Determining hybrid content of heavy quarkonia using lattice nonrelativistic QCD

Tommy Burch, Kostas Orginos*, and Doug Toussaint

aDepartment of Physics, University of Arizona,
Tucson, AZ 85721

Using lowest-order lattice NRQCD to create heavy meson propagators and applying the spin-dependent interaction, \(c_B \frac{\vec{\sigma} \cdot \vec{B}}{2m_q}\), at varying intermediate time slices, we compute the off-diagonal matrix element of the Hamiltonian for the quarkonium-hybrid two-state system. Diagonalizing this two-state Hamiltonian, the admixture of hybrid (|Q\bar{Q}g\rangle) in the ground state is found. We present results from a set of quenched lattices with an interpolation in quark mass to match the bottomonium spectrum.

1. INTRODUCTION

While experimental confirmation of the existence of hybrid states – hadrons with a gluonic excitation (qqg, qqqg, etc.) – remains elusive, much has been done theoretically, both on and off the lattice, to determine possible consequences of their existence. In particular, mixing of (non-exotic) hybrids with normal hadronic states (with the same \(J^{PC}\)) is expected to occur, necessitating a redefinition of the true hadronic ground state in terms of its constituents: e.g.,

\[
|\Upsilon\rangle = A_S|b\bar{b}\rangle + A_H|b\bar{b}g\rangle + \ldots
\]  

(1)

We offer the bottomonium example for a reason: it is this system which we study with our simulations.

Since this system is composed of relatively heavy quarks, we use the nonrelativistic approximation to QCD (NRQCD) [1] when we perform the lattice simulations. NRQCD has been used before to study heavy hybrids [2]. The main difference in the current study is the fact that we apply the lowest-order (in \(\frac{1}{m_q}\)) spin-dependent interaction (\(c_B \frac{\vec{\sigma} \cdot \vec{B}}{2m_q}\)) “perturbatively,” i.e., at a single time slice. Then, using different source and sink operators (quarkonium \(\rightarrow\) hybrid, and vice versa), we are able to extract the off-diagonal matrix element of the Hamiltonian for this quarkonium-hybrid two-state system. We then diagonalize this new Hamiltonian and find the admixture of hybrid within the true ground state [3].

2. METHOD

In our implementation of NRQCD, we use a time-step-symmetric form of the heavy quark evolution operator [1]:

\[
G(\vec{x}, t + a) = \left( 1 - \frac{aH_0}{2n} \right)^n U_t^{\dagger}(x) \left( 1 - \frac{aH_0}{2n} \right)^n \times (1 - \delta_{t',t}a\delta H) G(\vec{x}, t),
\]  

(2)

with a value of \(n = 2\) (more than sufficient for stability, \(n > \frac{3}{2m_q}\)). For simplicity, we use only the lowest-order term of the heavy quark expansion in the diagonal part of our Hamiltonian:

\[
H_0 = -\vec{D}^2 \frac{2m_q}{2m_q},
\]  

(3)

where \(\vec{D}\) is the covariant derivative. The spin-dependent term appears in the interaction

\[
\delta H = c_B \frac{-g}{2m_q} \vec{\sigma} \cdot \vec{B},
\]  

(4)

which is applied only at a single, intermediate time slice, \(t'\).

For our lattice mesons, we use an incoherent sum of point sources: at the source end, we
as the first exponential in this fit can be expressed causing the mixing between these configurations, mixed propagators and fitting this correlator in knowing the amplitudes and masses from the un-

\[
C(t) = A_{0\text{e}} e^{-m_{q,t}} + A_{1\text{e}} e^{-m_{q,t}}. 
\]

For the “mixed” propagators, the source and sink operators differ (quarkonium \(\rightarrow\) hybrid, and vice versa). These are also fit with a two-exponential form. However, since the source and sink operators are different, and since there is the single interaction \(\delta H\) at the intermediate time slice \(t'\) causing the mixing between these configurations, the first exponential in this fit can be expressed as

\[
C_{\text{mix}}(t) = A_{0\text{e} \text{src}} A_{0 \text{e} \text{snk}}^{1/2} \left( \frac{1}{1H} \left[ c_B \frac{-q}{2m_q} \cdot \vec{B} \right] 1S \right) 
\times e^{-m_{q,\text{src}} t'} e^{-m_{q,\text{snk}} (t-t')} + \ldots . 
\]

Knowing the amplitudes and masses from the unmixed propagators and fitting this correlator in the region \(t > t'\), we extract the off-diagonal matrix element. We then repeat this calculation for larger and larger values of \(t'\) in search of a plateau, indicating the decay of excited-state contributions (e.g., from \(2S\) or \(2H\), depending on the source) to the mixed amplitude.

3. RESULTS

The heavy meson propagators are evaluated on a set of quenched, improved lattices (100 configurations with Symanzik 1-loop-improved gauge action, \(20^3 \times 64, \beta = 8.0;\) see Ref. [4]). The lattice spacing is determined using the spin-averaged 1S-1P mass splitting for bottomonium; \(a^{-1} = (440\text{ MeV})/(a \Delta M_{SP}) = 1590(30)\text{ MeV}\). We also construct non-zero-momentum \(1^-\) S-wave propagators and use the resulting dispersion relation to determine the kinetic mass for this meson. This kinetic mass is calculated for two input values of the quark mass \((m_qa = 2.5\text{ and }2.8)\) and an interpolation is then performed to match this meson mass to the experimental value for the mass of the \(\Upsilon\) (9.46 GeV). We thus arrive at a (lattice-

\[
\begin{array}{c|c}
\text{Operator} & \text{Meson operators.} \\
\hline
\bar{Q} & Q \bar{Q} \\
\bar{Q}\sigma_i D_i Q & Q\sigma_i D_i Q \\
\bar{Q}\epsilon_{ijk}\sigma_j D_k Q & Q\epsilon_{ijk}\sigma_j D_k Q \\
\bar{Q}(\sigma_i D_j + \sigma_j D_i - \frac{2}{3}\delta_{ij}\sigma_k D_k)Q & Q(\sigma_i D_j + \sigma_j D_i - \frac{2}{3}\delta_{ij}\sigma_k D_k)Q \\
\bar{Q}\sigma_i B_i Q & Q\sigma_i B_i Q \\
\bar{Q}B_i Q & QB_i Q \\
\end{array}
\]

start with a given quark color and spin at all spatial points, without fixing the gauge; at the sink end, we sum over all the contributions where the quark and anti-quark are at the same spatial point. Since the lattices are not gauge-fixed, we expect the contributions from sources with the quark and anti-quark at different spatial points to average to zero. We combine the quark and anti-quark sources (propagators) with the appropriate spin matrices and gauge fields to construct the meson operators at the source (and sink) time slices. The meson operators we use are displayed in Table 1.

Using identical source and sink operators, “un-

\[
|\Upsilon\rangle = 0.99837(6) |1S(1^-)\rangle \\
- 0.057(1) |1H(1^-)\rangle, 
\]

\[
|\eta_b\rangle = 0.9953(1) |1S(0^+)\rangle \\
+ 0.097(2) |1H(0^+)\rangle, 
\]

with the tree-level value \(c_B = 1\).

In an attempt to determine \(c_B\) nonperturbatively, we perform additional runs with the interaction, \(\delta H\), applied at all intermediate time slices...
and with the values of $c_B = 1$ and 2. We then use the $\Upsilon - \eta_b$ mass difference (a quantity having a quadratic dependence upon the $\vec{\sigma} \cdot \vec{B}$ term) to set the value of $c_B^2$. Unfortunately, as there is no experimental value for the mass of the $\eta_b$, we rely on potential model results [5] which put this mass difference in the $30 - 60$ MeV range. Combining this with our lattice results for the mass difference $\Delta M_{\Upsilon - \eta_b} = 18.4(4)$ and $71.3(2.1)$ MeV for $c_B = 1$ and 2, respectively – suggests $c_B^2 \sim 1.7 - 3.4$. The resulting probability admixtures of hybrids within the bottomonium ground states thus become

$$|\langle 1H|T\rangle|^2 \sim 0.0054 - 0.011, \quad (9)$$

$$|\langle 1H|\eta_b\rangle|^2 \sim 0.016 - 0.032. \quad (10)$$

Obviously, the uncertainty is dominated by the lack of a reliable value for the $\Upsilon - \eta_b$ mass difference. Note the factor of $\sim 3$ enhancement for the mixing in the $0^{-+}$ channel, due to spin statistics [6]. Interested readers can find a more detailed discussion of the results for the $1^{--}$ channel in Ref. [3]. The simulations are to be repeated on lattices with a different value of the coupling in order to attempt a continuum extrapolation.

REFERENCES

6. T. Barnes, private communication.