The 2001 Superoutburst of WZ Sagittae: A Clue to the Dynamics of Accretion Disks

John K. Cannizzo

e-mail: cannizzo@stars.gsfc.nasa.gov

NASA/GSFC/Laboratory for High Energy Astrophysics, Emergent Information Technologies, Inc., Code 662, Greenbelt, MD 20771

to appear in the Astrophysical Journal (Letters)

Received 2001 September 7; accepted 2001 October 1
ABSTRACT

We examine the light curve of the July-August 2001 superoutburst of WZ Sagittae. During the decline from maximum light the locally defined decay time increases from $\sim 4 \text{ d mag}^{-1}$ to $\sim 12 \text{ d mag}^{-1}$ over the first $\sim 15 \text{ d}$ of the $\sim 25 \text{ d}$ superoutburst, as the system faded from $m_V \simeq 8.5$ to $m_V \simeq 10$. The superoutburst is caused by the sudden accretion of $\sim 10^{24} \text{ g}$ of gas onto the white dwarf, and the deviation from exponentiality in the decay light curve is expected qualitatively during a “viscous decay” in which the dominant mode of depletion of the gas stored in the accretion disk is accretion onto the central object. In other words, as the mass of the accretion disk decreases, the viscous time scale increases. We show that the data are also quantitatively consistent with the theoretical viscous decay time, both calculated via a simple scaling and also from time dependent calculations, when one adopts standard model parameters for WZ Sge.

Subject headings: accretion, accretion disks – binaries: close – stars: individual (WZ Sge, A0620-00)
1. Introduction

WZ Sagittae is a fascinating source which has been the subject of much research in the field of cataclysmic variables (CVs) — interacting binaries in which a Roche lobe filling K or M dwarf secondary transfers matter onto a white dwarf (WD) primary (Warner 1987, 1995). WZ Sge is a member of the dwarf novae — a subclass of the CVs characterized by semiperiodic outbursts — and furthermore it is the prototype of the “WZ Sagittae” stars which have short orbital periods ($P_{\text{orb}} = 81$ min for WZ Sge — Krzemiński 1962) and only exhibit very infrequent “superoutbursts” that last $\sim 30$ d. Previous superoutbursts from WZ Sge occurred in 1913, 1946, and 1978. The most recent superoutburst began 23 July 2001 and ended 18 Aug 2001. The time since the previous superoutburst (23 yr) is significantly less than the previous two recurrence times (both 33 yr).

Due to the widespread interest in WZ Sge and fortuitous timing of the 2001 superoutburst, a large number of amateur astronomers observed the system, and as a result the outburst light curve is particularly well defined. The obvious deviation from an exponential shape, which is rendered observable due to a combination of both the relatively large dynamic range in $m_V$ and also the long duration of a superoutburst, has implications for the physics of accretion of the gas in the accretion disk onto the WD.

Smak (1993) summarizes previous observational and theoretical work on WZ Sge. He discusses the superoutbursts in WZ Sge in terms of the accretion disk limit cycle model (Meyer & Meyer-Hofmeister 1981, Cannizzo 1993a), and estimates an accretion disk mass needed to power the superoutburst of $\sim 10^{24}$ g. There has been some debate on the quiescent evolution of the accretion disk in the WZ Sge stars. If the matter accumulates in the usual way during quiescence, Smak (1993) calculates that the quiescent value of the Shakura & Sunyaev (1973) alpha parameter $\alpha_{\text{cold}}$ must be smaller by a factor $\sim 10^2 - 10^3$ than for normal dwarf novae to accumulate this much gas. Hameury, Lasota, & Huré
(1997, see also Lasota, Kuulkers, & Charles 1999) present an alternate viewpoint in which outbursts are triggered by a fluctuation in the mass transfer from the secondary star, and the “normal” $\alpha_{\text{cold}}$ value of a few hundredths prevails in quiescence. If the matter is brought over from the secondary in $\sim 1 \text{ d}$, for example, this would imply a temporary mass transfer rate of $\sim 10^{19} \text{ g s}^{-1}$. Meyer-Hofmeister, Meyer, & Liu (1998) present a time dependent model using a small $\alpha_{\text{cold}} \sim 10^{-3}$, plus evaporation of matter from the inner disk to prevent inside-out outbursts. In connection with very evolved systems such as WZ Sge, Meyer & Meyer-Hofmeister (1999) discuss a possible physical mechanism for generating low $\alpha_{\text{cold}}$ in cool accretion disks with little or no partial ionization, which involves a much-reduced efficiency for the magneto-rotational instability for turbulent angular momentum transport in the accretion disk (Balbus & Hawley 1991, 1998, Hawley & Balbus 1991, see also Gammie & Menou 1998, Menou 2000, Balbus & Terquem 2001). In this work we restrict our attention to the outbursting state.

2. Background

In any time dependent physical system where processes which operate over a wide range in time scale mediate the equilibrium level of a given physical quantity — in this instance the mass of the accretion disk — the controlling or governing time scale is that which is the slowest. For our study this is the viscous time at the outer disk edge. Aside from complications such as the precessing, eccentric outer disk which is thought to develop and give rise to superhumps in the light curve (Whitehurst 1988, Hirose & Osaki 1990, Murray & Armitage 1998), the superoutbursting disk is relatively simple in that the surface density $\Sigma(r)$ at all radii far exceeds the critical surface density $\Sigma_{\text{min}}(r)$ below which the cooling thermal transition is thought to occur. Therefore the entire disk is in the hot, ionized state, and the disk mass can only decrease through accretion onto the central WD.
(There is also some mass loss from the outer edge due to the tidal action from the secondary star, and from intermediate radii through winds.)

One can obtain a simple estimate of the viscous time at the outer disk edge by using the “vertically-averaged” equations which give the radial disk structure (Shakura & Sunyaev 1973). Since the observed superoutburst energy constrains the $t = 0$ mass of the disk (Smak 1993), we require scalings of the disk variables in terms of surface density $\Sigma$ rather than the usual accretion rate $\dot{M}$. Eqn. [A6] of Cannizzo & Reiff (1992) gives the midplane disk temperature $T = T(\Omega, \alpha_{\text{hot}}, \Sigma)$, where the Keplerian frequency $\Omega(r) = (GM_{\text{WD}}r^{-3})^{1/2}$ and $\alpha_{\text{hot}}$ is the value of alpha in the outbursting disk ($\simeq 0.1 - 0.2$, Smak 1984, 1998, 1999).

If we adopt an opacity law for the ionized state $\kappa = 2.8 \times 10^{24} \, \text{g cm}^{-2} \rho T^{-3.5}$ and use the condition of local hydrostatic equilibrium $\Omega^2 h^2 = c_s^2 = RT\mu^{-1}$ (where $c_s$ is the local sound speed, $h$ is the local disk semi-thickness, $R$ the ideal gas constant, and $\mu$ the mean molecular weight = 0.67), we then get for the viscous time

$$\tau_v \equiv (\alpha \Omega)^{-1} (r/h)^2 = 5.7 \, \text{d} \, m_1^{5/14} \, r_{10}^{13/14} \, (\alpha_{\text{hot}}/0.1)^{-8/7} \, (\Sigma/2 \times 10^3 \, \text{g cm}^{-2})^{-3/7},$$

where $m_1 = M_{\text{WD}}/M_\odot$ and $r_{10} = r_{\text{outer}}/(10^{10} \, \text{cm})$. The value $2 \times 10^3 \, \text{g cm}^{-2}$ used in the $\Sigma$ scaling comes from Smak’s (1993) estimate of the mass $\Delta M = 10^{24} \, \text{g}$ in the disk at $t = 0$, taken to be distributed in a steady state profile $\Sigma(r) \propto r^{-3/4}$ extending to $10^{10} \, \text{cm}$. The other parameters have been scaled in rough accord with Smak’s summary: primary mass $0.45 \pm 0.2 M_\odot$ and outer disk radius $(1.1 \pm 0.2) \times 10^{10} \, \text{cm}$. The viscous time scale $\tau_v$ represents an $e$–folding time over which local surface density perturbations at a given radius are smoothed out in a dynamically evolving disk.

Assuming that the mass in the decaying disk is continually redistributed so that it always extends out to the (formal) outer disk radius, and that $\alpha_{\text{hot}}$ is constant, we see that basic disk theory predicts $\tau_v = \tau_0 (\Sigma/\Sigma_0)^{-3/7}$. As the mass drains onto the WD and the surface density in the disk decreases, one should see a corresponding increase in the locally
defined time scale associated with the decay light curve at a given point, given that the luminosity varies as the rate of change of disk mass, or $\partial \Sigma / \partial t$. Even if the global decay of the light curve does not have precisely an exponential form, one can always define a small local patch of it to be exponential, so that by definition the time scale defined from a local tangent to the light curve $\tau_{\text{e-fold}}(\Sigma) = \tau_{\text{e-fold}}(\partial \Sigma / \partial t)$. Standard values of $\alpha_{\text{hot}} \simeq 0.1 - 0.2$ imply a critical surface density for cooling $\Sigma_{\text{min}}$ at $r_{10} \sim 100$ g cm$^{-2}$. At this point the cooling front forms at the outer edge and the subsequent decay is much faster. Therefore, theory predicts that the superoutburst should end when the initial value of $\Sigma(r_{\text{outer}}) \sim 2000$ g cm$^{-2}$ has decreased to $\sim 100$ g cm$^{-2}$, a dynamic range of about a factor of 20. Since $\tau_v \propto \Sigma^{-3/7}$, this implies an increase by a factor $\sim 3 - 4$ in the locally defined decay time scale for the superoutbursting light curve from start to end.

3. Data and Modeling

3.1. Data

Figure 1 shows the $m_V$ data from the VSNET website of the July-Aug 2001 superoutburst of WZ Sge. The data are contributed by amateur astronomers spanning many organizations across the world. The first panel presents all the individual data points. One can see by eye a noticeable departure from exponentiality in the decay (i.e., from a straight line when plotted as $m_V$ versus time). The second panel shows moving two day averages of the light curve binned into 0.2 d bins. The third panel shows the decay time scale computed from the light curve in the second panel. From $\sim 25$ July 2001 to $\sim 8$ Aug 2001, the locally defined decay time scale increased from $\sim 4$ d mag$^{-1}$ to $\sim 12$ d mag$^{-1}$. This is consistent with our scaling law from the previous section. (One $e-$folding [i.e., $2.718 \times$] exceeds 1 mag [i.e., $2.512 \times$] by about 8%, a difference which we ignore given the
3.2. Time Dependent Modeling

As a final check, we utilize our time dependent accretion disk code to calculate a detailed light curve (for descriptions of the code see Cannizzo 1993b, 2001). We assume $M_{\text{WD}} = 0.45 M_\odot$, $r_{\text{inner}} = 9 \times 10^8$ cm, and $r_{\text{outer}} = 1 \times 10^{10}$ cm. At $t = 0$ we place a cold torus of gas at $0.8r_{\text{outer}}$ in the form of a Gaussian, with a width $r_{\text{FWHM}} = 1 \times 10^9$ cm. We perform two trials: one with $\Sigma_{\text{peak}} = 2 \times 10^4$ g cm$^{-2}$ (i.e., $\Delta M_{\text{disk}} = 1 \times 10^{24}$ g), and one with $\Sigma_{\text{peak}} = 6 \times 10^4$ g cm$^{-2}$ (i.e., $\Delta M_{\text{disk}} = 3 \times 10^{24}$ g). This is motivated by Smak’s inferred initial disk mass $\sim 10^{24}$ g. Figure 2 shows the light curves and associated decay time scales for these models. As expected, the deviations from exponentiality are seen here as well. It is interesting that the dynamic range over which $\tau(m_V)$ changes is about a factor of two, which is slightly less than seen in WZ Sge. This may be a reflection of the crude (Planckian) flux distributions which are used to model the local emissivity in the disk. The calculations of $\tau$ based on $\dot{M}_{\text{inner}}$ show a larger dynamic variation.

4. Discussion

Our result supports the standard model for dwarf nova outbursts in which the outbursting disk decays primarily via simple accretion onto the WD when in the “viscous plateau” (Cannizzo 1993b) stage, and $\alpha_{\text{hot}} \simeq 0.1 - 0.2$. The faster decay which began 18 Aug 2001 in WZ Sge would then be caused by the onset of the cooling front, its rapidness in comparison to the previous evolution stemming from the fact that the thermal time scale is shorter than the viscous time scale. It is fortunate that nature has given us the WZ Sge stars which remain in a viscous plateau long enough for one to be able to constrain
the functional form of the decay as being demonstrably different from exponential. For longer period dwarf novae such as SS Cyg ($P_{\text{orb}} = 6.6$ hr) in which one frequently sees flat-topped outbursts where the disk is also presumably entirely in the hot state, the short duration ($\sim 7 - 10$ d) of the viscous state due to a smaller initial surface density excess $\Sigma(\text{outer})/\Sigma_{\text{min}}(\text{outer})$ in the outer disk, combined with the slower viscous time at the outer disk (due to larger $r_{\text{outer}}$), conspire to produce a much smaller dynamic variation in $m_V$. In fact, the variations that are seen may be due almost entirely to sloshing action of the gas in the disk as it responds to the matter redistribution accompanying the outburst.

It is interesting to ponder the implications for other types of systems, for example the soft X-ray transients (SXTs) in which the accreting object is a neutron star or black hole. The similarity between the superoutbursts in the WZ Sge stars and the X-ray nova outbursts seen in the some of the brightest and best studied SXTs has been pointed out by Kuulkers et al. (1996, see also Kuulkers 1998). King & Ritter (1998, see also King 1998, Shahbaz, Charles, & King 1998) propose that the $\sim 30 - 50$ d $e-$folding decay times seen in these SXTs are viscous, insomuch as one expects strong irradiation in the outbursting state to prevent the cooling front from forming. Cannizzo (2000) argues that even though irradiation keeps the entire disk ionized, the viscous time scale would be too slow (the time dependent computations of Dubus et al. [2001, e.g. Fig. 16a] show an $e-$folding decay time of $\sim 100 - 200$ d during the viscous plateaus of their outbursts), and Cannizzo hypothesizes that some other agent such as strong evaporation must be at work in outburst to reduce the disk mass from the inner disk on a time scale shorter than that due to accretion acting alone.

A scaling of the viscous time at the outer edge applied to A0620-00, which had a bright outburst and exponential decay in 1975, gives

$$\tau_v \simeq 400 \text{ d} \left( \frac{M_1}{10 M_\odot} \right)^{5/14} \left( \frac{r_{11}}{0.1} \right)^{13/14} \left( \frac{\alpha_{\text{hot}}}{0.1} \right)^{-8/7} \left( \frac{\Sigma}{10^2 \text{ g cm}^{-2}} \right)^{-3/7}.$$
Such a long time is problematic in accounting for the $\sim 30 - 50$ d $\tau$-folding decay times seen in systems with a variety of orbital periods and hence outer disk radii (see Table 1 of Mineshige, Yamasaki, Ishizaka 1993), and more importantly, the rise times of $\sim 1 - 3$ d for the secondary maxima in the SXTs which, in the model of King & Ritter (1998), are due to the arrival of matter from the outer disk to the inner disk. In reality, matter added to the disk at the outer edge and diffusing inward would spread out and not produce such a short, well-defined rise in the X-ray light curve. Furthermore, it would be difficult to maintain an exponential decay over a dynamic range of $\sim 10^2 - 10^3$ in $L_X$ as observed, due to the decrease in $\Sigma(r_{\text{outer}})$ and the fact that $\tau_v \propto \Sigma(r_{\text{outer}})^{-3/7}$. (The dynamic ranges for viscous decays shown in Dubus et al. 2001 are only about a factor of 10.)

Could irradiation of the outer disk in the SXTs affect the midplane temperature enough to change significantly the viscous time scale? For irradiation to be strong enough to have a significant effect on the midplane temperature (which enters into the disk semithickness and hence viscous time scale), the local irradiation temperature needs to exceed not just the local non-irradiated disk photospheric temperature, but also the midplane temperature which is $\sim \tau^{1/4}$ times the effective temperature, where $\tau$ is the vertically integrated optical depth. Starting again with eqn. [A6] from Cannizzo & Reiff (1992), one can show

$$\tau = \kappa \Sigma \simeq 3000(\frac{M_1}{10 M_\odot})^{-1/14} \frac{r_{11}^{3/14}}{r_{\text{hot}}/0.1} (\frac{\alpha_{\text{hot}}}{0.1})^{-4/7} (\frac{\Sigma}{100 \text{ g cm}^{-2}})^{2/7},$$

an optical depth of $\sim 10^3 - 10^4$ through the vertical structure, so that one would need a local irradiation temperature about an order of magnitude greater than the effective temperature. Realistic calculations of the irradiation effect in SXTs show that the change in midplane temperature is small (Dubus et al. 1999). In other words, irradiation indeed keeps the entire disk ionized during the “viscous” decay (because the irradiation temperature exceeds $\sim 10^4$ K at the outer disk edge), but because of the large optical depth one does not see a significant change in midplane temperature.
5. Conclusion

We have examined the 2001 superoutburst light curve of WZ Sge. We find that the deviation from exponentiality during the time the system was in superoutburst is roughly characterized by a linear increase in the local decay time from $\sim 4 \text{ d mag}^{-1}$ to $\sim 12 \text{ d mag}^{-1}$ up to about two-thirds of the way through superoutburst. This is consistent with standard limit cycle accretion disk theory, adopting systemic parameters for WZ Sge taken from Smak (1993), namely an initial disk mass $10^{24} \text{ g}$, a central mass $0.45 M_\odot$, an outer disk radius $10^{10} \text{ cm}$, and an alpha value $\alpha_{\text{hot}} = 0.1$. Among the dwarf novae, only the WZ Sge stars show a large enough dynamic range in $m_V$ in their stages of viscous decay during outburst to allow this type of study.

An application of the viscous time $\tau_v(\alpha_{\text{hot}}, r_{\text{outer}}, \Sigma)$ scaling law to outbursts in the soft X-ray transients appears to lead to decay times which are slower than observed in several of the bright, well-studied systems (see Fig. 9 of Kuulkers 1998). Also, the rise times of $\sim 1 - 3 \text{ d}$ for the secondary maxima seen in these systems are especially difficult to reconcile with the class of theories in which material added at the outer disk edge must diffuse to the inner edge (Chen et al. 1993, Augusteijn et al. 1993, King & Ritter 1998) since the diffusion time and the smearing time would both be $\sim \tau_v(r_{\text{outer}})$.

The data were obtained from the VSNET website www.kusastro.kyoto-u.ac.jp/vsnet/etc/searchobs.cgi?text=SGEWZ. We thank Erik Kuulkers for pointing us to this link. We also thank Tom Marsh for organizing a stimulating ad hoc session on WZ Sge at the August 2001 CV meeting in Goettingen, Germany ("The Physics of Cataclysmic Variables and Related Objects").
REFERENCES


Krzemiński, W. 1962, PASP, 74, 66
Kuulkers, E. 1998, New Astronomy Rev., 42, 1
Mineshige, S., Yamasaki, T., & Ishizaka, C. 1993, PASJ, 45, 707
Smak, J. 1984, Acta Astr., 34, 161

This manuscript was prepared with the AAS \LaTeX macros v3.0.
FIGURE CAPTIONS

Figure 1. The amateur $m_V$ data from the VSNET website for the July-August 2001 superoutburst of WZ Sge (top panel), the data binned into 0.2 d bins and smoothed in 2 d moving averages (middle panel), and the decay time scale associated with the smoothed light curve (bottom panel). The two straight lines in the second panel indicate a decay time of 10 d mag$^{-1}$.

Figure 2. The model light curve for the outburst described in the text, using parameters relevant for WZ Sge. Shown are the $M_V$ light curve (top panel), the disk mass (middle panel), and the locally defined decay time scale along the light curve (bottom panel). The two curves given in each panel are for an initial disk mass of $10^{24}$ g and $3 \times 10^{24}$ g, with the latter curve extending further to the right. The two straight lines in the top panel indicate a decay time of 10 d mag$^{-1}$. The additional two sets of curves which appear toward the bottom of the third panel show the decay time scale computed not from the $M_V$ light curve but rather from the rate of mass loss from the inner edge onto the WD. It appears that the effect of computing a $V$ band light curve introduces a flattening into the deviation from exponentiality: the change in $\tau$ in the model light curve is only about a factor of two, versus about a factor of three for the observed WZ Sge superoutburst light curve.