Introduction

Polarized light-antilight distributions in a meson-cloud model
are some semi-inclusive data [11] which could be sensitive to the light antiquark flavor asymmetry, they are not accurate enough to provide strong constraint for the polarized antiquark flavor asymmetry [10, 11]. However, the Relativistic Heavy Ion Collider (RHIC) [12] and the Common Muon and Proton Apparatus for Structure and Spectroscopy (COMPASS) [13] experiments should clarify the details of the polarized antiquark distributions in a few years. It is the right time to investigate the antiquark flavor asymmetry $\Delta \bar{u}/\Delta \bar{d}$ by possible theoretical models and to summarize various predictions.

In this paper, we intend to shed light on the virtual meson model which has been successful in the unpolarized studies [14]. The purpose of this paper is to extend the studies of the virtual $\rho$-meson contributions by Fries and Schäfer in Ref. [7]. In particular, we point out that the $g_2$ part of the polarized $\rho$ contributions to the polarized flavor asymmetry in addition to the ordinary longitudinal part, which was calculated in Ref. [7]. Because this paper shows only $g_2$ terms in this paper and because the situation is still confusing in the sense that another $\rho$-meson paper [8] claims major differences from Ref. [7] in supposedly the same $\rho$ contributions, the detailed formalism is shown in the following sections. The meson model was extended recently to a different direction in Ref. [8] by including $\tau - \rho$ interference terms, however, this paper is intended to investigate a different kinematical aspect within the meson model.

The paper consists of the following. The formalism of $\rho$ contributions to $\Delta \bar{u} - \Delta \bar{d}$ is presented in Sec. II. Meson momentum distributions are obtained in Sec. III, and numerical results of $\Delta \bar{u} - \Delta \bar{d}$ are shown in Sec. IV. Our studies are summarized in Sec. V.

II. VECTOR-MESON CONTRIBUTIONS

The cross section of polarized electron-nucleon scattering is generally written in terms of lepton and hadron tensors:

$$\frac{d\sigma}{dE_\ell' dG'_{\ell'}} = \frac{\alpha^2}{m_e (q^2)^2} L^{\mu\nu}(p_e, s_e, q) W_{\mu\nu}(p_N, s_N, q),$$

(2.1)

where $\alpha$ is the fine structure constant, $E_\ell'$ and $G'_{\ell'}$ are the scattered electron energy and solid angle, and $p_e$, $\mu_e$, $p_N$, and $q$ are initial electron, final electron, nucleon, and virtual photon momenta, respectively. The electron and nucleon spins are expressed by $s_e$ and $s_N$ with the normalization $s_e^2 = s_N^2 = -1$. Throughout this paper, the convention $g_{00} = g_{11} = g_{22} = g_{33} = +1$ is used so as to have, for example, $p_N^2 = (p_N^\mu)^2 = m_N^2$. Furthermore, the Dirac spinor is normalized as $u_{\mu'}^\dagger = E_\ell'/m_e$ or $E_N/m_N$, where $E_\ell$ and $E_N$ are electron and nucleon energies, and $m_e$ and $m_N$ are their masses. The polarized lepton and hadron tensors are given by

$$L^{\mu\nu}(p_e, s_e, q) = 2 \left[ p_e^\mu p_e'^\nu + p_e'^\mu p_e'^\nu - (p_e \cdot p_e') m_e^2 \right] g^{\mu\nu} - i \varepsilon^{\mu\nu\rho\sigma} m_e q_\rho s_{e\sigma},$$

(2.2)

$$W_{\mu\nu}(p_N, s_N, q) = \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(p_N + q - p_X) \times <p_N, s_N| J_{\mu}(0)| X > < X| J_{\nu}(0)| p_N, s_N >,$$

(2.3)

where the factor $\varepsilon^{\mu\nu\rho\sigma}$ is the antisymmetric tensor with the convention $\varepsilon_{123} = +1$.

![FIG. 1: Virtual vector-meson contribution.](image)

Next, we consider the process in Fig. 1, where the nucleon splits into a virtual vector meson and a baryon, then the virtual photon interacts with the polarized meson. Because scalar mesons do not contribute directly to the polarized structure functions due to spinless nature, the lightest vector meson, namely $\rho$, is taken into account in this paper. In future, we may extend the present studies by including heavier vector mesons. As the final state baryon, the nucleon and $\Delta$ are considered. Expressing the $V N B$ vertex multiplied by the meson propagator as $J_{V N B}(k, s_V, p_N, s_N, p_B, s_B)$ and calculating the cross section due to the process in Fig. 1, we obtain

$$\frac{d\sigma}{dE_\ell' dG'_{\ell'}} \propto \frac{\alpha^2}{2\pi E_B} \sum_{X, s_V, \lambda_B} \left| J_{V N B} \right|^2 <k, s_V| J_{\rho}(0)| X > \times < X| J_{\rho}(0)| k, s_V > (2\pi)^4 \delta^4(k + q - p_X),$$

(2.4)

Here, $k$ and $s_V$ indicate the meson momentum and spin. This equation has the same form as Eq. (2.1). Therefore, the last part is identified with a vector-meson contribu-
tion to the nucleon tensor:

\[
W_{\mu\nu}(p_N, s_N, q) = \int \frac{d^3p_B}{(2\pi)^3} \frac{2m_V m_B}{E_B} \sum_{\lambda_V, \lambda_B} |J_{VNB}|^2 \mathcal{W}^{(V)}_{\mu\nu}(k, s_V, q),
\]

where \(m_V\) is the meson mass, and the meson tensor is defined by

\[
\mathcal{W}^{(V)}_{\mu\nu}(k, s_V, q) = \frac{1}{4\pi m_V} \sum_X (2\pi)^4 \delta(k + q - p_X)
\times \langle k, s_V | J_\mu(0) | X > < X | J_\nu(0) | k, s_V \rangle .
\]

(2.6)

In this way, the vector-meson contribution to the nucleon tensor is expressed in terms of the \(VNB\) vertex and the meson tensor. Because we are interested in meson effects on the polarized parton distributions in the nucleon, we try to project the \(g_1\) part out from the nucleon tensor. The definition of the \(g_1\) and \(g_2\) structure functions is given in the asymmetric part of the nucleon tensor:

\[
W^{A}_{\mu\nu}(p_N, s_N, q) = i \varepsilon_{\mu\nu\alpha\beta} q^\alpha \left[ \frac{g_1}{p_N \cdot q} + \frac{g_2}{(p_N \cdot q)^2} \right] .\]

(2.7)

In order to discuss each structure function separately, a projection operator

\[
P^{\mu\nu} = \frac{m_N^2}{2p_N \cdot q} \frac{m_N}{p_N \cdot q},
\]

is then applied to give

\[
P^{\mu\nu} W^{A}_{\mu\nu}(p_N, s_N, q) = \frac{m_N^2}{(p_N \cdot q)^2} \left[ q^2 + (s_N \cdot q)^2 \right] g_1 - \gamma^2 g_2.
\]

(2.9)

Here, \(\gamma\) is defined by

\[
\gamma^2 = \frac{4x^2 m_N^2}{Q^2}.
\]

(2.10)

with \(Q^2 = -q^2\). In the same way, \(g_1\) and \(g_2\) structure functions of the vector meson are defined in the asymmetric part of the tensor. Operating the projection also on the meson tensor, we obtain

\[
P^{\mu\nu} W^{(V)}_{\mu\nu}(k, s_V, q) = \frac{m_N}{m_V} \left[ A_1 g_1^V(k, q) + A_2 g_2^V(k, q) \right],
\]

(2.11)

where \(A_1\) and \(A_2\) are given by

\[
A_1 = \frac{m_N m_V}{p_N \cdot q} (s_N \cdot q s_V \cdot q - q^2 s_N \cdot s_V),
\]

(2.12)

\[
A_2 = \frac{m_N^2 m_V q^2}{p_N \cdot q} (s_N \cdot k s_V \cdot q - k \cdot q s_N \cdot s_V).
\]

(2.13)

From Eqs. (2.5), (2.9), and (2.11), the meson contribution to the nucleon structure functions becomes

\[
\frac{m_N^2}{(p_N \cdot q)^2} \left[ q^2 + (s_N \cdot q)^2 \right] g_1(p_N, q) - \gamma^2 g_2(p_N, q)
\]

\[
= \int \frac{d^3p_B}{(2\pi)^3} \frac{2m_N m_B}{E_B} \sum_{\lambda_V, \lambda_B} |J_{VNB}|^2 \times \left[ A_1 g_1^V(k, q) + A_2 g_2^V(k, q) \right].
\]

(2.14)

Then, the above integration variables \(p_B^\mu, p_B^\nu, p_B^\gamma\) are changed for the meson momentum fraction \(y\), the transverse momentum \(k_{\perp}\) of the meson, and the angle \(\phi\) between \(\vec{k}_{\perp}\) and the transverse spin vector of the nucleon \(\vec{s}_N\):

\[
y = \frac{k \cdot q}{p_N \cdot q}, \quad \vec{k}_{\perp} \cdot \vec{s}_N = k_{\perp} \tau_N \cos \phi.
\]

(2.15)

with \(\tau_N = |\vec{s}_N|^2\). Then, the meson contribution is expressed as

\[
\frac{m_N^2}{(p_N \cdot q)^2} \left[ q^2 + (s_N \cdot q)^2 \right] g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)
\]

\[
= \int_0^1 dy \left[ B_1(y) g_1^V(x/y, Q^2) + B_2(y) g_2^V(x/y, Q^2) \right].
\]

(2.16)

The upper limit of the \(y\)-integration range is taken as 1 by considering the vector-meson mass smaller than the nucleon mass. However, one should be careful in extending the present studies to other mesons with larger masses. The meson momentum distributions are expressed as

\[
B_{1,2}(y) = \int_0^{\frac{2}{\gamma}} d\vec{k}_{\perp} \int_0^{2\pi} d\phi \frac{|\vec{p}_N| m_N m_B}{(2\pi)^3 E_B} \frac{\partial y'}{\partial y} \sum_{\lambda_V, \lambda_B} |J_{VNB}|^2 A_{1,2},
\]

(2.17)

where \(y'\) is the longitudinal momentum fraction defined in the meson momentum

\[
\vec{k} = \vec{k}_{\perp} + y' \vec{p}_N.
\]

(2.18)

In the infinite momentum frame \(|\vec{p}_N| \to \infty\), \(y\) and \(y'\) are related by

\[
y = \frac{y}{1 + \sqrt{1 + \frac{\gamma^2}{y \cdot m_N^2 (k_{\perp}^2 + m_V^2)}}}.
\]

(2.19)

Because time-ordered perturbation theory is used for the reaction in Fig. 1 as explained in Sec. III, the vector meson is taken as an on-shell particle: \(k^2 = m_V^2\) in the above derivation. The partial derivative \(\partial y'/\partial y\) can be calculated from this expression. In the infinite momentum frame, the momentum fraction \(y'\) has to satisfy the kinematical condition \(0 \leq y' \leq 1\), namely the meson \(V\)
and the baryon $B$ should move in the forward direction. The maximum transverse momentum is given by
\[
(\vec{k}_T^2)_{\text{max}} = \frac{m_N^2}{\gamma^2} \left( \sqrt{1 + \gamma^2 + 1} \right) \left( \sqrt{1 + \gamma^2 + 1 - 2y} \right) - m_N^2.
\]
(2.20)

Practically, it does not matter to take the upper bound $\vec{k}_T^2_{\text{max}} \rightarrow \infty$ in Eq. (2.17) at small-$x$ where the antiquark distributions play a major role, because $\vec{k}_T^2_{\text{max}} \sim Q^2(1-y)/x^2 \gg m_N^2$ is beyond the vertex momentum cutoff region discussed in Sec. III. The contribution to the integral between $(\vec{k}_T^2)_{\text{max}}$ and $\vec{k}_T^2 = \infty$ is extremely small in general. Furthermore, half of the upper bound becomes $(\vec{k}_T^2)_{\text{max}} \rightarrow \infty$ in the limit $Q^2 \rightarrow \infty$, and it is consistent with the previous publications [7, 8]. In this way, the meson contribution is expressed in terms of the nucleon structure functions convoluted with the meson momentum distributions in the nucleon.

Using the integration variables $y$, $\vec{k}_T$, and $\phi$, we express the coefficients $A_1$ and $A_2$ as
\[
A_1 = \lambda_N \lambda_V \left[ 1 + \frac{\vec{k}_T^2}{y y' m_N^2} \left( \sqrt{1 + \gamma^2 - 1} \right) \right]
- \frac{\gamma^2 \lambda_N \lambda_V \cos \phi \frac{k_T}{y m_N}}{y m_N^2},
A_2 = \frac{\gamma^2 m_N^2}{y y' m_N^2} \left[ - \lambda_N \lambda_V + \frac{\gamma^2 \lambda_N \lambda_V \cos \phi \frac{k_T}{y m_N}}{y m_N} \left( \sqrt{1 + \gamma^2 - 1} \right) \right]
\]
(2.21)
in the limit $|\vec{p}_N| \rightarrow \infty$. Preliminary calculations indicate that $\phi$ dependence can be extracted out from another part of the integrand in Eq.(2.17) as
\[
|\vec{p}_N| m_N m_B \frac{\partial y \sum_{\nu} J_{\nu NB}}{(2\pi)^3 E_B} \frac{\partial y}{\partial y} \equiv C_L^\lambda \lambda_V + \lambda_N \lambda_V C_L^\lambda_T.
\]
(2.22)

Then, after the $\phi$ integration, Eq. (2.17) becomes
\[
B_1(y) = \sum_{\nu} \lambda_V \left[ \lambda_N f_{1L}^{\lambda V}(y) - \frac{\gamma^2}{\gamma_N} f_{1}^{\lambda V}(y) \right],
\]
(2.23)
\[
B_2(y) = \sum_{\nu} \lambda_V \left[ - \lambda_N f_{2L}^{\lambda V}(y) + \frac{\gamma^2}{\gamma_N} f_{2}^{\lambda V}(y) \right],
\]
(2.24)
where $\lambda_N$ dependence is explicitly denoted in meson momentum distributions, which are defined by
\[
f_{\lambda L}^{\lambda V}(y) = \int_0^{(\vec{k}_T^2)^{\text{max}}} d\vec{k}_T^2 2\pi C_L^\lambda \left[ 1 + \frac{\vec{k}_T^2}{y y' m_N^2} \left( \sqrt{1 + \gamma^2 - 1} \right) \right]
\]
(2.25)
\[
f_{1T}^{\lambda V}(y) = \int_0^{(\vec{k}_T^2)^{\text{max}}} d\vec{k}_T^2 2\pi C_T^\lambda \left[ \frac{k_T}{y m_N} \right]
\]
(2.26)
\[
f_{2T}^{\lambda V}(y) = \int_0^{(\vec{k}_T^2)^{\text{max}}} d\vec{k}_T^2 2\pi C_T^\lambda \left[ \frac{m_N}{y m_N} \right]
\]
(2.27)
\[
f_{2T}^{\lambda V}(y) = \int_0^{(\vec{k}_T^2)^{\text{max}}} d\vec{k}_T^2 2\pi C_T^\lambda \left[ \frac{m_N}{y m_N} \right]
\]
(2.28)
Because the functions $f_{\lambda L}^{\lambda V}$, $f_{1T}^{\lambda V}$, and $f_{2T}^{\lambda V}$ are proportional to $\gamma^2$, they vanish in the limit $Q^2 \rightarrow \infty$. As it is obvious from Eq. (2.16), it is necessary to consider both longitudinal and transverse polarizations for the nucleon in order to extract the $g_1$ part. In addition, the $g_2$ structure function of the meson contributes. The function $f_{1L}^{\lambda V}(y)$ is the ordinary meson momentum distribution with the momentum fraction $y$ in the longitudinally polarized nucleon. The function $f_{2T}^{\lambda V}(y)$ is the distribution in the transversely polarized nucleon. On the other hand, $f_{2T}^{\lambda V}(y)$ and $f_{1T}^{\lambda V}(y)$ are the distributions associated with $g_2$ of the vector meson. Expressing Eq. (2.16) in terms of the nucleon and meson helicities, $\lambda_N$ and $\lambda_V$, we obtain
\[
(\beta_N^2 - \gamma_N^2 \gamma^2) g_1(x, Q^2) - \gamma^2 g_2(x, Q^2) = \sum_{\lambda_N} \lambda_N \int_{x'}^{1} \frac{dy}{y} \left[ \left\{ f_{1L}^{\lambda N}(y) - \frac{\gamma_N^2}{\gamma_N} f_{1T}^{\lambda N}(y) \right\} g_1^{\lambda N}(x/y, Q^2) + \left\{ - \lambda_N f_{2L}^{\lambda N}(y) + \frac{\gamma_N^2}{\gamma_N} f_{2T}^{\lambda N}(y) \right\} g_2^{\lambda N}(x/y, Q^2) \right].
\]
(2.29)

Combining the longitudinal polarization $\lambda_N = 1$ ($\gamma_N = 0$) with the transverse polarization $\lambda_N = 1$ ($\lambda_N = 0$), we can extract the $g_1$ part as
\[
g_1(x, Q^2) = \frac{1}{1 + \gamma^2} \int_{x'}^{1} \frac{dy}{y} \left[ \left\{ f_{1L}(y) + f_{1T}(y) \right\} g_1^{Y}(x/y, Q^2) + \left\{ - \lambda_N f_{2L}(y) + \lambda_N f_{2T}(y) \right\} g_2^{Y}(x/y, Q^2) \right].
\]
(2.30)
The $g_2$ part is obtained in the same way as
\[
g_2(x, Q^2) = \frac{1}{1 + \gamma^2} \int_{x'}^{1} \frac{dy}{y} \left[ \left\{ f_{2L}(y) + f_{2T}(y) \right\} g_2^{Y}(x/y, Q^2) + \left\{ - \lambda_N f_{2L}(y) + \lambda_N f_{2T}(y) \right\} g_2^{Y}(x/y, Q^2) \right].
\]
(2.31)
In the limit $Q^2 \rightarrow \infty$, namely $\gamma^2 \gamma_N \rightarrow 0$, only the momentum distribution $\Delta f_{1L}(y)$ remains finite, and Eq. (2.30) agrees with the expression in Ref. [7].

In Eq. (2.30), there are additional terms associated with $g_2$ of the meson. For discussing these $g_2$ type contributions to $\Delta \bar{g}_1 - \Delta \bar{g}_2$ is approximated by the Wandzura-Wikaczew (WW) relation [15] by neglecting higher-twist terms:
\[
g_2^{(WW)}(x, Q^2) = -g_1^{Y}(x, Q^2) + \int_{x'}^{1} \frac{dy}{y} g_1^{Y}(y, Q^2).
\]
(2.32)
Then, providing the leading-order expression for \( g_2^{VW}(x, Q^2) \), we have
\[
g_2^{VW}(x, Q^2) = \frac{1}{2} \sum_i e_i^2 \left[ \Delta q_i^{VW}(x, Q^2) + \Delta \tilde{q}_i^{VW}(x, Q^2) \right].
\]
The above WW distributions are defined by
\[
\Delta q_i^{VW}(x, Q^2) = -\Delta \tilde{q}_i^{VW}(x, Q^2) + \int_x^1 \frac{dy}{y} \Delta q_i^{VW}(y, Q^2),
\]
and the same equation for \( \Delta \tilde{q}_i^{VW}(x, Q^2) \). From these equations, we obtain a vector meson contribution to the polarized antiquark distribution \( \Delta \tilde{q}_i \) in the proton as
\[
\Delta \tilde{q}_i^{VNB}(x, Q^2) = \frac{1}{1 + \gamma^2} \int_x^1 \frac{dy}{y} \times \left[ \{ \Delta f_{1L}(y) + \Delta f_{2T}(y) \} \Delta \tilde{q}_i^{V}(x/y, Q^2) - \{ \Delta f_{2L}(y) + \Delta f_{2T}(y) \} \Delta \tilde{q}_i^{VW}(x/y, Q^2) \right].
\]
If this kind of vector-meson contribution is the only source for the polarized flavor asymmetry, the \( \Delta \tilde{q} = \Delta \tilde{d} \) distribution is then calculated by taking the difference \( \Delta \tilde{q}_i^{VNB} - \Delta \tilde{q}_i^{VNB} \) in the above equation.

### III. MESON MOMENTUM DISTRIBUTIONS

In order to estimate the meson distributions numerically, it is necessary to calculate the momentum distributions \( \Delta f_{1V}(y) \) of the meson. We calculate them by considering the vector-meson creation processes \( N \to VN' \) and \( N \to VN' \) through the interactions
\[
V_{VNN} = \tilde{\phi}_V \cdot \tilde{T} F_{VNN}(k) \gamma^\mu \bar{u}(p_N, s_N) \gamma^*_\mu \times \left[ g_V \gamma_\mu - \frac{f_V}{2 m_N} i \sigma_{\mu\nu} \tilde{K}^\nu \right] u(p_N, s_N), \quad (3.1)
\]
\[
V_{VN\Delta} = \tilde{\phi}_V \cdot \tilde{T} F_{VN\Delta}(k) \gamma^\mu \bar{u}(p_\Delta, s_\Delta) \gamma^*_\mu \frac{f_{VN\Delta}}{m_V} \gamma_\mu \times \left[ \tilde{K}^\mu \gamma^*_\mu - \tilde{K}^\nu \gamma^*_\nu \right] u(p_N, s_N), \quad (3.2)
\]
where \( u(p_N, s_N) \) is the Dirac spinor, \( U^\mu(p_\Delta, s_\Delta) \) is the Rarita-Schwinger spinor, and \( \tilde{T} \) is the polarization vector of the vector meson. The \( VNN \) and \( VNN \) coupling constants are denoted by \( g_V, f_V \), and \( F_{VNN} \), \( F_{VN\Delta} \), and \( f_{VN\Delta} \), and form factors are denoted as \( F_{VNN}(k) \) and \( F_{VN\Delta}(k) \). Isospin dependence is taken into account by the factor \( \tilde{\phi}_V \cdot \tilde{T} \), and it is defined in terms of a reduced matrix element and a Clebsch-Gordan coefficient. [16, 17]

\[
< B | \tilde{\phi}_V \cdot \tilde{T} | N > = (-1)^{M_N} \sqrt{T_B \parallel T_N \parallel T_B + 1} \times < T_N M_N : 1 - M_N T_B M_B > \quad (3.3)
\]
The neutron polarization vector is given by

\[ \mathbf{\lambda} = \mathbf{\lambda} \cdot \mathbf{\hat{z}} \cdot (\beta + \mp \frac{1}{2} \hat{\lambda} \cdot \mathbf{\hat{z}}) \]

\[ \varphi = x \pm \frac{1}{2} \mathbf{\hat{z}} \cdot \mathbf{\lambda} - \mp \frac{1}{2} \mathbf{\lambda} \cdot \mathbf{\hat{z}} \]

where \( \mathbf{\lambda} \) is the neutron polarization vector, \( \beta \) is the momentum transfer, \( x \) is the scattering angle, and \( \varphi \) is the scattering angle. The polarization vector is defined as

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\[ \mathbf{\lambda} = \mathbf{\lambda} \cdot \mathbf{\hat{z}} \cdot (\beta + \mp \frac{1}{2} \hat{\lambda} \cdot \mathbf{\hat{z}}) \]
For each configuration, the partition function is
(\text{2.1})
\lambda^{2D} \propto \mathcal{Z}_A \propto \lambda^2 \mathcal{Z}_0

where \lambda = \frac{\beta}{\beta_0} is the ratio of the heat parameters. Therefore, the order parameter
(\text{2.2})
\delta = \frac{\mathcal{Z}_A}{\mathcal{Z}_0} = \lambda^2

The order parameter \delta is related to the partition function \mathcal{Z}_A by a simple quadratic equation. This equation is derived by setting the derivative of the partition function with respect to \lambda equal to zero.

\section{Results}

As expected, the partition function is a quadratic equation in \lambda. The solutions for \lambda are given by
(\text{2.3})
\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}

where a, b, and c are coefficients determined by the specific problem. The roots of this equation provide the critical points of the partition function, which are the values of \lambda that maximize (or minimize) the partition function. These critical points are important in understanding the phase transitions in the system.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The partition function as a function of \lambda. The critical points are indicated by the peaks in the graph.}
\end{figure}
butions in the $\rho$ meson:

\[
(\Delta \bar{V} - \Delta \bar{d})_{\rho^+} = -\Delta V_{\rho},
\]
\[
(\Delta \bar{V} - \Delta \bar{d})_{\rho^0} = 0,
\]
\[
(\Delta \bar{V} - \Delta \bar{d})_{\rho^-} = +\Delta V_{\rho},
\]

(4.3)

For the $g_2$ part of $\rho$, the Wandzura-Wilczek relation is used as discussed in Sec. II:

\[
\Delta V_{\rho}^{WW}(x, Q^2) = -\Delta V_{\rho}(x, Q^2) + \int_{x^0}^1 \frac{dy}{y} \Delta V_{\rho}(y, Q^2).
\]

(4.4)

Both the valence-quark distribution and the WW distribution are shown at $Q^2=1$ GeV$^2$ in Fig. 6.

\[\text{FIG. 6: Assumed polarized valence-quark distribution in the }\rho\text{ meson and the WW distribution at }Q^2=1\text{ GeV}^2.\]

Necessary isospin factors are calculated from Eq. (3.3) as

\[
|<p| \tilde{\Delta}_{\rho^+} T \tilde{V}_{\rho}|p>|^2 = 2,
\]
\[
|<\Delta | \tilde{\Delta}_{\rho^+} T \tilde{V}_{\rho}|p>|^2 = 1/3,
\]
\[
|<\tilde{\Delta} | \tilde{\Delta}_{\rho^+} T \tilde{V}_{\rho}|p>|^2 = 1.
\]

(4.5)

Using Eqs. (2.36), (4.3), and (4.5), we obtain

\[
(\Delta \bar{V} - \Delta \bar{d})_{\rho^0} = \sum_{\rho, B} \left\{ (\Delta f_{1L} + \Delta f_{1T}) \otimes (\Delta \bar{V} - \Delta \bar{d})_\rho \right. \\
\left. - \left\{ (\Delta f_{2L} + \Delta f_{2T}) \otimes (\Delta \bar{V} - \Delta \bar{d})_\rho^{WW} \right\} \right.
\]
\[
= \left[ -2 \Delta f_{1L}^{\rho NN} + \frac{2}{3} \Delta f_{1L+1T}^{\rho NN} \right] \otimes \Delta V_{\rho}
\]
\[
- \left[ -2 \Delta f_{2L}^{\rho NN} + \frac{2}{3} \Delta f_{2L+2T}^{\rho NN} \right] \otimes \Delta V_{\rho}^{WW},
\]

(4.6)

where $\otimes$ indicates the convolution integral in Eq. (2.36);

\[
a \otimes b = \frac{1}{1 + \gamma^2} \int_{x^0}^1 \frac{dy}{y} a(y) b(x/y).
\]

(4.7)

The meson momentum distributions $\Delta f_{1L}^{pNN}$ and $\Delta f_{2L+1T}^{pNN}$ are defined by extracting the isospin factors:

\[
\Delta f_{1L+1T}^{pNN} = \frac{\Delta f_{1L}^{pNN} + \Delta f_{1T}^{pNN}}{2}, \quad i=1, 2.
\]

(4.8)

The expression of Eq. (4.6) may seem to be different from Refs. [7, 8] even in the limit $Q^2 \to \infty$; however, it is just the matter of the definition of the meson momentum distributions. They included the isospin factor

\[
|<p| \tilde{\Delta}_{\rho^+} T \tilde{V}_{\rho}|p>|^2 + |<\Delta | \tilde{\Delta}_{\rho^+} T \tilde{V}_{\rho}|p>|^2 = 3,
\]

(4.9)

in the distribution $\Delta f(y)$ for the $\rho N N$ process and the factor

\[
|<\Delta | \tilde{\Delta}_{\rho^+} T \tilde{V}_{\rho}|p>|^2 + |<\Delta^+ | \tilde{\Delta}_{\rho^+} T \tilde{V}_{\rho}|p>|^2 = 2
\]

(4.10)

for the $\rho N \Delta$. Therefore, our expression certainly agrees on those in Refs. [7, 8] at $Q^2 \to \infty$.

The remaining quantities are the vector form factors. They are roughly known from the studies of one-boson-exchange potentials (OEPs); however, a slight change of the cutoff parameter could result in a large difference of antiquark distributions. Furthermore, there is an issue of the charge and momentum conservations for the splitting process [24] if a $t=(p_N-p_B)^2$ dependent form factor is used. A possible solution is to use the $t$ dependent form factor multiplied by a $u$ dependent one [25]. For this purpose, it is more convenient to take an exponential form factor so as to become the additional form $t+u$ within the form factor:

\[
F_{\rho NN}(k) = F_{\rho NN}(k) = \exp \left[ \frac{m_{VB}^2 - m_{VB}^2}{2 \Lambda_0^2} \right],
\]

(4.11)

where $m_{VB}^2$ is defined in Eq. (3.15), and the cutoff parameter $\Lambda_0$ is taken as $\Lambda_0=1$ GeV in the following numerical results. In Ref. [18], the cutoff parameters are obtained by fitting baryon-production cross sections $pp \to BX$: $\Lambda_F^{pNN}=1.10$ GeV and $\Lambda_G^{pNN}=0.98$ GeV. However, the parameters are not well determined in general. We discuss the dependence on the cutoff value at the end of this section. The form factors are the same as the ones in the previous publications [7, 8], so that we could compare our results with theirs.

Using these form factors and the parton distributions in $\rho$, we obtain the $\rho N N$ and $\rho N \Delta$ process contributions to the $\Delta \bar{V} - \Delta \bar{d}$ in the nucleon. In Fig. 7, the $1L$, $2L$, $1T$, and $2T$ type distributions from the $\rho N N$ process are shown at $Q^2=1$ GeV$^2$ together with their total. The ordinary $1L$ term is the dominant contribution; however, the $2L$ term becomes important at $x > 0.3$. It is as large as the $1L$ distribution in the medium $x$ region although it is fairly small at $x < 0.05$. The $1T$ and $2T$ distributions are very small in the whole $x$ range. Because $p \to p^+ N$ is the only contributing process in which the valence $\bar{d}$ distribution in $p^+$ plays the main role, the $\rho N N$ contributions are negative in the $\Delta \bar{V} - \Delta \bar{d}$ in the nucleon.
where the numerator and denominator are defined as

\[
\frac{1}{A_1} = \frac{1}{V^2 \sqrt{2 g \nu}} = \frac{1}{V^2 \sqrt{2 g}}.
\]

(1.2)

The expression for the vertex and the denominator is used in the process of calculating the total cross section. The expression for the vertex and denominator is used in the process of calculating the total cross section.
APPENDIX A: ANALYTICAL EXPRESSIONS

OF MEAN MOMENT DISTRIBUITIONS

ACKNOWLEDGEMENTS

CONCLUSIONS

The results of (B) and (C) show that the correction factor of the mean moment distribution is given by:

\[ f(x) = \frac{1}{\Delta} \]

where \( \Delta \) is the correction factor.

The correction factor can be expressed as:

\[ \Delta = \frac{1}{\frac{1}{N} \sum_{i=1}^{N} f_i} \]

where \( f_i \) is the correction factor for each data point.

In general, the correction factor is given by:

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The correction factor is the ratio of the total correction to the number of data points.

For example, if there are 10 data points, and the correction factors are 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0, then the correction factor is:

\[ \Delta = \frac{1}{\frac{1}{10} \times (0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 + 0.8 + 0.9 + 1.0)} = \frac{1}{0.55} = 1.82 \]

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where \( f_i \) is the correction factor for each data point.
In spite of their claim, we believe that the FS results are right with the following reasons. We also checked the helicity amplitudes in Ref. [18], which is referred to as Jülich in the following. In addition to obvious types, our results differ from the Jülich expressions. First, complex conjugate should be taken if their expressions are given for the process \( N \rightarrow \rho B \) as indicated in their appendix. Second, \( f_6 \) terms have different sign. If the Jülich amplitudes were written for \( N_B \rightarrow B \) for \( B \rightarrow \rho N \), we would agree on their expressions. Depending on the \( \rho \) momentum direction, the \( f_6 \) term in Eq. (3.1) becomes either positive or negative, which could lead to the different sign of the \( g_6, f_6 \) terms. However, it is obvious that the outgoing meson is considered in the formalism. Furthermore, taking summation over the helicity amplitudes, we reproduce the unpolarized momentum distributions of Melnitchouk and Thomas [19, 24], whereas the CS results are inconsistent. The \( VN \) vertex in Eq. (3.1) is also consistent with the one in Ref. [29].

We also tested \( \rho N \Delta \) helicity amplitudes in the vertex momentum (B), but the results disagree on the Jülich expressions. However, if the momentum (A) is used, our results agree on them. It seems that the helicity amplitudes are chosen for the vertex (B) in \( \rho N \) and for (A) in \( \rho N \Delta \). Therefore, as far as we investigated, we believe that the FS calculations are right also for the \( \rho N \Delta \) process.

In the following, we show the helicity dependent meson momentum distributions. Because the distributions with \( \lambda_V = 0 \) are irrelevant for calculating \( \Delta f(y) \), so that they are not shown. The isospin factors are extracted from the expressions.

\begin{align}
\frac{\partial y'}{\partial y} y \frac{y'}{y'} &= \left( 1 + \frac{k_1^2}{y/\gamma m_N} \right)^2 \frac{D_{VNN}^{I \lambda}(y', k_1^2)}{F_{VNN}^{I \lambda}(y', k_1^2)} ,
\end{align}

(\text{A1})

\begin{align}
\frac{\partial y'}{\partial y} y \frac{y'}{y'} &= \left( 1 + \frac{k_1^2}{y/\gamma m_N} \right)^2 \frac{D_{VNN}^{I \lambda}(y', k_1^2)}{F_{VNN}^{I \lambda}(y', k_1^2)} ,
\end{align}

(\text{A2})

\begin{align}
\frac{\partial y'}{\partial y} y \frac{y'}{y'} &= \left( 1 + \frac{k_1^2}{y/\gamma m_N} \right)^2 \frac{D_{VNN}^{I \lambda}(y', k_1^2)}{F_{VNN}^{I \lambda}(y', k_1^2)} ,
\end{align}

(\text{A3})

\begin{align}
\frac{\partial y'}{\partial y} y \frac{y'}{y'} &= \left( 1 + \frac{k_1^2}{y/\gamma m_N} \right)^2 \frac{D_{VNN}^{I \lambda}(y', k_1^2)}{F_{VNN}^{I \lambda}(y', k_1^2)} ,
\end{align}

(\text{A4})

Here, the partial derivative is given by

\begin{align}
\frac{\partial y'}{\partial y} y \frac{y'}{y'} &= \left( 1 + \frac{2}{y \gamma/m_N} \right)^{-1/2} \left( k_1^2 + m_N^2 \right) .
\end{align}

(\text{A5})

The distributions \( D_{VNN}^{I \lambda}(y', k_1^2) \) are calculated for the prescription (A) as

\begin{align}
D_{VNN}^{I \lambda}(y', k_1^2) &= \frac{2}{y^{1/2}(1 - y)^{1/2}} \left[ g_2 \left( k_1^2 + y^2 m_N^2 \right) + g \left( 1 - y \right) \left\{ \left( 1 - y \right) m_N^2 \right\} \right] + \frac{k_1^2}{4y^2 m_N^2} \left[ y^2 m_N^2 k_1^2 + \left( y^{2} m_N^2 - (1 - y) m_N^2 \right)^{2} \right] ,
\end{align}

(\text{A6})

\begin{align}
D_{VNN}^{I \lambda}(y', k_1^2) &= \frac{2k_1^2}{y^{3/2}(1 - y)^{1/2}} \left[ g_2 (1 - y)^2 - g \left( 1 - y \right) \left\{ \left( 1 - y \right) m_N^2 \right\} + \frac{k_1^2}{4y^2 m_N^2} 2y^2 m_N^2 \left( k_1^2 + y^2 m_N^2 - (1 - y) m_N^2 \right) \right] .
\end{align}

(\text{A7})

In the \( VN \Delta \) process, the distributions are calculated for the vertex momentum (A) as

\begin{align}
D_{VNN}^{I \lambda}(y', k_1^2) &= \frac{2}{3y^{3/2}(1 - y)^{1/2} m_N^2 m_V^2} \left[ k_1^4 + k_1^2 \left\{ 3 - 4y^2 (1 - y)^2 \right\} m_N^2 \right] + \frac{k_1^2}{2y^2 m_N^2} \left[ y^{2} (1 - y)^{3} m_N m_N m_N \right] + \frac{k_1^2}{2y^2 m_N^2} \left[ (1 - y)^{2} m_N^2 - (1 - y)^{2} m_N^2 \right] .
\end{align}

(\text{A9})

In the following, we show the helicity dependent meson momentum distributions. Because the distributions with \( \lambda_V = 0 \) are irrelevant for calculating \( \Delta f(y) \), so that they are not shown. The isospin factors are extracted from the expressions.
\[ D^\nu_{\Delta N}(y', k_2^2) = \frac{\nu_{\Delta N} k_1}{3 g^2 (1 - y') m^2_{\Delta N} \bar{m}^2_{\gamma} \bar{m}^2_{\nu}} \left[ k_1^4 m_N \right. \\
- 2y k_1 \left\{ (1 - 2y) m_N m^2_{\Delta} + (1 - y') m_N m^2_{\nu} \right\} \\
- 2 (2 - 3y) y^3 m_N^2 m^2_{\Delta} - 2 y^3 (1 - y') m^2_{\nu} \bar{m}^2_{\nu} \right] \\
+ 2y (1 - y') m^2_{\Delta} m_N m^2_{\nu} - (1 - y') m^2_{\nu} m^2_{\nu} \right). \quad (A11) \]

In the same way, the distributions are obtained for the prescription (B) as

\[ D^\nu_{\Delta N}(y', k_2^2) = \frac{2 y^3 (1 - y') m^2_{\nu}}{3 g^2 (1 - y') m^2_{\Delta N} \bar{m}^2_{\gamma} \bar{m}^2_{\nu}} \left[ 2 y^3 (1 - y') \right] \\
+ g_y f_y f_y \left\{ (1 + y') k_1^2 + 2 y^3 m^2_{\nu} \right\} \\
+ \frac{f^2_y}{4 m^4_{\nu}} \left( k_1^2 + y^2 m^2_{\nu} \right) \left( k_1^2 + 4 y^2 m^2_{\nu} \right) \right]. \quad (A12) \]

\[ D^T_{\Delta N}(y', k_2^2) = \frac{\nu_{\Delta N} k_1}{y^3 (1 - y') m^2_{\Delta N} \bar{m}^2_{\gamma} \bar{m}^2_{\nu}} \left[ 2 y^3 (1 - y') m^2_{\nu} \right. \\
- g_y f_y f_y \left\{ (1 - y') y m^2_{\nu} \right\} \\
+ \frac{f^2_y}{4 m^4_{\nu}} \left[ 4 y^3 m^2_{\nu} \right] \left( k_1^2 + y^2 m^2_{\nu} \right) \right]. \quad (A13) \]

\[ D^T_{\Delta N}(y', k_2^2) = \frac{\nu_{\Delta N} k_1}{y^3 (1 - y') m^2_{\Delta N} \bar{m}^2_{\gamma} \bar{m}^2_{\nu}} \left[ 2 y^3 (1 - y') m^2_{\nu} \right. \\
+ g_y f_y f_y \left\{ (1 - y') y m^2_{\nu} \right\} \\
+ \frac{f^2_y}{4 m^4_{\nu}} \left[ 4 y^3 m^2_{\nu} \right] \left( k_1^2 + y^2 m^2_{\nu} \right) \right]. \quad (A14) \]

\[ D^T_{\Delta N}(y', k_2^2) = \frac{\nu_{\Delta N} k_1}{y^3 (1 - y') m^2_{\Delta N} \bar{m}^2_{\gamma} \bar{m}^2_{\nu}} \left[ 2 y^3 (1 - y') m^2_{\nu} \right. \\
+ k_1 \left\{ (3 - 4y') y^2 m^2_{\nu} + 4 y^3 y m^2_{\nu} \right\} \\
+ k_1 \left\{ (1 - y') y^2 m^2_{\nu} \right\} \\
- 2 y^3 (1 - y') m_N m^2_{\nu} + 2 y^3 (1 - y') m^2_{\nu} m^2_{\nu} \right] \\
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+ y^2 (1 - y') m^2_{\nu} \right]. \quad (A15) \]

\[ D^T_{\Delta N}(y', k_2^2) = \frac{\nu_{\Delta N} k_1}{3 y^3 (1 - y') m^2_{\Delta N} \bar{m}^2_{\gamma} \bar{m}^2_{\nu}} \left[ k_1^4 \right. \\
+ k_1 \left\{ (3 + 2y') y^2 m^2_{\nu} + 4 y^3 y m^2_{\nu} \right\} \\
+ 2y^3 (1 - y') m_N m^2_{\nu} + 2 y^3 (1 - y') m_N m^2_{\nu} \right] \\
- 2 y^3 (1 - y') m_N^2 m^2_{\nu} - 2 y^3 (1 - y') m^2_{\nu} m^2_{\nu} \right] \\
+ 3 y^2 (1 - y') m^2_{\nu} m^2_{\nu} \right]. \quad (A16) \]

\[ D^T_{\Delta N}(y', k_2^2) = \frac{\nu_{\Delta N} k_1}{3 y^3 (1 - y') m^2_{\Delta N} \bar{m}^2_{\gamma} \bar{m}^2_{\nu}} \left[ k_1^4 \right. \\
+ k_1 \left\{ (3 + 2y') y^2 m^2_{\nu} + 4 y^3 y m^2_{\nu} \right\} \\
+ 2y^3 (1 - y') m_N m^2_{\nu} + 2 y^3 (1 - y') m_N m^2_{\nu} \right] \\
- 2 y^3 (1 - y') m_N^2 m^2_{\nu} - 2 y^3 (1 - y') m^2_{\nu} m^2_{\nu} \right] \\
+ 3 y^2 (1 - y') m^2_{\nu} m^2_{\nu} \right]. \quad (A17) \]

The longitudinal distributions agree on the FS results in the limit \( Q^2 \rightarrow \infty \) except for a term in Eq. (A16). The factor \( 1/y^3 (1 - y')^3 \) is written as \( 1/y^2 (1 - y')^3 \) in Ref. [7]. It is possibly a misprint [28].

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