A Simple Solution to the Strong $CP$ Problem

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The problem is simply stated. The minimal three-family standard model involves two distinct CP-violating phases: the $\delta$ parameter in the Kobayashi-Maskawa matrix and $\bar{\theta}$, the measure of strong CP violation, both of which can be generated by complex terms in the quark mass matrices. The former angle is of order unity, whereas the latter is known to be less than $10^{-9}$. The strong CP problem arises if this disparity is regarded as other than coincidental.

There have been many proposed resolutions to this problem of which three classics are: a massless up quark [1], an invisibilized version of the Peccei-Quinn axion [2], and the Barr-Nelson implementation of spontaneous CP violation [3]. Although none of these $\bar{\theta}$-avoidance mechanisms are decisively excluded, neither is convincingly true. Consequently there have been many other suggested remedies which would be too tedious to enumerate. Some models appeal to supersymmetry [4], others appeal to technicolor [5], and still others invoke extra dimensions [6]. In addition, many solutions have been put forward in which CP is softly broken [7]. The last-cited models (of which I am a coauthor) are decisively ruled out by currently available experimental data. In this note I present yet another model making use of soft CP violation, one which is both simpler than its predecessors and seemingly compatible with experiment.

We begin by assigning a flavor quantum number $F$ to each quark family. We assign $F = +1$ to the right- and left-handed quarks of a nominal first family, $F = 0$ to those of a second family, and $F = -1$ to those of the third family. (Similar $F$ assignments could be made to the leptons.) In addition, we introduce three doublets of Higgs bosons, $h_0$, $h_1$ and $h_2$, where the subscripts indicate their $F$ quantum numbers. Each of these doublets contributes both to the up and down quark mass matrices. Although the Glashow-Weinberg constraint [8] is not respected, unacceptable flavor-changing effects should be avoidable with a modest degree of fine tuning.

All dimension-4 terms in the Lagrangian are assumed to be both CP and flavor invariant. As a result of this hypothesis, the Yukawa couplings of the $h_i$ to quarks (as well as their quartic self-couplings) must be real and must conserve the flavor quantum number $F$. Furthermore, the CP-violating Chern-Simons term cannot be present in the Lagrangian. Quadratic expressions in the Higgs fields are not constrained by the above hypothesis. These dimension-2 mass terms are permitted to be complex and $F$-violating. This is our proposed mechanism for the explicit but soft violation of both CP invariance and the flavor symmetry.
The parameters of the Higgs portion of the Lagrangian are chosen so that each of the three electrically neutral Higgs bosons develops a complex vacuum expectation value (vev). An appropriate weak-hypercharge rotation lets us choose \( \langle h_0 \rangle \) to be real with no loss of generality. The sum of the squared magnitudes of the vevs is constrained to take its conventional electroweak value. We assume that all three vevs are similar in magnitude to avoid the appearance of light surviving bosons (i.e., with masses less than \( \sim 100 \) GeV) such as are known not to exist. Observed violations of CP invariance are caused by the complexity of the Higgs vevs, which lead to the irremovable complexity of the quark mass matrices \( M_U \) and \( M_D \). Furthermore, the \( F \) invariance of the Yukawa couplings ensures that in tree approximation \( M_D \) is an upper triangular matrix whilst \( M_U \) is lower triangular. All diagonal entries of these matrices are real and the phases of the off-diagonal entries are constrained in a manner to be discussed.

The triangular mass matrices we obtain are sufficiently general to produce any spectrum of quark masses and can result in any desired Kobayashi-Maskawa matrix. Furthermore, the determinants of these mass matrices are evidently real, so that there is no strong CP problem in tree approximation.

With one possible identification of the observed quarks with the flavor quantum number, we obtain:

\[
M_D = \begin{pmatrix}
    m_d & \epsilon_{12} & \epsilon_{13} \\
    0 & m_s & \epsilon_{23} \\
    0 & 0 & m_b
\end{pmatrix}
\quad \text{and} \quad
M_U = \begin{pmatrix}
    m_u & 0 & 0 \\
    \epsilon_{21}^* & m_c & 0 \\
    \epsilon_{31}^* & \epsilon_{32}^* & m_t
\end{pmatrix},
\]

where the \( m_i \) are the quark masses in the absence of mixing. Quark mixing is induced by smaller off-diagonal entries, whose complex phases are constrained as follows:

\[
\arg \epsilon_{12} = \arg \epsilon_{21} = \arg \epsilon_{23} = \arg \epsilon_{32}, \quad \text{and} \quad \arg \epsilon_{13} = \arg \epsilon_{31}.
\]

(2)

Note that the complexity of the mass matrices is irremovable except under the coincidental circumstance that

\[
\chi \equiv \arg(\epsilon_{12}^2 \epsilon_{13}^*) = 0.
\]

(3)

It is illustrative to estimate the Kobayashi-Maskawa parameters in the small-mixing approximation. We obtain:

\[
\theta_{12} \approx |\epsilon_{12}/m_s - \epsilon_{21}^* m_u/m_c^2|, \quad \theta_{23} \approx |\epsilon_{23}/m_b - \epsilon_{32}^* m_c/m_t^2|, \quad \theta_{31} \approx |\epsilon_{13}/m_b - \epsilon_{31}^* m_u/m_t^2|,
\]

(4)
with the $\delta$ parameter assuming any desired value. Note that the KM parameters are essentially determined by the off-diagonal terms of $M_D$, provided that the $\epsilon_{ij}$ with $j > i$ are comparable to those with $i > j$. Putting in numbers for quark masses and mixings, we obtain the rough estimates:

\[ \epsilon_{12} \approx 25 \text{ MeV}, \quad \epsilon_{13} \approx 13 \text{ MeV}, \quad \epsilon_{23} \approx 150 \text{ MeV}. \]  \tag{5}

While our model does not suffer a strong CP problem in tree approximation, radiative corrections to quark masses will modify the quark mass matrices and can thereby lead to a non-vanishing value for $\bar{\theta}$. Are these corrections small enough to avert a problem? An examination of one-loop corrections to the quark mass matrices reveals that the only such terms are those explicitly involving $\chi$, as defined in Eq. (3). The most threatening term by far is a complex contribution to the up-quark mass mediated by charged Higgs bosons, which may be estimated to yield:

\[ \Delta \bar{\theta} = \left( \frac{1}{4\pi} \right)^2 \frac{\epsilon_{13} \epsilon_{23}^* \epsilon_{21}^*}{(\text{vev})^2 m_u} K, \]  \tag{6}

where $K$ is a dimensionless integral which could be small for an appropriate choice of the spectrum and mixing pattern of the Higgs bosons. In any case, its prefactor is of order $10^{-9}$.

Several distinctive features of this model may be testable. In particular, it requires the existence of two surviving charged Higgs bosons, which should be readily detectable at a future linear collider. Furthermore, should one or both of these particles lie much below the top quark mass, we should expect a significant (although not readily calculable) branching ratio for the decay $t \to b + h^+$. The predicted existence of five neutral bosons should offer interesting challenges to experimenters. Although flavor violation via their exchange can be made small, it could be large enough to yield measurable departures from the standard-model description of CP violation.

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References