Dual Superconductivity and Chiral Symmetry in Full QCD

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1. Introduction

Confinement is experimentally a well established property of strong interactions. In the quenched approximation a phase transition driven by temperature from a confined to a deconfined phase has been observed by lattice simulations, an order parameter being the v.e.v. of the Polyakov line, which is zero in the confined (low temperature) phase and nonzero in the deconfined (high temperature) phase, thus signalling the spontaneous breaking of the $Z_3$ centre symmetry. The situation is different in full QCD, where the presence of dynamical quarks makes the $Z_3$ centre symmetry explicitely broken, so that the v.e.v. of the Polyakov line ceases to be an order parameter. One speaks instead of a phase transition from a spontaneously broken to a restored chiral symmetry, the chiral condensate being the order parameter. Anyway, at the chiral phase transition a peak in the Polyakov line susceptibility is still observed, indicating a sort of deconfining crossover. However up to now the relation between chiral symmetry and confinement is still not well understood.

On the other hand, confinement in the quenched theory is now well understood in terms of an order/disorder phase transition [1,2], with the QCD vacuum behaving like a dual superconductor at low temperatures and undergoing a phase transition to a normal conducting state at the critical temperature $T_c$. A disorder parameter which is the v.e.v. of a magnetically charged operator has been developed and used to demonstrate dual superconductivity of the vacuum for U(1), SU(2) and SU(3) Yang-Mills theory [3–7]. Contrary to the v.e.v. of the Polyakov line, the definition of the disorder parameter detecting dual superconductivity can be extended to full QCD. It is then possible to test whether there is any transition from a dual superconducting to an ordinary vacuum also in presence of dynamical fermions and how it is related to the chiral phase transition. This is the aim of our study and we will present some preliminary results in this paper.

2. The disorder parameter in full QCD

The definition of the disorder parameter in pure gauge theories has been discussed in [3–7], so we will only recall here the main points. It is the
v.e.v. of a gauge invariant operator, $\mu$, which carries magnetic charge. The $U(1)$ magnetic subgroup is selected by the so called abelian projection. A gauge fixing is performed, usually by diagonalizing a local operator in the adjoint representation, which leaves a $U(1) \times U(1)$ gauge freedom (for $SU(3)$), corresponding to two possible definitions of the magnetic charge. It has been proved in [7] that the dual superconductivity mechanism for confinement works independently of the abelian projection chosen, as already suggested in [2].

On the lattice $\mu$ can be written as [3–7]

$$ \mu \equiv \exp \frac{\beta}{3} \sum_{\vec{x}, \vec{t}} \text{Re} \text{Tr} \left( \hat{U}_0(\vec{x}, t_0) - U_0(\vec{x}, t_0) \right), $$  \hspace{1cm} (1)

where the sum in the exponential is extended only to the temporal plaquettes $U_0$ in the time slice $t = t_0$ where the magnetic monopole is created, and $U_0(\vec{x}, t_0)$ indicates a plaquette modified by the insertion of a monopole field, defined as an abelian field in the abelian projected gauge. The disorder parameter is then defined as

$$ \langle \mu \rangle = \frac{\int (DU) e^{-S_G}}{\int (DU) e^{-S}}, $$

where $S_G$ is the standard Wilson action and $S$ is a modified action obtained by inserting the monopole field in the temporal plaquettes on time slice $t_0$. It can be shown [4], by a change of integration variables, that the insertion of a monopole field in the temporal plaquettes on time slice $t_0$ is equivalent to creating a monopole at $t_0$ which then propagates at all times $t > t_0$. For this reason, periodic boundary conditions in time direction are not consistent with the definition of $\langle \mu \rangle$, and $C^*$-periodic boundary conditions have to be used instead:

$$ U_\mu(\vec{x}, t = N_t) = U_\mu^*(\vec{x}, t = 0), $$ \hspace{1cm} (3)

$N_t$ being the temporal extension of the lattice and $U_\mu^*$ being the complex conjugated of $U_\mu$.

A lattice determination of $\langle \mu \rangle$ is difficult, since it is the average of a quantity which fluctuates exponentially with the square root of the physical volume. It is much more convenient to study

$$ \rho = \frac{\partial}{\partial \beta} \log \langle \mu \rangle = \langle S_G \rangle - \langle S_M \rangle, $$ \hspace{1cm} (4)

where $\langle \cdot \rangle$ indicates the v.e.v. obtained using the action $S$ to weight configurations. From the knowledge of $\rho$ the relevant physical information on $\langle \mu \rangle$ can be extracted. The result in pure gauge theory is that $\langle \mu \rangle \neq 0$ in the confined, low temperature phase, while it goes to zero at $T_c$, with critical indices which can be extracted from the finite size scaling behaviour of $\rho$.

In full QCD the definitions of $\langle \mu \rangle$ and $\rho$ are modified in a very simple way:

$$ \langle \mu \rangle = \frac{\int (D\bar{\psi}D\psi DU) e^{-S_M-S_F}}{\int (D\bar{\psi}D\psi DU) e^{-S_G-S_F}}, $$ \hspace{1cm} (5)

where $S_F$ is the fermionic action, and

$$ \rho = \langle S_G \rangle - \langle S_M \rangle. $$ \hspace{1cm} (6)

Unlike the $Z_3$ centre symmetry, the $U(1)$ magnetic symmetry defined after abelian projection is still a good symmetry also in presence of dynamical fermions. Therefore $\langle \mu \rangle$ can be a correct disorder parameter for the dual superconductivity transition also in full QCD.

3. Algorithm implementation and simulation details

We have used two flavours of staggered fermions and the R version of the HMC algorithm for our simulations. Some technical complications arise in the computation of the second member on the right hand side of Eq. (6). In the evaluation of $\langle S_M \rangle$, the use of $C^*$-periodic boundary conditions in time direction for the gauge fields requires $C^*$ boundary conditions in temporal direction also for fermionic variables (in addition to the usual antiperiodic ones), in order to ensure gauge invariance of the fermionic determinant. This implies relevant changes in the formulation and implementation of the HMC algorithm which are explained in detail in Ref. [8].

We have chosen the Polyakov line as the local adjoint operator which defines the abelian projection. Actually, calling $L(\vec{x}, t)$ the Polyakov line starting at point $(\vec{x}, t)$, the abelian projection is defined by the operator $L(\vec{x}, t)L^*(\vec{x}, t)$, which transforms in the adjoint representation when using $C^*$ boundary conditions.
The use of a modified gauge action also implies changes in the molecular dynamics equations. One has to maintain constant the modified hamiltonian containing $S_M$. A change in any temporal link indeed induces a change in $L(\vec{x},t)$ and hence in the abelian projection defining the monopole field. Therefore the dependence of $S_M$ on temporal links is non trivial and the equations of motion for the temporal momenta become more complicated. Details of the changes necessary in the molecular dynamics equations will be published in a forthcoming paper [9]. We have used a temporal lattice extent $N_t = 4a$ and spatial extents $N_s = 16, 32$. We have chosen to vary the temperature, $T = 1/(N_t a(\beta, m_\rho))$, moving in the ($\beta, m_\rho$) plane while keeping a fixed value of $\frac{m_\pi}{m_\rho}$. To do this and to extract the physical scale we have used fits to the $m_\rho$ and $m_\pi$ masses published in [10]. In particular we present here results obtained at $\frac{m_\pi}{m_\rho} \simeq 0.505$.

4. Results and Outlook

Results obtained for $\rho$ on the $16^3 \times 4$ and $32^3 \times 4$ lattices are shown in Fig. 1. A clear peak is present in both cases and, as can be seen in Fig. 2, it is located exactly at the chiral phase transition, at a critical temperature $T_c \sim 150$ MeV. Moreover, as is visible in Fig. 1, the peak scales roughly by a factor of eight when going from $N_s = 16$ to $N_s = 32$, i.e. $\propto N_s^2$. Previous studies of full QCD at a similar set of physical parameters [11] have found a crossover rather than a phase transition at $T_c$. From our present results we can draw the following conclusions: i) the disorder parameter shows a behaviour very similar to that observed in pure Yang-Mills theories, thus indicating the presence of a transition from a dual superconducting to an ordinary vacuum state also for full QCD. This transition clearly happens in correspondence of the chiral phase transition; ii) our results seem to scale roughly with a critical index $\nu \sim 1/3$. Of course we still need to collect more data near the critical point and at different values of $N_s$ in order to draw more precise conclusions about the critical behaviour of $\rho$ at $T_c$.

REFERENCES