Temporal quark and gluon propagators: measuring the quasiparticle masses

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We calculate the Coulomb gauge temporal quark and gluon propagators in quenched QCD. From the temporal quark and gluon propagators, dispersion relations and quasiparticle masses are determined by means of the Maximum Entropy Method.

It is well known that QCD undergoes a phase transition to a deconfined phase where it is believed that the dominant degrees of freedom are quasiparticles with quantum numbers of quarks and gluons. Quasiparticles show up as poles in the retarded quark and gluon propagators. The corresponding dispersion relations (the position of the poles) was studied in so-called HTL perturbation theory which is valid if the separation of different scales holds \( 1/T \ll 1/gT \ll 1/g^2T \) (see e.g. [1] for a review). In the interesting temperature region, however, the coupling is large \( g \gg 1 \).

Nevertheless, the corresponding quasiparticle picture finds application in refined perturbative calculations of the bulk thermodynamic properties where it helps to improve the convergence of perturbative series [2] as well as in more phenomenological approaches [3]. In view of these facts a non-perturbative study of quark and gluon dispersion relations in the deconfined phase is highly desirable.

We have calculated the quark and gluon propagators in quenched QCD with Wilson fermions on \( 64^3 \times 16 \) lattice at two different temperatures, \( T = 1.5T_c \) using 20 configurations and \( T = 3T_c \) using 40 configurations (\( T_c \) is the critical temperature of the deconfinement phase transition). We have used the standard Wilson action for the gauge fields and an \( O(a) \) improved fermion action (clover action). Furthermore we have performed our simulations at the critical value of the hopping parameter \( \kappa = \kappa_c(T = 0) \) determined at zero temperature. The values of the gauge coupling \( \beta \) corresponding to the two values of the temperatures are \( \beta = 6.872 \) for \( T = 1.5T_c \) and \( \beta = 7.457 \) for \( T = 3T_c \). The non-perturbative value of the clover coefficient \( c_{sw} \) and of the value of the critical hopping parameter \( \kappa_c \) for these \( \beta \)-values were obtained from interpolations of results given in [4]: \( c_{sw} = 1.412488, \kappa_c = 0.13495 \) for \( T = 1.5T_c \) and \( c_{sw} = 1.338924, \kappa_c = 0.13390 \) for \( T = 3T_c \).

Lattice calculations can provide information on the imaginary-time (Matsubara) propagator \( D(i\omega_n, p) \) (\( \omega_n \) being the Matsubara frequencies). This is related to the retarded propagator by analytic continuation \( D_R(p_0, p) = -D(p_0 + i\epsilon, p) \). This implies

\[
D(i\omega_n, p) = -\int_{-\infty}^{+\infty} d\omega \frac{\rho(\omega, p)}{i\omega_n - \omega}, \tag{1}
\]

where \( \rho(\omega, p) = \frac{1}{\pi} \text{Im} D_R(\omega + i\epsilon, p) \) is the spectral function.

As quark and gluon propagators are gauge dependent quantities it is necessary to fix a particular gauge which we chose to be the Coulomb gauge. In this gauge the condition \( \partial_\tau A_i(\tau, \vec{x}) = 0 \) is imposed independently on different time slices which allows to construct a transfer matrix [5].

As the consequence of this the spectral function of quarks and gluons is positive which allows to use the Maximum Entropy Method (MEM) to reconstruct the spectral functions (see Ref. [7] for a review). Though the quark and gluon propagators are gauge dependent quantities the positions of the peaks in the spectral function (poles in

\[\text{To fix the gauge completely an additional time-dependent gauge transformation is necessary. This, however, depends on temporal links } \mathcal{U}_0(\tau, x) \text{ only [6].}\]
the retarded propagators) is gauge independent at any order of perturbation theory \[8\]. Gauge independence of the peak position can be proven also non-perturbatively in a class of gauges allowing the construction of a transfer matrix \[5\]. At finite temperature there are two kinds of quasiparticle excitations (corresponding to two branches in the dispersion relation \[1\]): the real quasiparticles (quarks and transverse gluons), which are the analog of partonic degrees of freedom at zero temperature, and collective excitations (plasmino and longitudinal gluons), which have exponentially small residues for momenta \( p > T \).

We have calculated the quark and gluon propagators in the mixed \((\tau,p)\)-representation, \( D(\tau,p) = \sum_n D(\omega_n,p) \exp(-i\omega_n \tau) \) for several values of the 3-momenta \( p \). Since the smallest non-zero momentum available on our lattices is \( p_{\text{min}} = 1.5 T \), the particular behavior of the collective excitations at small momenta cannot be resolved. Using Eq. (1) the gluon propagator in the mixed representation can immediately be written in terms of the spectral function,

\[
D_g^{(T,L)}(\tau,p) = \int_0^{\infty} d\omega \rho_g(\omega,p) \frac{\cosh(\omega(\tau - 1/2 T))}{\sinh(\omega/2 T)},
\]

where \( T \) and \( L \) refer to transverse and longitudinal gluons respectively. For the longitudinal propagators at non-zero momentum we have found \( D_L(\tau,p) = 0 \) within statistical accuracy of our calculations. Using Eq. (2) we determine the gluon spectral function using MEM and from the position of the peak the gluon dispersion relation \( \omega_g(p) \). We note, however, that the gluon spectral function has a significant continuum contribution above the light cone \((\omega > p)\) which is absent in HTL perturbation theory.

We also have studied the behavior of the temporal gluon propagators in terms of the effective masses (see e.g. \[6\] for the definition). Usually effective masses reach a plateau which is the zero temperature mass or the screening mass at finite temperature. This need not be the case for the temporal effective masses at finite temperature. In fact, the effective masses \( m_g(\tau,p) \) extracted from the gluon propagators are always larger than the position of the peak in the gluon spectral function. This can be attributed to the presence of the continuum contribution in the spectral function.

The temporal gluon propagators are influenced by a zero Matsubara mode contribution \( D_g^{(T,L)}(i\omega_n = 0, p) \), which is the static magnetic propagator in momentum space studied in detail in Ref. \[6\]. At \( p = 0 \) the magnetic propagators are strongly volume dependent and very large lattices are needed to perform a reliable infinite volume extrapolation \[6\]. As a consequence the dispersion relation at zero momentum , i.e the plasmon frequency \( \omega_P \), cannot be reliably determined from our present calculations.

The most general form of the temporal quark
propagator is

$$D^0(\tau, p) = \gamma_0 F(\tau, p) + \vec{\gamma} \cdot \vec{n} G(\tau, p) + H(\tau, p),$$  \hspace{1cm} (3)

with $\vec{n} = \vec{p}/p$. In the chiral limit the last term vanishes. Indeed, we have found that $H(\tau, p) = 0$ within statistical errors. Using the most general form for the retarded quark propagator [9] and Eq.(1) one can derive the following representation for the functions $F$ and $G$:

$$F(\tau, p) = \int_0^\infty d\omega \rho_F(\omega, p) \frac{\cosh(\omega(\tau - 1/2T))}{\cosh(\omega/T)},$$  \hspace{1cm} (4)

$$G(\tau, p) = \int_0^\infty d\omega \rho_G(\omega, p) \frac{\sinh(\omega(\tau - 1/2T))}{\cosh(\omega/T)}. $$  \hspace{1cm} (5)

It turns out that our data on $G$ are too noisy to apply the MEM analysis to them. The local masses extracted from $F$ and $G$, however, are identical within statistical errors. In fact, if the quark propagators are dominated by a single quasiparticle contribution, the local masses extracted from $F$ and $G$ should be identical. We therefore applied the MEM analysis only for $F$. The reconstructed spectral function $\rho_F(\omega, p)$ shows only a single peak indicating the absence of the plasmino branch for the values of the momenta studied by us (at zero momentum there is no distinction between the quasiparticle (quark) and plasmino branch). In contrast to the gluon spectral function the quark spectral function extracted from $F$ has negligible continuum contribution above the light cone. As a consequence the corresponding local masses $m_q(\tau, p)$ show a plateau already at $\tau a = 1$ ($a$ denotes the lattice spacing). The position of the peak in the spectral function $\omega_q(p)$ agrees with the average value of the local masses $m_q(\tau, p)$.

Our numerical results for the dispersion relations of quark and gluons $\omega_q(p)$ and $\omega_g(p)$ are summarized in Fig. 1. While at $T = 3T_c$ the dispersion relation both for quarks and gluons is close to the free dispersion relation $\omega^2(p) = p^2$ one sees large deviations from it at $T = 1.5T_c$. In order to quantify the deviations from the free propagation we have fitted the dispersion relations to $\omega^2(p) = p^2 + m^2_{q,g}$ and determined the values of quasiparticle masses $m_q$ and $m_g$. For $T = 3T_c$ we have found $m_q/T = 1.7 \pm 0.1$ and $m_g/T = 1.2 \pm 0.1$. The value of the gluon mass is compatible with the leading order result of HTL perturbation theory $m_g^2 = g^2 T^2/2$ if we assume for the gauge coupling $g(3T_c) \sim 1.6$ as suggested by the short distance behavior of the heavy quark potential [10]. With the same value of $g$ the leading HTL result for $m_q$ is roughly a factor 2 smaller than the value found by us. For $T = 1.5T_c$ we have obtained $m_q/T = 3.9 \pm 0.2$, $m_g/T = 3.4 \pm 0.3$. Here the value of $m_q$ was obtained by omitting the first three values of $\omega_q(p)$ from the fit. The values of the quasiparticle masses at $T = 1.5T_c$ are considerably larger than the corresponding ones at $3T_c$ and those expected from perturbation theory. This is consistent with the temperature dependence of quasiparticle masses $m_{q,g}/T$ used in quasiparticle models for the equation of state [3].

Acknowledgments: The work has been supported by the TMR network ERBFMRX-CT-970122 and by the DFG under grant FOR 339/1-2. The numerical calculations have been performed on Cray T3E at the NIC, Jülich, and at the HLRS in Stuttgart.

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