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ISSN 1344-3879
RIKEN-AF-NP-377

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November 2000

Description of Gamow-Teller Resonance within Phonon Damping Model

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(November 24, 2000)

Abstract

An approach to the damping of charge-exchange resonances is proposed using the quasiparticle representation of the phonon damping model. The formalism is applied to calculate the strength distribution of the Gamow-Teller resonance (GTR) in ^{90}Nb . The results are found in reasonable agreement with the recent experimental data. It is also demonstrated that the spreading of GTR up to the excitation energy of 60–70 MeV is caused by two factors, the presence of high-lying two-quasiparticle configurations to which the GTR is coupled, and the strength of this coupling. The effect of the latter on the GTR spreading is shown to be stronger than the one of the former.

PACS numbers: 24.30.Cz, 25.40.Kv

Key words: Gamow-Teller resonance, quenching, charge-exchange reactions

The Gamow-Teller resonance (GTR) is a charge-exchange resonance as it is caused by exchanging neutron with proton in a nucleus. The total transition strength of GTR is given by the model-independent sum rule (GTR-SR), $S_{\beta_-} - S_{\beta_+} = 3(N - Z)$, which should be nearly exhausted by β_- transition strength summed over all Gamow-Teller (GT) states in the daughter nucleus $(Z + 1, N - 1)$ formed after the (p,n) reactions. The quenching of GTR had been a problem addressed to the missing of GTR strength for many years as the GTR had been observed only with about 60% of GTR-SR for many nuclei all over the periodic table [1]. Two physically different mechanisms haven been brought forward to resolve the problem of GTR quenching. The first mechanism (A) [2-6], which considers only structureless nucleons, suggests that the coupling of GTR state to more complicated configurations such as $2p2h$ using a tensor force spreads the GTR strength to the region of high-excitation energies E . This leads to a high-lying tail of GTR, which extends up to $E \simeq 50 - 60$ MeV. Until very recently, it was experimentally impossible to extract this continuous and structureless tail from the background. This explains why only about half of GTR was experimentally seen. The second mechanism (B) [7], which involves subnucleonic degrees of freedom, proposes that the $\Delta(1232)$ isobar - neutron-hole admixtures in the wave function attracts a substantial part of GTR strength to the region of Δ -hole states at around 300 MeV of excitation energy, causing the quenching of GTR strength in the low-energy region. In 1997, Wakasa et al. [8] succeeded to extract the high-lying tail of GTR, which extends up to 50 MeV in the excitation energy of the daughter nucleus ^{90}Nb formed after the reaction $^{90}\text{Zr}(p,n)^{90}\text{Nb}$. This result sort of brought the long-time debate to an end as it confirms the validity of mechanism (A) above, leaving little room to mechanism (B). The experimental data of [8] are supported by results of several theoretical studies, which treat the coupling of $1p1h$ GT states with $2p2h$ configurations in a microscopic manner [4-6]. From these theoretical approaches, the one proposed by Dinh Dang et al. [6] is most recent and found in best quantitative

agreement with the entire GTR strength distribution for ^{90}Nb reported in [8].

Recently, the phonon damping model (PDM) has been proposed to study damping of giant dipole resonance (GDR) in highly excited nuclei [9]. The PDM later was applied successfully in the study of multiple phonon resonances such as double GDR [10]. The recent inclusion of the superfluid pairing correlations in the quasiparticle representation of PDM allows one to apply the PDM also to study pygmy dipole resonances in neutron-rich nuclei [11]. The PDM treats the damping of a giant resonance (GR) as the result of coupling of the GR phonon to many noncollective ph configurations (at zero temperature T), while at $T \neq 0$ coupling to pp and hh configurations are also taken into account. This mechanism (at $T = 0$) has been shown to be effectively equivalent to mechanism (A) mentioned above [6].

The aim of the present paper is to test the validity of PDM in a further development of this model, namely, the extension of its quasiparticle representation to the study of charge-exchange resonances. The calculations are carried out for GTR in ^{90}Nb and the results obtained are discussed in comparison with the experimental data of [8].

We use the quasiparticle representation of the PDM Hamiltonian in the form [11]

$$H = \sum_{jm} \epsilon_j \alpha_{jm}^\dagger \alpha_{jm} + \sum_{\lambda\mu i} \omega_{\lambda i} b_{\lambda\mu i}^\dagger b_{\lambda\mu i} + \frac{1}{2} \sum_{\lambda\mu i} \frac{(-)^{\lambda-\mu}}{\hat{\lambda}} \sum_{jj'} f_{jj'}^{(\lambda)} \times \\ \left\{ u_{jj'}^{(+)} \left[A_{jj'}^\dagger(\lambda\mu) + A_{jj'}(\lambda\tilde{\mu}) \right] + v_{jj'}^{(-)} \left[B_{jj'}^\dagger(\lambda\mu) + B_{jj'}(\lambda\tilde{\mu}) \right] \right\} \left(b_{\lambda\mu i}^\dagger + b_{\lambda\tilde{\mu} i} \right), \quad (1)$$

where $\hat{\lambda} = \sqrt{2\lambda+1}$. The first term at the right-hand side (RHS) of Hamiltonian (1) corresponds to the independent-quasiparticle field. The second term stands for the phonon field described by phonon operators, $b_{\lambda\mu i}^\dagger$ and $b_{\lambda\mu i}$, with multipolarity λ , which generate the harmonic collective vibrations such as GR. Phonons are ideal bosons within PDM, i.e. they have no fermion structure. The last term is the coupling between quasiparticle and phonon fields, which is responsible for the microscopic

damping of collective excitations. In Eq. (1) the following standard notations are used

$$A_{jj'}^\dagger(\lambda\mu) = \sum_{mm'} \langle jmj'm' | \lambda\mu \rangle \alpha_{jm}^\dagger \alpha_{j'm'}^\dagger, \quad (2)$$

$$B_{jj'}^\dagger(\lambda\mu) = - \sum_{mm'} \langle jmj'm' | \lambda\mu \rangle \alpha_{jm}^\dagger \alpha_{j'm'}, \quad (3)$$

with $(\lambda\tilde{\mu}) \longleftrightarrow (-)^{\lambda-\mu}(\lambda-\mu)$. Functions $u_{jj'}^{(+)} \equiv u_j v_{j'} + v_j u_{j'}$ and $v_{jj'}^{(-)} \equiv u_j u_{j'} - v_j v_{j'}$ are combinations of Bogolyubov's u and v coefficients. The quasiparticle energy ϵ_j is calculated from the single-particle energy E_j as

$$\epsilon_j = \sqrt{(\tilde{E}_j - E_F)^2 + \Delta^2}, \quad \tilde{E}_j \equiv E_j - Gv_j^2, \quad (4)$$

where the pairing gap Δ and the Fermi energy E_F are defined as solutions of the BCS equations. The operators (2) and (3), and their Hermitian conjugates satisfy the following commutation relations within the ensemble average

$$\left\langle [A_{jj'}(\lambda\mu), A_{j_1j_1'}^\dagger(\lambda'\mu')] \right\rangle = (1 - n_j - n_{j'}) \delta_{\lambda\lambda'} \delta_{\mu\mu'} \left[\delta_{j_1j_1'} \delta_{j'j_1} + (-)^{j'-j+\lambda} \delta_{j_1j_1'} \delta_{j'j_1} \right], \quad (5)$$

$$\left\langle [B_{jj'}(\lambda\mu), B_{j_1j_1'}^\dagger(\lambda'\mu')] \right\rangle = (n_{j'} - n_j) \delta_{\lambda\lambda'} \delta_{\mu\mu'} \delta_{j_1j_1'} \delta_{j'j_1}, \quad (6)$$

where it is assumed that $\langle \alpha_j^\dagger \alpha_{j'} \rangle = 0$ if $j \neq j'$. The quantity $n_j = \langle \alpha_j^\dagger \alpha_j \rangle$ is the quasiparticle occupation number, which is defined in the ensemble average $\langle \dots \rangle \equiv \text{Tr}[\dots \exp(-\beta H)] / \text{Tr} \exp(-\beta H)$ at a given temperature $T = \beta^{-1}$.

The equation for the propagation of a collective phonon, which generates the GR and which is damped due to coupling to quasiparticle field, has been derived explicitly in [11] making use of the double-time Green's function method [12]. Only final equations necessary for numerical application in the present work are quoted below. The equation for the Green's function of phonon propagation $G_{\lambda l}(E)$ has the form

$$G_{\lambda i}(E) = \frac{1}{2\pi} \frac{1}{E - \omega_{\lambda i} - P_{\lambda i}(E)} , \quad (7)$$

where the explicit form of the polarization operator $P_{\lambda i}(E)$ is

$$P_{\lambda i}(E) = \frac{1}{\hat{\lambda}^2} \sum_{jj'} [f_{jj'}^{(\lambda)}]^2 \left[\frac{(u_{jj'}^{(+)})^2 (1 - n_j - n_{j'}) (\epsilon_j + \epsilon_{j'})}{E^2 - (\epsilon_j + \epsilon_{j'})^2} - \frac{(v_{jj'}^{(-)})^2 (n_j - n_{j'}) (\epsilon_j - \epsilon_{j'})}{E^2 - (\epsilon_j - \epsilon_{j'})^2} \right]. \quad (8)$$

The phonon damping $\gamma_{\lambda i}(\omega)$ (ω real) is obtained as the imaginary part of the analytic continuation of the polarization operator $P_{\lambda i}(E)$ into the complex energy plane $E = \omega \pm i\varepsilon$. Its final form is

$$\gamma_{\lambda i}(\omega) = \frac{\pi}{2\hat{\lambda}^2} \sum_{jj'} [f_{jj'}^{(\lambda)}]^2 \left\{ (u_{jj'}^{(+)})^2 (1 - n_j - n_{j'}) [\delta(E - \epsilon_j - \epsilon_{j'}) - \delta(E + \epsilon_j + \epsilon_{j'})] - \right. \\ \left. (v_{jj'}^{(-)})^2 (n_j - n_{j'}) [\delta(E - \epsilon_j + \epsilon_{j'}) - \delta(E + \epsilon_j - \epsilon_{j'})] \right\}. \quad (9)$$

The quasiparticle occupation numbers n_j appear in these equations due to (5) and (6). At $T = 0$, all n_j are zero, so the factor $1 - n_j - n_{j'}$ becomes 1, while the last terms at the RHS of Eqs. (8) and (9), which contain $(v_{jj'}^{(-)})^2$, vanish. The essential step in extending these results to the study of charge-exchange resonances is to consider the coupling of GR phonon to only proton (π) particle - neutron (ν) hole states, which correspond to β_- transition. In the quasiparticle representation above, this is tantamount to the summation over only $(jj') = (j_\pi, j_\nu)$ in Eqs. (8) and (9).

The GR energy $\bar{\omega}$ is found as the pole of the Green's function (7), i.e. as the solution of

$$\bar{\omega} - \omega_{\lambda i} - P_{\lambda i}(\bar{\omega}) = 0 . \quad (10)$$

The GR width Γ_λ is calculated as twice of the damping $\gamma_\lambda(\omega)$ at $\omega = \bar{\omega}$, i.e.

$$\Gamma_\lambda = 2\gamma_\lambda(\bar{\omega}), \quad (11)$$

where $\lambda = 1$ corresponds to the dipole case. The GR strength function $S_{\text{GR}}(\omega)$ is calculated using the analytic properties of the Green function (7) as [9]

$$S_{\text{GR}}(\omega) = \frac{1}{\pi} \frac{\gamma_{\text{GR}}(\omega)}{(\omega - \bar{\omega})^2 + \gamma_{\text{GR}}^2(\omega)} . \quad (12)$$

For GTR, the integrated strength, or the non-energy weighted sum of strength (NEWSS), $\mathcal{S}$ is normalized to the GTR-SR as

$$\mathcal{S} \equiv C_{\text{GT}} \int_0^{E_{\text{max}}} S_{\text{GR}}(\omega) d\omega = 3(N - Z), \quad (13)$$

with the normalization factor C_{GT} , neglecting a small contribution of β_+ process. This factor takes the contribution of the GT-transition operator effectively into account.

We calculated the β_- strength distribution in ^{90}Nb formed after the $^{90}\text{Zr}(\text{p},\text{n})^{90}\text{Nb}$ reaction. The single-particle energies of ^{90}Zr are obtained within the Hartree-Fock method using SGII interaction [13]. The proton spectrum consists of 82 levels within -41 MeV and 58.4 MeV. The neutron spectrum has the same number of levels located between -50.7 MeV and 51.2 MeV. The BCS equation can be solved either by choosing the pairing constant G to find the Fermi energy E_{F} and the gap Δ , or by choosing Δ to find E_{F} and G . We follow the latter, choosing the value of proton-pairing gap $\Delta_{\pi} = 1.3$ MeV in agreement with the trend $\Delta = 12/\sqrt{A}$. The procedure of selecting the PDM parameters has been described in detail in [9]. The same principle holds for GTR, according to which, the unperturbed phonon energy ω_{λ} ($\lambda = 1$) in Eqs. (7) and (10), and the quasiparticle-phonon coupling vertex $f_1 \equiv f_{j_{\pi}j_{\nu}}^{(1)}$ for all (j_{π}, j_{ν}) are chosen so that the GTR energy, found as the solution of Eq. (10), and the GTR width, calculated from Eqs. (9) and (11), reproduce their corresponding experimental values. As no experimental value for the spreading width of the total GTR strength distribution is available, the second criterion of parameters' selection is replaced with obtaining such a GTR width so that the overall GTR line shape is reproduced. This procedure yields the following values for the PDM parameters, $\omega_1 = 15.5$ MeV, and $f_1 = 0.4031$ MeV. The δ -function in Eq. (9) is smoothed using the representation $\delta(x) = [(x - i\varepsilon)^{-1} - (x + i\varepsilon)^{-1}]/(2\pi i)$ with $\varepsilon = 0.3$ MeV.

FIGURES

FIG. 1. Strength function of GTR for ^{90}Nb . Excitation energy E are measured with respect to the ground state of ^{90}Zr . The solid line is the result obtained within PDM. The dotted line represents the result of the microscopic calculations of Ref. [6] using a smearing parameter of 2 MeV. The data points are the experimental results of Ref. [8].

The solid curve in Fig. 1 is the normalized strength function $S(E) \equiv C_{\text{GT}} S_{\text{GT}}(E)$ of GTR for ^{90}Nb , calculated using Eqs. (12) and (13) and plotted as a function of excitation energy E , which is measured from the ground state of ^{90}Zr . The normalization factor C_{GT} is 35.65, with which we get the NEWSS = $3(N - Z) = 30$ at $E_{\text{max}} = 70$ MeV. The experimental strength distribution from [8] (data points) and the result of microscopic calculations of [6] with coupling to $2p2h$ states via two-RPA phonon configurations (dotted curve) are also shown for comparison. This figure shows that the theoretical predictions agree reasonably well with the data within the region of the GTR peak up to $E \simeq 22 - 25$ MeV. In the region beyond the GTR ($E > 25$ MeV), the tail obtained within PDM is more enhanced than the one of microscopic calculations in [6] up to 35 MeV. At $E > 35$ MeV, the results by both of calculations

nearly coincide. We also carried out a test calculation with $\Delta_\pi = 0$. This shifts up the GTR peak by around 1.5 MeV. The strength between 16 and 22 MeV increases slightly. The high-lying tail remains nearly unchanged. However, the low-lying part at $E < 12$ MeV becomes depleted significantly, worsening the agreement with the data. Increasing f_1 by around 20% restores the position of the GTR peak, but the depletion of the strength in the low energy region remains. This shows the importance of pairing within PDM in improving the agreement with the entire experimental GTR distribution. Both of theoretical calculations in the present work and of [6], which exhaust the GTR-SR up to $E = 70$ MeV, underpredict the data in the region of the high-lying tail ($E > 30$ MeV). The experimental NEWSS, however, exceeds the GTR-SR because of the contribution from the isovector-spin monopole resonance and the uncertainty involved in the multipole decomposition analysis used in extracting the data. This is clearly seen in Fig. 2, where the NEWSS in Eq. (13) are displayed as a function of excitation energy E_{max} . The experimental NEWSS reaches the GTR-SR at $E_{\text{max}} \simeq 47$ MeV, and continues to increase at $E_{\text{max}} > 47$ MeV. The theoretical predictions approach the GTR-SR at $E = 70$ MeV. The NEWSS, obtained within PDM in the region up to $E = 22$ MeV, is 65.6% of GTR-SR to be compared with the value 62.5% found in the experimental result in Fig. 2. This agrees well with the fraction of the GTR-SR, observed experimentally before the measurement of [8], and known as the quenched GTR.

FIG. 2. NEWSS as a function of E_{max} . The dashed line is the experimental result of Ref. [8]. The notation for solid and dotted lines is as in Fig. 1.

FIG. 3. Strength function of GTR for ^{90}Nb . The solid line is the PDM result as in Fig. 1. The dashed line is the PDM result obtained using a reduced value of the quasiparticle-phonon coupling parameter f_1 as described in the text. The dotted line is the PDM result obtained when the phonon is coupled to a truncated quasiparticle space, in which the two-quasiparticle energies do not exceed 16 MeV.

In order to see the effect of coupling strength between phonon and two-quasiparticle states, and the role of high-lying two-quasiparticle configurations in the spreading of GTR to high-energy region, we have carried out two test calculations. In the first test, the squared value $(f_1)^2$ of quasiparticle-phonon coupling parameter f_1 has been reduced by 3 times. In the second test, the two-quasiparticle configurations, to which the phonon is coupled, are truncated so that they are not located higher than the GTR peak. This leads to the constraint that the sum of their energies (the two-quasiparticle energy) $\epsilon_j + \epsilon_{j'}$ does not exceed 16 MeV. The strength functions obtained in these tests are shown in Fig. 3 as the dashed (first test) and dotted (second test) lines together with the original PDM result (solid curve from Fig. 1). It is seen that both of the effects of coupling strength and the high-lying

two-quasiparticle configurations contribute to the enhancement of spreading of GTR toward high-energy region. However, the effect of coupling strength is stronger as the result with reduced coupling strength shows much narrower strength distribution with a high-lying tail depleted about twice. Compared to this effect, the change in the GTR strength distribution, caused by the constraint of coupling of phonon to only low-lying two-quasiparticle states, turns out to be much weaker. This result within PDM agrees with the predictions of previous microscopic calculations [4-6] on the important role of tensor force in pushing a substantial part of GTR strength to the high-energy region beyond the GTR.

In conclusion we have extended the PDM in its quasiparticle representation to apply it in the study of charge-exchange resonances. The results of its application in numerical calculations of the GTR strength distribution are found in reasonable agreement with our previous calculations within an explicit microscopic approach [6] as well as the recent experimental data [8]. This shows the validity of PDM in the description of GTR damping. The results obtained in this work also confirms the important role of the coupling strength between GTR phonon and two-quasiparticle configurations. Using a reasonably strong coupling strength, one can spread the GTR within PDM up to excitation energy $E = 60 - 70$ MeV in such a way that the strength localized within the GTR region ($E \leq 22$ MeV) is around 65% of the total GT sum rule, while the rest of strength is found in the region beyond the GTR, but below 60 - 70 MeV.

N.D.D. thanks V. Kim Au for assistance. The numerical calculations have been carried out using the Alpha AXP work-station at the RIKEN computer and information center.

REFERENCES

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- [1] C. Gaarde, Proceedings of the Niels Bohr Centennial Conference on Nuclear Structure, Copenhagen (North-Holland, Amsterdam, 1985) p. 449c.
- [2] K. Shimizu, M. Ichimura, and A. Arima, Nucl. Phys. A **226**, 282 (1974).
- [3] I.S. Towner and F.C. Khanna, Phys. Rev. Lett. **42**, 51 (1979).
- [4] G.F. Bertsch and I. Hamamoto, Phys. Rev. C **26**, 1323 (1982).
- [5] S. Drożdż, V. Klempt, J. Speth, and J. Wambach, Phys. Lett. B **166**, 18 (1986).
- [6] N. Dinh Dang, A. Arima, T. Suzuki, and S. Yamaji, Phys. Rev. Lett. **79**, 1638 (1997); Nucl. Phys. A **621**, 719 (1997).
- [7] M. Ericson, A. Figureau, and C. Thevenet, Phys. Lett. N **45**, 19 (1973); M. Rho, Nucl. Phys. A **231**, 493 (1974); E. Oset and M. Rho, Phys. Rev. Lett. **42**, 51 (1979).
- [8] T. Wakasa et al., Phys. Rev. C **55**, 2909 (1997).
- [9] N. Dinh Dang and A. Arima, Phys. Rev. Lett. **80** (1998) 4145, Nucl. Phys. A **636** (1998) 427; N. Dinh Dang, K. Tanabe, and A. Arima, Phys. Rev. C **58** (1998) 3374, Nucl. Phys. A **645** (1999) 536.
- [10] N. Dinh Dang, V. Kim Au, and A. Arima, Phys. Rev. Lett. **85**, 1827 (2000).
- [11] N. Dinh Dang, V. Kim Au, T. Suzuki, and A. Arima, Preprint RIKEN-AF-NF **374** (2000) 13 pages (submitted).

[12] D.N. Zubarev, Sov. Phys. Uspekhi **3**, 320 (1960).

[13] N. Van Giai and H. Sagawa, Nucl. Phys. A **371**, 1 (1981).