PARAMETRIZATION OF THE ENERGY SPECTRUM IN THE TRITIUM BETA DECAY.

J. STUDNIK AND M. ZRALEK

Department of Field Theory and Particle Physics, Institute of Physics, University of Silesia, Uniwersytecka 4, PL-40-007 Katowice, Poland

Abstract

Taking into account mixings among neutrino states, the end of the energy spectrum of the electron in the tritium beta decay is investigated. It is shown that for real energy resolutions of a spectrometer, $\Delta E > 0.08 \text{ eV}$, the effective electron neutrino mass should not be taken in the form

$$m_{\beta}^{(1)} = \sqrt{\sum |U_{ei}|^2 m_i^2} \quad \left[ m_{\beta}^{(2)} = \sqrt{\sum |U_{ei}|^2 m_i} \right].$$

Last atmospheric and solar experiments convince us that neutrinos are massive particles. However, the problem of absolute values of their masses is still waiting for a solution. Apparently three kinds of neutrino experiments have a chance to determine the light neutrino masses. These are:

1. neutrino oscillation experiments,
2. the tritium beta decay experiment,
3. neutrinoless double beta decay experiments.

Among them the tritium $\beta$ decay

$${}_1^3 \text{H} \rightarrow {}_2^3 \text{He} + e^- + \bar{\nu}_e,$$  \hspace{1cm} (1)

is of great importance. Here the end of the electron energy spectrum in the absence of the lepton mixing is described by [1]

$$\frac{dN}{dE} = R(E)(E_0 - E) \sqrt{(E_0 - E)^2 - m_{\beta}^2}. \hspace{1cm} (2)$$

$E$ is the electron kinematic energy ($E = E_{\text{tot}} - m_e \approx \frac{p^2}{2m_e}$), $E_0 = M(\text{}_1^3 \text{H}) - M(\text{}_2^3 \text{He}) - m_e \approx 18572.1 \text{ eV}$, and

$$R(E) = G_F^2 \frac{m_e^5 \cos \theta_c}{2\pi^3} |M|^2 F(E) \sqrt{2m_eE(E + m_e)}, \hspace{1cm} (3)$$

1
where $G_F$ is the Fermi constant, $\theta_c$ is the Cabibbo angle and M is the nuclear matrix element. $F(E)$ is neutrino mass independent, smooth function of $E$, which takes into account radiative corrections to the produced final state electron (see [1] for details). The effective mass $m_\beta$ of the produced neutrino is determined, regardless of the fact if it is a Dirac or a Majorana particle.

Presently only the upper limit on $m_\beta$ is available. Two experiments in Mainz [2] and Troitsk [3] have recently found

$$m_\beta < 2.2 \text{ eV} \ [2],$$
$$m_\beta < 2.5 \text{ eV} \ [3].$$

The real problem is how this effective mass $m_\beta$, extracted from the experiment, is connected to the realistic neutrino masses $m_i$. In the case of three light neutrinos (i=1,2,3) which mix ($|\nu_e\rangle = \sum U_{ei} |\nu_i\rangle$), the spectrum of the electron energy is given by [4]

$$\frac{dN}{dE} = R(E)(E_0 - E) \sum_{i=1}^{3} |U_{ei}|^2 \sqrt{(E_0 - E)^2 - m_i^2} \Theta(E_0 - E - m_i).$$

$\Theta(E_0 - E - m_i)$ is the step function. This formula depends on five parameters, namely three neutrino masses and two mixings because $\sum |U_{ei}|^2 = 1$. If only the upper bound on $m_\beta$ is determined (present situation, Eq. 4) then a precise relation

$$m_\beta = f(|U_{ei}|^2, m_i)$$

is not so important. However, future experiments, as KATRIN [5] with the sensitivity $m_\beta \sim 0.3 - 0.35 \text{ eV}$ have a chance to find $m_\beta \neq 0$ and then a form of Eq. 6 becomes crucial for neutrinos mass determination. Two parametrizations have been suggested in literature. The first one [6]

$$m_\beta^{(1)} = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2},$$

is a result of Taylor expansion of the spectrum (5) around the point

$$\left(\frac{m_i}{E_0 - E}\right)^2 \approx 0.$$  

The second [7]

$$m_\beta^{(2)} = \sum_{i=1}^{3} |U_{ei}|^2 m_i,$$

follows from the approximation of the precise distribution (Eq. 5) by the effective one (Eq. 2) near the end of the electron energy spectrum $E \to E_0$. We would like to elucidate the situation and decide which parametrization, (7) or (9) is better and should be used for future neutrinos mass determination.
Among five parameters \(|U_{ei}|\) and \(m_i\) only one is actually unknown, it is the mass of the lightest one \(m_1\). From neutrino oscillation experiments the mixing matrix elements \(|U_{ei}|^2\), \((i=1,2,3)\) \(\delta m_{\text{atm}}^2 = m_3^2 - m_2^2\) and \(\delta m_{\text{solar}}^2 = m_2^2 - m_1^2\) are determined. For the LMA MSW solution of the solar neutrino problem the best fit values are \([8]\)

\[
|U_{e1}|^2 = 0.55, \quad |U_{e2}|^2 = 0.43, \quad |U_{e3}|^2 = 0.02, \quad (10)
\]

and \(\delta m_{\text{solar}}^2 = 3.5 \times 10^{-5} \text{ eV}^2\). Atmospheric neutrino oscillations give \(\delta m_{\text{atm}}^2 = 3.1 \times 10^{-3} \text{ eV}^2\) \([8], [9]\). Uncertainties in the determination of the oscillation parameters from solar neutrino experiments are large \([8]\), but our main conclusions depend only weakly on them. The masses of heavier neutrinos are the function of the lightest neutrino \(m_1\)

\[
m_2 = \sqrt{m_1^2 + \delta m_{\text{solar}}^2}, \quad m_3 = \sqrt{m_1^2 + \delta m_{\text{solar}}^2 + \delta m_{\text{atm}}^2}. \quad (11)
\]

As \(R(E)\) is a smooth function of \(E\) at the end of \(\beta\) spectrum, we can approximate \(R(E) \approx R(E_0 - m_1)\). Then we can plot scaled energy distribution as:

\[
\frac{1}{R(E_0 - m_1)} \frac{dN}{dE} \equiv f_i(E). \quad (12)
\]

In Fig. 1 the full \(f_0(E)\) (Eq. 5) and two effective distributions ( \(f_1(E)\) with \(m_\beta = m^{(1)}_\beta\) and \(f_2(E)\) with \(m_\beta = m^{(2)}_\beta\) ) are depicted as a function of energy \(E_0 - E\) for three particular values of the lightest neutrino masses \((m_1 = 0.001 \text{ eV}, m_1 = 0.01 \text{ eV} \text{ and } m_1 = 0.1 \text{ eV})\). To compare both approximations, the ratio

\[
g(E) = \frac{|f_0(E) - f_2(E)|}{|f_0(E) - f_1(E)|} \quad (13)
\]

is also shown. We can see that for small values of \(x = E_0 - E\) the effective distribution \(f_1(E)\) with \(m^{(1)}_\beta\) approximates the full spectrum in a better way \((g(x) < 1)\). For larger \(x\), \(g(x) > 1\), and \(m^{(2)}_\beta\) gives better result. This conclusion is general, independent of the lightest neutrino mass \(m_1\) and values of the other oscillation parameters. To answer the question which effective neutrino mass \(m^{(1)}_\beta\) or \(m^{(2)}_\beta\) should be used in future experimental searches, the number of events in a possible small interval \(\Delta E\) which still can be resolved by a detector

\[
(E_0 - m_1 - \Delta E, E_0 - m_1) \quad (14)
\]

should be calculated. The integral

\[
n_i(\Delta E) = \int_{E_0 - m_1 - \Delta E}^{E_0 - m_1} f_i(E) \delta E \quad (15)
\]

can be done analytically,

\[
n_0(\Delta E) = \frac{1}{3R(E_0 - m_1)} \{ |U_{e1}|^2 B^{3/2} + |U_{e2}|^2 \left( B - \delta m_{\text{solar}}^2 \right)^{3/2} \times \}
\]
\[
\times \Theta (\Delta E - (m_2 - m_1)) + |U_{e3}|^2 \left( B - \delta m_{\text{solar}}^2 - \delta m_{\text{atm}}^2 \right)^{3/2} \Theta (\Delta E - (m_3 - m_1)), \quad (16)
\]
\[ n_i(\Delta E) = \left( B - (m_{\beta}^{(i)})^2 \right)^{3/2} \Theta (\Delta E - (m_{\beta}^{(i)} - m_1)) , \] (17)

with

\[ B = \Delta E(\Delta E + 2m_1). \]

To compare both approximate spectra the ratio

\[ h(\Delta E) = \frac{|n_0(\Delta E) - n_2(\Delta E)|}{|n_0(\Delta E) - n_1(\Delta E)|} \] (18)

is plotted on Fig. 2 for three different neutrino masses \( m_1 = 0.001 \text{ eV}, m_1 = 0.01 \text{ eV} \) and \( m_1 = 0.1 \text{ eV} \). We can see that independently of chosen \( m_1 \) and for \( \Delta E > m_3 - m_1 \), \( h(\Delta E) > 1 \). We know that \( m_3 - m_1 < 0.08 \text{ eV} \) [10]. It will be very difficult to get such a small energy spectrum resolution. So, let us conclude. In practice \( \Delta E \gg m_3 - m_1 \), and approximate spectrum with \( m_{\beta} = \sqrt{\sum |U_{e\beta}|^2 m_i^2} \) should be used in future searches of neutrino masses in the tritium \( \beta \) decay.
FIG. 1. The scaled electron energy distribution $f_i(E)$ at the end of the spectrum for $\bar{\nu}_H$ decay for three different masses of the lightest neutrino (a) $m_1 = 0.001$ eV, (b) $m_1 = 0.01$ eV, (c) $m_1 = 0.1$ eV. $f_0(E)$ describes the full (dashed line) and $f_i(E)$ (i=1,2) describes approximate effective energy distribution for $m_{1\beta}^{(1)}$ (tick solid line) and $m_{1\beta}^{(2)}$ (thin solid line). The function $g(E)$ compare both approximations (see text).
FIG. 2. $h(\Delta E)$ as a function of $\Delta E$ for minimal neutrino mass (a) $m_1 = 0.001 \text{ eV}$, (b) $m_1 = 0.01 \text{ eV}$, (c) $m_1 = 0.1 \text{ eV}$.

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