Luminosity and Variability of Collimated Gamma-ray Bursts

Shiho Kobayashi\textsuperscript{1}, Felix Ryde\textsuperscript{2} and Andrew MacFadyen\textsuperscript{3}

\textsuperscript{1}Department of Earth and Space Science, Osaka University, Toyonaka, Osaka 560, Japan
\textsuperscript{2}Center for Space Science and Astrophysics, Stanford University, Stanford, CA 94305, USA
\textsuperscript{3}Astronomy Department, University of California, Santa Cruz, CA 95064, USA

ABSTRACT

Within the framework of the internal shock model, we study the luminosity and the variability in gamma-ray bursts from collimated fireballs. In particular we pay attention to the role of the photosphere due to $e^\pm$ pairs produced by internal shock synchrotron photons. It is shown that the observed Cepheid-like relationship between the luminosity and the variability can be interpreted as a correlation between the opening angle of the fireball jet and the mass included at the explosion with a standard energy output. We also show that such a correlation can be a natural consequence of the collapsar model. Using a multiple-shell model, we numerically calculate the temporal profiles of gamma-ray bursts. Collimated jets, in which the typical Lorentz factors are higher than in wide jets, can produce more variable temporal profiles due to smaller angular spreading time scales at the photosphere radius. Our simulations quantitatively reproduce the observed correlation.

Subject headings: gamma rays: bursts; shock waves

1. Introduction

Gamma-ray bursts (GRBs) and the afterglows are well described by the fireball model (e.g. see Piran 2000), in which an explosive flow of relativistic matter (ejecta) is released from a central engine. The collision of fast-moving ejecta with slower ejecta results in a GRB. Thereafter, the ejecta shocks and sweeps up a large amount of ambient matter. The shocked matter powers the long-lived afterglow.

The temporal profiles of GRBs are often very variable and each profile looks very different. Quantitative measures of the variability have been suggested allowing for a systematic study of their morphology. Several studies have explored the possibility that quantities directly measurable in GRB light curves could be related to the luminosities
of the bursts. Stern, Poutanen & Svensson (1999) concluded that there is an intrinsic correlation between luminosity and the complexity of GRBs. Fenimore & Ramirez-Ruiz (1999,2000) found that the luminosities of seven bursts with known redshifts are correlated with the variabilities. Based on this work, Reichart et al. (2001) also reported a possible Cepheid-like luminosity estimator for long bursts.

Recent afterglow observations allow us to determine the geometry of ejecta, whether it is spherical or conical. There is excellent observational evidence for ejecta having conical geometry (jet), and there appears to be a correlation in the sense that the bursts with the largest gamma-ray fluences have the narrowest opening angles. The correlation is improved when the fluences are all scaled to the same distance by using the redshift measurements. Frail et al. (2001) reported that the gamma-ray energy releases, corrected for geometry, are narrowly clustered around $10^{51}$ erg, and suggested that the wide variation in fluences and luminosities of GRBs is due entirely to a distribution of the opening angles. If this is the case, the Cepheid-like relation implies a correlation between the opening angle and the variability: variable bursts are radiated from highly collimated jets. Such a correlation is actually found in the observations (see Figure 1).

As a result of relativistic beaming, an observer can see only a limited portion of the ejecta. There should be no observable distinction between a spherical ejecta and a conical ejecta until the ejecta has slowed down in the afterglow phase. Therefore, the correlation between variability and opening angle should be attributed to the properties of the central engine. In this Letter, we will explore the relations among variability, luminosity and jet opening angle in the framework of the internal shock model. We show that the variability-opening angle relation and the Cepheid-like relation can be interpreted as a correlation between the opening angle and the mass included at the explosion. In §2 we first discuss the collapsar model as the central engine of GRBs. In §3 we make some comments on the typical GRB energy. In §4 we discuss the internal shocks and the temporal profile. Numerical results are compared with the observations. In §5 we give conclusions.

## 2. Central Engine: Collapsar

While the nature of the GRB progenitors is still unsettled, it now appears likely that at least some bursts originate in explosions of very massive stars (main sequence mass $M_{\text{ms}} > 25 M_\odot$). The model currently favored for long bursts is the collapsar model in which GRBs are caused by relativistic jets expelled along the rotation axes of collapsing massive stars. The jets are powered by a black hole with a surrounding accreting torus. Energy from the accretion is pumped into jets via electrodynamic processes or by neutrino
annihilation. In addition spin energy of the rotating black hole may be tapped by magnetic fields anchored in the accretion disk.

According to Frail et al. (2000) and Piran et al. (2001), the wide variation in the fluences of GRBs originates from the differences in opening angles of the jets. A wide jet radiates gamma-ray photons into a large solid angle, resulting in a dim burst. Though we do not know yet what physical mechanism results in the wide variation of the opening angles, it is likely that a wide jet involves a large mass $M$ at the explosion, and that it results in a flow with a lower Lorentz factor $\Gamma \sim E/M \propto \theta^{-2}$.

In the collapsar model, the duration of a GRB is set by the collapse timescale of a massive stellar core, typically $\sim 10$ sec for helium cores of $8 < M_\alpha < 15$. The progenitor has a mass $> 25 M_\odot$ on the main sequence but loses its envelope to a binary companion and/or stellar wind before core collapse. The stellar core is assumed to collapse to a black hole, due to its large mass, and to be rotating sufficiently rapidly to form an accretion disk around the newly born black hole (Woosley 1993; MacFadyen & Woosley 1999).

Typical accretion disk timescales are milliseconds and gas typically resides briefly in the accretion disk. The key point is that the disk is continuously fed by the collapsing star. Short time scales are available and indeed expected due both to variability in accretion rate and the jet instabilities as the hot jet material expands along the polar axis of the star and interacts with the surrounding star.

For disks forming with radius $\sim 10^7$ cm, typical accretion timescales are $t_{\text{acc}} \sim 0.01$ sec. Fluctuations in accretion rate due to instabilities were shown to exist with 50 msec timescales (MacFadyen & Woosley 1999). In models where accretion energy is tapped to power relativistic polar jets, these fluctuations in accretion may translate into variation in jet Lorentz factor perhaps leading to variation in the GRB light curves. However time scales calculated so far in numerical simulations are probably too short to produce the observed variations.

A more promising source of time structure in GRBs are instabilities arising as a relativistic jet propagates through the stellar mantle. Instabilities in the flow due to shear between the jet and the star or backflow from the jet head result in fluctuating jet speeds. Recent numerical simulations of relativistic jets propagation through collapsars demonstrate the presence of the variability some with timescales of $\sim 0.1$ sec (Zhang, Woosley & MacFadyen).

MacFadyen, Woosley & Heger, 2001 (hereafter MWH) showed that jets of equivalent energy injected into a stellar envelope can be focussed by the pressure of the star. The degree of focusing is a function of the assumed entropy of the jet which was parametrized
by $E_{\text{int}}/E_{\text{tot}}$. “Colder” jets were squeezed into tightly collimated flow while “hotter” jets of the same energy expanded sideways (See MWH Fig 10). The result was that “hotter” jets are broader and sweep up a larger mass of stellar material along the rotation axis of the star. While the calculations in MWH were non-relativistic, the results should hold in special relativity since the asymptotic lorentz factor is an inverse function of the swept up mass. Current fully relativistic numerical simulations indicate the the results do hold (Zhang, Woosley & MacFadyen).

MWH also experimented with the effect of the collapsar density structure on jet collimation and found a large difference in jet opening angle for two density structures corresponding to low and high disk mass. The difference was due to the assumed viscosity parameter, but may also result from initial angular momentum and density structure of the progenitor stars. Calculations of the collapsar density structures and their effect on relativistic jet opening angle are currently underway.

3. Typical GRB energy

In the collapsar model, the energy powering the GRB originates is an accretion torus surrounding a rotating black hole of several solar masses. The stellar progenitor must lose its envelope in order to have a sufficiently small radius to allow a relativistic jet formed near the accreting black hole to pierce the star in a GRB timescale i.e. $R_* < 10\text{sec} \ (c/2) \approx 1.5 \times 10^{11}$ cm. In addition the range of angular momentum conducive to GRB formation may be narrow. It is constrained on the low end by the angular momentum sufficient to form a disk and on the upper end, so that the disk is small enough to cool at least partially by neutrino emission (though these neutrino need not participate directly in jet formation). Disks which are inefficiently cooled are convective leading to outflows and inefficient accretion (MacFadyen & Woosley 1999; Narayan et. al). The successful GRB-producing collapsars may therefore be expected to have roughly similar energetics.

Another requirement for GRB production is that the jets have sufficient momentum flux to overcome the ram pressure of the column of gas accreting along the stellar rotation axis. Since the momentum flux $\sim \theta^{-2}$ for a given injection rate, there is a maximum opening angle which will not be swamped by the accretion ram. Equivalently for a given $\theta$ there is a minimum momentum flux capable of launching a jet. Give an observed maximum jet opening angle, this sets a lower limit for energy flux capable of producing a GRB. Given a similar typical timescale this implies a similar total energy.
4. Internal Shocks and the Temporal Structure

Internal shocks arise in a relativistic wind with a nonuniform velocity when the fast moving flow catches up the slower one. The wind can be modeled by a succession of relativistic shells (Kobayashi, Piran & Sari 1997, hereafter KPS). A collision of two shells is the elementary process, and produces a single pulse of gamma-ray. Three time scales, the cooling time, the hydrodynamic time and the angular spreading time, are relevant to the pulse width. With the relevant parameters the cooling time is negligible compared to the other two time scales (Sari, Narayan & Piran 1996). Let \( d \) and \( D \) be the width and separation of the shells. Then the hydrodynamic time scale \( \sim d/c \) and the angular spreading time scale \( \sim D/c \) determine the rise and the decay time of the pulse, respectively. Since most observed pulses rise more quickly than they decay, the pulse width is mainly determined by the angular spreading time \( \sim D/c \) (Norris et al. 1996; Ryde & Petrosian 2001).

The whole light curve of a GRB is given by the superposition of the resulting pulses from each collision. Since all shells are moving towards us with almost speed of light, it is possible to show that we observe pulses arising from collision between shells mostly according to their positions inside the wind, i.e., according to the time when those shells were emitted by the central engine (KPS; Nakar & Piran 2001). The relative positions of the thin shells inside the wind are also determined by the separations. Then, the variability time scales in a temporal profile reflect well the shell separations at the central engine, provided that we can observe all collisions.

However, it has recently been shown that the Thomson optical depth due to \( e^\pm \) pairs produced by synchrotron photons plays an important role in the internal shock model (Guetta, Spada & Waxman 2001; Asano & Kobayashi 2001). In order to obtain high radiative efficiency and the characteristic clustering of spectral break energies of GRBs in the range 0.1-1 MeV, the collision radii are required to be similar to the photosphere radius. Since collisions producing narrow pulses occur at small radii \( \propto D \), the photosphere might obscure these, leaving only the wider pulses visible. This will make the temporal profile smooth. In a wider jet, the typical Lorentz factor \( \Gamma \) is smaller, a larger fraction of the collisions occur at small radii \( \propto \Gamma^2 \) below the photosphere, therefore, the smoothing effect is expected to be stronger.
4.1. The Multiple-Shell Model

We here discuss the smoothing effect by using a multiple-shell model. We represent the irregular wind by \( N \) relativistic shells in a manner similar to that in KPS. Because of the relativistic beaming effect, we can study the emission from a jet by using the spherical shells. We assign an index \( i (i = 1, N) \) to each shell according to the order of the emission from the central engine. Each shell is characterized by four variables: Lorentz factor \( \Gamma_i \), mass \( m_i \), width \( d_i \), and the distance to the outer neighbor shell \( D_i \). We assume that the Lorentz factors and the separations are distributed uniformly in logarithmic spaces; between \( \Gamma_{\min} \) and \( \Gamma_{\max} \) and between \( D_{\min} \) and \( D_{\max} \), respectively. The width is assumed to be a constant value \( d = D_{\min} \). Since the internal shock process is very efficient only when the central engine produces shells with comparable masses (Kobayashi and Sari 2001), we consider this equal mass case. The mass is determined by the isotropic explosion energy \( E_{\text{iso}} \) as \( m = E_{\text{iso}} / \sum \Gamma_i c^2 \). Assuming the correlation between the Lorentz factor and the mass involved in the jet: \( \Gamma_{\max} = a \times \theta^{-2} \), the isotropic explosion energy itself is related to the geometrically corrected explosion energy, \( E_{\text{iso}} = 2a^{-1} \Gamma_{\max} E \).

Consider a two shell collision. A rapid shell with \( \Gamma_r \) catches up to a slower one with \( \Gamma_s \) and the two merge to temporarily form a single shell. Using conservation of energy and momentum we calculate the Lorentz factor of the merged shell to be \( \Gamma_m \sim \sqrt{\Gamma_r \Gamma_s} \). The internal energy of the merged shell is the difference of kinetic energy before and after the collision, \( E_{\text{int}} \sim mc^2(\Gamma_r + \Gamma_s - 2\Gamma_m) \). A fraction \( \epsilon_e \) of this internal energy goes into the electrons, and it is radiated via synchrotron emission. If \( \epsilon_e < 1 \), the merger stays hot after the emission. As a result, the merger will spread, transforming the remaining internal energy back to kinetic energy. A simplified description of this process is to assume that the two shells reflect with a smaller relative velocity (Kobayashi & Sari 2001).

The synchrotron photons can be scattered by electrons within the merger. The Thomson optical depth is increased significantly when taking into account \( e^\pm \) pairs produced by internal shocks (Guetta, Spada & Waxman 2001; Asano & Kobayashi 2001). The typical energy of the synchrotron emission, in the shell frame, is well below the pair production threshold. However, since the photon spectrum extends to high energy as a power law with spectral index \( \sim -1 \), there exists an equal energy of photons per logarithmic energy intervals, and there may exist a large number of photons beyond the threshold. The pairs produced by these photons could contribute significantly to the Thomson optical depth. The photosphere radius \( R_{\pm} \), where the Thomson optical depth becomes unity, can be estimated by assuming that a significant fraction, \( \sim 1/2 \), of the radiative energy \( \epsilon_e E_{\text{int}} \) is converted to pairs. Since the number density of the pairs is given
by \( n_\pm \sim \epsilon_e E_{int}/8\pi m_e c^2 R^2 \Gamma_m^2 d \), the photosphere radius is

\[
R_\pm \sim \left( \frac{\sigma_T}{4\pi m_e c^2} \frac{\epsilon_e E_{iso}}{N\Gamma_{max}} \right)^{1/2} \left( \frac{\Gamma_r}{\Gamma_s} \right)^{1/4} \sim 10^{14} \text{cm} \quad \epsilon_e^{-1/2} (E_{iso,54}/N_2)^{1/2} (\Gamma_{max,3} \Gamma_{min,2})^{-1/4}
\]

(1)

where \( f = 10^x f \), \( E_{iso} \) is in units of erg and we have assumed \( \Gamma_r \sim \Gamma_{max} \) and \( \Gamma_s \sim \Gamma_{min} \).

If a collision happens below the photosphere, the whole internal energy produced by the collision is converted to kinetic energy again via the shell spreading. A simplified description of this process is to assume that the two shells reflect with the same relative velocity.

Numerous collisions happen during the evolution of the multiple-shells. Each collision produces a pulse. However, the main pulses are produced by collisions between the fastest shells and the slowest shells at \( R \sim \Gamma_{min}^2 D_i \), for which the photosphere radii take almost the same value. Then, we can define two characteristic collision radii, \( \Gamma_{min}^2 D_{min} \) and \( \Gamma_{min}^2 D_{max} \), and a typical photosphere radius \( R_\pm \). If \( R_\pm > \Gamma_{min}^2 D_{max} \), the first collisions for most shells occur below the photosphere, but the shells reflect each other with the same velocity. Then, it is still possible to produce bright pulses when the shells propagate beyond the photosphere and collide into other shells. However, if an enormous number of collisions and reflections happen below the photosphere, the shells are ordered with increasing values of the Lorentz factors and no collision happens any more. Especially, in order to produce bright pulses, the shells need to come out from the photosphere before the slowest shells are sorted out at the tail of the wind with a width \( \Delta \sim N D_{max}/\log(D_{max}/D_{min}) \). Using this sorting radius \( \sim \Gamma_{min}^2 \Delta \) and the minimum collision radius \( \Gamma_{min}^2 D_{min} \), GRBs are classified into the following three cases.

(1) Collimated Jet Case: \( \Gamma_{min} > (R_\pm/D_{min})^{1/2} \). Since all collisions occur above the photosphere, the variability should be less dependent on \( \Gamma_{min} \) and only on the distribution of the separations at the central engine.

(2) Intermediate Jet Case: \( (R_\pm/\Delta)^{1/2} < \Gamma_{min} < (R_\pm/D_{min})^{1/2} \). Some collisions occur above the photosphere to produce bright pulses wider than \( \sim R_\pm/c \Gamma_{min}^2 \). The variability of the temporal profile should highly depend on \( \Gamma_{min} \).

(3) Wide Jet Case: \( \Gamma_{min} < (R_\pm/\Delta)^{1/2} \). The shells are almost ordered with increasing values of the Lorentz factors within the photosphere, only minor collisions happen. The resulting bursts are very dim and it could be difficult to detect.

### 4.2. Numerical Results

The gamma-ray energy released from a GRB is narrowly clustered around \( 10^{51} \text{erg} \) (Frail et al. 2001), and the conversion efficiency from the explosion energy into the gamma-rays is
about 10% (Guetta, Spada & Waxman 2001; Kobayashi & Sari 2001). Then, the standard explosion energy is \( \sim 10^{52} \text{erg} \). The observed wide jets with \( \theta \sim 0.2 - 0.3 \) also should have ultra-relativistic Lorentz factors to be optically thin to high energy photons, assuming \( \Gamma_{\text{max}} = 100 (\theta/0.2)^{-2} \), the isotropic explosion energy is given by \( E_{\text{iso}} \sim 10^{54} (\Gamma_{\text{max}}/200) \text{erg} \).

The numerical temporal structures are plotted in Figure 2a-c for a model with different minimum Lorentz factors \( \Gamma_{\text{min}} = 20, 50 \) or 200. As we expected, the temporal profile is smooth for the low Lorentz factor as in Figure 2a, and more variable for higher Lorentz factor as in Figure 2b and 2c. We have assumed \( N = 100, \Gamma_{\text{max}}/\Gamma_{\text{min}} = 10, D_{\text{min}}/c = 1 \text{msec}, D_{\text{max}}/c = 1 \text{second} \) and \( \epsilon_e = 0.1 \). Though we evaluate the photosphere radius for each collision in numerical simulations, the parameter dependence is weak especially for the main pulses. The typical photosphere radius is about \( R_{\pm} \sim 10^{14} \text{cm} \), with which the characteristic Lorentz factors are \( (R_{\pm}/\Delta)^{1/2} \sim 10 \) and \( (R_{\pm}/D_{\text{min}})^{1/2} \sim 10^3 \).

The definition of variability by Fenimore & Ramirez-Ruiz (2000) and that by Reichart et al. (2001) are slightly different, but both relate the variability to the square of the time history after removing low frequencies by smoothing. To evaluate the variability of numerical light curves, we use a simplified version of the Reichart et al. (2001) variability,

\[
V = \frac{\sum (C_i - \langle C \rangle)^2}{\sum C_i^2} \tag{2}
\]

where \( C_i \) is the photon count at time \( t_i \) with 64 msec resolution (the BATSE resolution), and \( \langle C \rangle \) is the count smoothed with a boxcar window with a time scale equal to the smallest fraction of the burst time history that contains a fraction \( f \) of the total counts. Reichart et al. (2001) found that \( f = 45\% \) gives a robust definition of variability. Using the formula (2) with \( f = 0.45 \), we evaluated the variability measures as \( V = 0.04, 0.17 \) and 0.32 for Figure 2a, 2b and 2c, respectively.

Reichert et al. (2001) found that the isotropic peak luminosities \( L \) are correlated with the variability measures, \( L \propto V^{3.3 \pm 1.1} \). We now show that such a correlation exists in our numerical simulations also. A numerical temporal profile depends on the specific realizations of random Lorentz factors and random separations that are assigned to each shell, as well as on the model parameters. For a given \( \Gamma_{\text{min}} \) and the same model parameters as in Figure 2, we calculate the temporal profiles for 100 realizations, and evaluate the mean isotropic peak luminosity and the mean variability measure. Here the peak luminosity is calculated with 1 sec resolution as Reichart et al. (2001) had analyzed. In Figure 3, the solid line shows the mean values, while the dashed lines depict the 1\( \sigma \) error. We plot the opening angle-variability relation also in Figure 1. The numerical results reasonably fit the observational data. Since the numerical variability are calculated with \( \Delta t = 64 \text{msec} \) bins to compare with the BATSE data, the mean variability reaches an asymptotic value around
\[ \Gamma_{\min} \sim (R/c \Delta t)^{1/2} \sim 200. \] The breaks around \( V \sim 0.3 \) in Figure 1 and Figure 3 are due to this low time resolution. If \( \Gamma_{\min} \) is small, the slowest shells are sorted out at the tail before the shells reaches the photosphere, so the luminosity rapidly decreases for \( V < 0.05 \) in Figure 3. In range of \( 0.05 < V < 0.3 \), power law fits to numerical results give \( L \propto V^{2.2} \) and \( \theta \propto V^{-0.65} \). Our numerical results also give a power law similar to ones reported by Fenimore & Ramirez-Ruiz (2000) and by Reichart et al. (2001).

5. Conclusions

We have shown that there exists a correlation between the jet opening angle \( \theta \) and the gamma-ray light curve variability \( V \). Though the correlation is based on only seven events at present and needs to be further confirmed with more events, it is naturally expected if the luminosity \( L \) is correlated with the variability (Fenimore & Ramirez-Ruiz 2000; Reichart et al. 2001), and if GRBs have a standard energy output (Frail et al. 2001; Piran et al. 2001). This correlation might give us a way to measure the opening angle for a long burst directly from the GRB light curve.

We have shown that the \( \theta - V \) relation, or equivalently, due to the constancy of burst energy, the \( L - V \) relation can be interpreted as the correlation between the opening angle of a fireball jet and the included mass at the explosion. We also show that such a correlation can be a natural consequence of the collapsar model. Using a multiple-shell model, we numerically calculate the temporal profile and estimate the luminosity and the variability. Our simulations quantitatively reproduce the observed correlation. We obtain \( L \propto V^{2.2} \) for jets with moderate opening angles.

If the typical Lorentz factor of a jet is large enough, internal shocks could happen well above the photosphere, the produced internal energy density \( \propto R^{-2} \) becomes very low. Since the magnetic energy density behind the shock is usually assumed to be a constant fraction of the shock energy, the typical synchrotron energy \( E_p \) also becomes very low. However, if the baryon load in the jet is extremely low at the central engine, the jet becomes optically thin during the acceleration phase and the Lorentz factor saturates around a few thousand (Mészáros & Rees 2000). Therefore, if the minimum separation \( D_{\min}/c \) is smaller than \( \sim 1 \) msec, the “Collimated Jet Case” might not happen. It assures the clustering of the spectral break energies of GRB in the range 0.1-1 MeV (Guetta, Spada & Waxman 2001; Asano & Kobayashi 2001). On the other hand, if the typical Lorentz factor is small enough, the shells are ordered within the photosphere, only minor collisions happen. The resulting dim burst is smooth and has a soft spectrum. GRB980425 and recently reported Fast X-ray Transients (Heise et al. 2001; Kippen et al 2001) could be classified into this
Norris, Marani & Bonnell (2000) found that the isotropic peak luminosity is inversely proportional to the spectral lag which is defined as the time lag of the peak luminosity between different energy bands. The detailed study on the peak lag requires to discuss the spectrum of the internal shock emission and it is beyond the scope of this Letter. However, if the time lag is also determined by the angular spreading time scale, which is the key time scale in our model, the relation can be derived qualitatively. In our model the isotropic peak luminosity is scaled by the opening angle $L \propto \theta^{-2}$, while the angular spreading time is related to the opening angle as $t_{\text{ang}} \propto \Gamma^{-2} \propto \theta^4$. Then we obtain $L \propto t_{\text{ang}}^{-1/2}$.

We would like to thank the Aspen Center for Physics for their hospitality and for providing a pleasant working environment where this work was initiated. We would also like to thank Nicole M. Lloyd-Ronning for useful discussions. S.K. acknowledges support from the Japan Society for the Promotion of Science. F.R. acknowledges support from the Swedish Foundation for International Cooperation in Research and Higher Education (STINT). A.M. acknowledges support from DOE ASCI (B347885).
References

Fig. 1.— Variability vs. Opening Angle: Measurements (crosses) and upper or lower limits (squares) are shown from Reichart et al (2001) and Frail et al (2001). Numerical result (solid) with 1 $\sigma$ error bars of 100 random simulations (dashed).
Fig. 2.— Temporal structures for different Lorentz factors. (a) $\Gamma_{\text{min}} = 20$. (b) $\Gamma_{\text{min}} = 50$. (c) $\Gamma_{\text{min}} = 200$. The dashed lines show the structures smoothed with the boxcar wind. The conversion efficiency from the kinetic energy of shells to radiation are 9%, 15% and 16% for a, b and c, respectively.
Fig. 3.— Variability vs. Peak Luminosity with 1 σ error bars of 100 random simulations. Observational data: measurements (crosses) and upper or lower limits (squares) are shown from Reichart et al (2001) and Frail et al (2001).