if $0 \neq \theta < \theta_{\text{max}}$, with $\theta_{\text{max}}$ somewhere in the neighborhood of $10^{-30} \text{cm}^2$. (Note that just like the negative results of dispersion searches, discussed in the previous Subsection, would lead, perhaps counter-intuitively for some readers, to lower bounds on $\theta$, attempts to interpret the observed violations of the Markarian-501 threshold as a manifestation of canonical noncommutative spacetime require an upper bound on $\theta$. Both features are of course due to the peculiar momentum dependence associated with the unfamiliar infrared structure of the theory.)

C. Cosmic-ray threshold

Observations of ultra-high-energy cosmic rays present us with a puzzle that can be described in close analogy to the one considered in the previous subsection, for multi-TeV photons from Markarian 501. Cosmic rays can interact with the Cosmic Microwave Background Radiation (CMBR), producing pions ($p + \gamma \rightarrow p + \pi$). Taking into account the typical energy of CMBR photons, and assuming the validity of the kinematic rules for the production of particles in our present, classical and continuous, description of spacetime (conventional relativistic kinematics), one finds that these interactions should lead to an upper limit $E < 5 \times 10^{19}\text{eV}$, the GZK limit [50], on the energy of observed cosmic rays. Essentially, cosmic rays emitted with energies in excess of the GZK limit should loose energy on the way to Earth by producing pions, and, as a result, should still satisfy the GZK limit when detected by our observatories. Instead, several cosmic-rays above the GZK limit (with energies as high as $3 \times 10^{20}\text{eV}$) have been observed [51].

As for the case of the Markarian-501 paradox, one can advocate a deformed dispersion relation, which would explain these puzzling observations by leading to an accordingly shifted prediction for the $p + \gamma \rightarrow p + \pi$ threshold. Again we refer the reader interested in the analysis of the cosmic-ray paradox within other quantum-spacetime frameworks to previous results in the literature [45,14,30,27,52,53]; here we just intend to focus on canonical noncommutative spacetimes and estimate the range of values of the $\theta_{\mu,\nu}$ parameters that could significantly affect the GZK cosmic-ray threshold.

The process $p + \gamma \rightarrow p + \pi$ does not involve any hard photons, and, for the reasons discussed above, we will assume that the soft CMBR photon should be analyzed according to the conventional special-relativistic dispersion relation. The most significant dispersion-relation deformation relevant for this process could be attributed to the neutral pion.$^{11}$ Assuming that $(p\theta)^{-2}$ corrections to dispersion relations do indeed characterize the kinematics of this process, we observe that in order for these deformations to be significant at the GZK scale it is necessary to have

$^{11}$While the particle identification (as photons) is clearly robust and the distance to Markarian-501 is also reasonably well known ($\sim 150\text{Mpc}$), the $10\text{TeV}$-cutoff estimate is still subject to some scrutiny because of its dependence on the reliability of recent data on the FIBR and associated theoretical analyses [31]. The ongoing debate on the puzzling observations of Markarian-501 photons with energies above $10\text{TeV}$ might still be overruled by more refined FIBR measurements. We here adopt, as done, e.g. in Ref. [31], as working assumption the $10\text{TeV}$ limit, pending the outcome of future FIBR measurements.

$^{12}$Our wording here is rather prudent as a result of the fact that the pion is a composite particle. In Lie-algebra noncommutative spacetimes it appears [14] that composite particles are subject to a deformed dispersion relation that is different from the one for fundamental particles. We are not aware of analogous results in canonical noncommutative spacetimes, but of course this issue would be important for the kinematics of $p + \gamma \rightarrow p + \pi$.
result could provide clean signatures of spacetime-induced dispersion. Interaction charges. They are therefore mostly insensitive to this conventional dispersion-inducing effects, and as a case for the particles that reach our detectors from far-away galaxies. Neutrinos are only endowed with weak-be obstructed by the fact that most particles also interact with more conventional electro-magnetic media, as indeed the relatively large distances travelled ($10^{20}$ cm) on deformations of the pair-production threshold large enough to open the possibility of finding explanations for the puzzling results of observations of multi-TeV Markarian-501 photons and cosmic rays above the GZK cutoff. This may provide motivation for experimental searches aimed at this interesting range for the $\theta_{\mu,\nu}$ parameters. Since our estimates have been very preliminary (and we even did not take into account the numerical coefficients $\zeta$ introduced in Section 2) it is probably legitimate to soften the upper and lower limits of the range identified by our preliminary analysis: experimental searches (and theoretical analyses) could perhaps aim at the range $10^{-42} cm^2 < \theta < 10^{-36} cm^2$.

D. 1987a-type supernovae

Our final proposal for experimental studies of the dispersion-relation deformations that emerge in canonical noncommutative spacetimes concerns neutrinos. We have seen in Section 2 that also neutral spin-1/2 particles could acquire a dispersion-relation deformation, but with a somewhat softer (logarithmic, rather than inverse-square power) dependence on “$p\theta$”. This might mean that, if any of these experimental searches ends up being successful, the first positive results are unlikely to come from data on neutrino kinematics. Still, one should keep in mind that neutrinos are a particularly clean spacetime probe. The identification of quantum-spacetime effects that can be qualitatively described as “spacetime-medium effects”, such as the ones induced by canonical noncommutativity, can sometime be obstructed by the fact that most particles also interact with more conventional electro-magnetic media, as indeed is the case for the particles that reach our detectors from far-away galaxies. Neutrinos are only endowed with weak-interaction charges. They are therefore mostly insensitive to this conventional dispersion-inducing effects, and as a result could provide clean signatures of spacetime-induced dispersion. In this respect supernovae of the type of 1987a might provide a interesting laboratory because of the relatively high energy of the observed neutrinos ($\sim 100$ MeV), the relatively large distances travelled ($\sim 10^8$, $10^4$ light years), and the short (below-second) duration of the bursts.

V. CLOSING REMARKS

We have argued that certain types of experimental tests which were previously considered in the literature on Lie-algebra noncommutative spacetimes, can also be significant for investigations of canonical noncommutative spacetimes. The applicability of these tests comes from the fact that they rely on deformed dispersion relations, a feature that is present in both types of noncommutative spacetimes. The tests involve particles of relatively high energies; this is welcome in Lie-algebra noncommutative spacetimes because the effects are fully confined to the high-energy regime, and we argued that it should also be welcome in canonical noncommutative spacetimes in light of the severe infrared problems of field theories in these spacetimes. The conceptual and technical understanding of these infrared problems is still “in progress”, and we have argued that, as a result of this unsettled theory issues, low-energy limits on the $\theta_{\mu,\nu}$ parameters might have to be still perceived with some skepticism.

Perhaps the most intriguing part of our analysis is the one concerning the cosmic-ray and Markarian-501 paradoxes. There is already a lively debate on the possibility that these puzzling observations might be due to deformations of the dispersion relations. It appears natural to attempt to seek solutions of these paradoxes in the context of canonical noncommutative spacetimes. Here we just provided a rough estimate of the range of the $\theta_{\mu,\nu}$ parameters that could

13Let us emphasize that this requirement is necessary but of course not sufficient for a solution of the cosmic-ray paradox. We are just estimating the sensitivity here, while an actual solution of the paradox requires a detailed dedicated analysis. The reader should interpret with analogous prudence the estimate we derived in the previous Subsection from Markarian-501 data.

14As mentioned, canonical noncommutative spacetimes require the existence of a preferred frame, and most of their features, especially concerning the associated dispersion-relation deformations, can be described in close analogy with ordinary (commutative-spacetime) physics in presence of a medium (think, for example, of the laws that govern propagation of light in certain crystals).

15Interestingly, if spacetime was indeed described by a canonical noncommutative geometry, neutrinos would acquire at once both a deformed dispersion relation and the ability to interact, however softly, with electromagnetic media.
lead to such solutions. The range $10^{-42} \, cm^2 < \theta < 10^{-36} \, cm^2$ (i.e. $(10^4 TeV)^2 < \theta < (10TeV)^2$) that is favoured by our very preliminary analysis certainly deserves more careful investigation, since one might find there the first ever manifestation of a non-classical property of space-time.

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