One-loop renormalization of heavy-light currents

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We calculate the mass dependent renormalization factors of heavy-light bilinears at one-loop order of perturbation theory, when the heavy quark is treated with the Fermilab formalism. We present numerical results for the Wilson and Sheikholeslami-Wohlert actions, with and without tree-level rotation. We find that in both cases our results smoothly interpolate from the static limit to the massless limit. We also calculate the mass dependent Brodsky-Lepage-Mackenzie scale \( q^\star \), with and without tadpole-improvement.

1. INTRODUCTION

Although lattice QCD offers a nonperturbative method of calculating weak matrix elements from first principles, in practice a perturbative renormalization is also required to extract the continuum quantities for heavy-light systems. In this talk we discuss the renormalization of heavy-light vector and axial vector currents. These currents are needed for heavy quark phenomenology, such as the calculation of the decay constants and semi-leptonic form factors of heavy-light mesons. Here we calculate explicitly the mass dependent renormalization factors at one-loop order of perturbation theory.

In view of the mass dependence, we write
\[
e^{-m_{\text{lat}}[0]a/2} Z_{J_G} = 1 + \frac{\sum_{l=1}^{\infty} g_{10}^{(l)} Z_{J_G}^{(l)}}{Z_{J_G}^{\text{lat}}},
\]
so that the \( Z_{J_G}^{(l)} \) are only mildly mass dependent.

2. ONE-LOOP RESULTS

The renormalization factors \( Z_{J_G} \) of heavy-light currents are simply the ratio of the lattice and continuum radiative corrections:
\[
Z_{J_G} = \frac{\lvert Z_{2h}^{1/2} \Lambda_1 Z_{2l}^{1/2} \rvert_{\text{cont}}}{\lvert Z_{2h}^{1/2} \Lambda_1 Z_{2l}^{1/2} \rvert_{\text{lat}}},
\]
where \( Z_{2h} \) and \( Z_{2l} \) are wave-function renormalization factors of the heavy and light quarks, and the vertex function \( \Lambda_1 \) is the sum of one-particle irreducible three-point diagrams. We calculate explicitly \( Z_A \) and \( Z_V \) at one-loop order of perturbation theory.

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Fig. 1 plots the full mass dependence of the renormalization factors for the axial vector current \( Z_A^{1/2} \). These numerical results are for the SW action with and without rotation, and also for Wilson action without rotation. Our results agree with those previously obtained, for \( c_{SW} = 0 \) [2].
and for \( c_{SW} = 1, d_1 = 0 \) [3]. We find that in both cases our results smoothly interpolate from the static to massless limit. The resulting analytical expressions are in Ref. [4]. Fig. 2 plots the tadpole-improved renormalization factor for \( Z_{A_4}^{[1]} \). From this figure, we can see that tadpole-improvement significantly reduces the one-loop coefficients of renormalization factors. Results for \( Z_{A_4}^{[1]} \) and \( Z_{V_4}^{[1]} \) are given in Ref. [4].

The slope of our mass-dependent renormalization factors in the massless limit is related to the result of \( b \Gamma \) and \( c \Gamma \) for the heavy-light current. Here we calculate the mass dependence of \( q^* \) for the heavy-light current. Results are plotted in Fig. 3. For Wilson action case, our results agree with Ref. [9] in the massless limit. From Fig. 3, we can see that the mass dependence of \( q^* \) is weak from massless limit to \( m_{0a} \sim 1 \), especially for clover with rotation case. The original BLM prescription of \( q^* \) breaks down at larger masses, because \( Z_{A_4}^{[1]} \) (denominator in Eq. (8)) goes through zero at there. A prescription for \( q^* \) in this case is given in Ref. [12]. We also calculate tadpole-improved \( q^* \) and results are plotted in Fig. 4. We can see that plaquette tadpole-improvement significantly reduces \( q^* \), on the other hand, the reduction is rather small for \( \kappa_c \) tadpole-improvement. We summarize the results in the massless limit in Table 1.

We can also obtain the BLM scale for improvement coefficients \( b_J \) and \( c_J \) [5]. Then it is interesting to compare BLM perturbation theory with non-perturbative calculations of these coefficients [7][13]. We will present these results for \( q^* \) and the mentioned comparison in another publication [14].
4. CONCLUSIONS

We have obtained one-loop results of \( Z_A \) and \( Z_V \) with tree-level rotation, which should be useful for lattice calculations of \( f_B \) and of form factors for \( B \to \pi l \nu \). We have also obtained the BLM scale \( q^* \) for arbitrary masses, which should reduce the uncertainty of one-loop calculations.

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### Table 1

<table>
<thead>
<tr>
<th></th>
<th>no improvement</th>
<th>plaquette through ( \kappa_c )</th>
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<tbody>
<tr>
<td>( Z_A^{[1]} )</td>
<td>-0.116457(2)</td>
<td>-0.033124(2)</td>
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<tr>
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<td>-0.133375(2)</td>
<td>-0.050042(2)</td>
</tr>
<tr>
<td>( Z_V^{[1]} )</td>
<td>-0.129430(2)</td>
<td>-0.046097(2)</td>
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<tr>
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<td>-0.174086(2)</td>
<td>-0.090752(2)</td>
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<td>( q_{Z,A}^* )</td>
<td>2.839</td>
<td>1.802</td>
</tr>
<tr>
<td></td>
<td>2.533</td>
<td>1.550</td>
</tr>
<tr>
<td>( q_{Z,V}^* )</td>
<td>2.845</td>
<td>2.060</td>
</tr>
<tr>
<td></td>
<td>2.370</td>
<td>1.700</td>
</tr>
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</table>

### REFERENCES

13. S. Collins et al., hep-lat/0109036.