We review the current status of theoretical study of non-leptonic two body B decays. There are two independent directions for this purpose. One is the so-called QCD factorization approach (or BBNS approach), which is based on naive factorization approach. The other one is named perturbative QCD approach. We list the different ideas and applications of the two methods, and make a comparison between the two.

1 Introduction

The current running B factories at KEK and SLAC arouse many interests of theoretical studies of B decays. The mechanism of CP violation and signal of new physics are among the most important issues in B physics. Most of B topic involve the study of non-leptonic B decays, such as CKM angle measurements and rare decays etc. Quantum Chromodynamic is one of the successful theory of particle physics, however, the non-perturbative behavior of QCD is still the unsolved problem. Unlike perturbative QCD, the hadronization of non-leptonic B decays can not be done from the first principal. Thus it is model dependent. The most successful approach recently is the factorization approach (FA), which can explain many of the decay channels by very few parameters.

Although the predictions of branching ratios agree well with experiments in most cases, there are still some theoretical points unclear in FA. First, it relies strongly on the form factors, which cannot be calculated by FA itself. Secondly, the generalized FA shows that the non-factorizable contributions are important in a group of channels. The reason of this large non-factorizable contribution needs more theoretical studies. Thirdly, the strong phase, which is important for the CP violation prediction, is quite sensitive to the internal gluon momentum. This gluon momentum is the sum of momenta of two quarks, which go into two different mesons. It is difficult to define exactly in the FA approach. To improve the theoretical predictions of the non-leptonic B decays, we try to improve the factorization approach, and explain the size of the non-factorizable contributions in a new way.

There are mainly two directions toward this improvement: One is the so
called QCD factorization or BBNS \(^5\), which is based on the naive factorization approach. In this approach, the non-factorizable contribution can be directly calculated if it is not dominant over the factorizable one. The other is the perturbative QCD (PQCD) approach which is based on Brodsky and Lepage’s idea \(^6\). In this direction, people try to calculate the form factors and also the non-factorizable and annihilation type contributions in a systematic way. In the next section, we will first explain the idea of naive and generalized factorization approach, which has been studied for years. In section 3, we will introduce the idea of QCD factorization. And in section 4, PQCD approach is applied. Finally, in section 5, we will give a brief comparison between the two approaches and summarize at the end.

## 2 Generalized Factorization Approach

The calculation of non-leptonic decays involves the short-distance Lagrangian and the calculation of hadronic matrix elements which are model dependent. The short-distance QCD corrected Lagrangian is calculated to next-to-leading order. The popular method to calculate the hadronic matrix elements is using the factorization method where the matrix element is expressed as a product of two factors \( \langle h_1 h_2 | H_{eff} | B \rangle = \langle h_1 | J_1 | B \rangle \langle h_2 | J_2 | 0 \rangle \). The first factor is proportional to the \( B \rightarrow h_1 \) form factor, while the second one is proportional to the decay constant of \( h_2 \) meson.

First, let us discuss the short distance part. The non-leptonic decays of \( B \) mesons are induced by the weak interaction in the quark level, which gives the effective four-quark operator. Together with the QCD corrections, the effective Hamiltonian for the charmless non-leptonic \( B \) decays is

\[
H_{eff} = \frac{G_F}{\sqrt{2}} \left[ V_{q'b} V_{qb}^* \left( \sum_{i=1}^{10} C_i O_i + C_g O_g \right) \right],
\]

(1)

where \( q = d, s \) and \( V_{q'b} \) denotes the CKM factors. The operators \( O_1, O_2 \) are tree level current operators. The operators \( O_3, \ldots, O_6 \) are QCD penguin operators. \( O_7, \ldots, O_{10} \) arise from electroweak penguin diagrams, which are suppressed by \( \alpha/\alpha_s \). Only \( O_9 \) has a sizable value whose major contribution arises from the \( Z \) penguin.

For example, let us consider \( B \) meson decays to two pseudoscalar mesons. In the factorization approach, using the effective Hamiltonian, we write the required matrix element in its factorized form

\[
\langle P_1 P_2 | H_{eff} | B \rangle = i \frac{G_F}{\sqrt{2}} V_{q'b} V_{q1}^* a_i f_{P_2} (m_B^2 - m_{P_1}^2) F_0^{P_1 - P_2} (m_{P_2}^2).
\]

(2)
The dynamical details are coded in the quantities $a_i$, which we define as $a_i \equiv C_{i}^{eff} + C_{j}^{eff}/N_c$, where $\{i, j\}$ is any of the pairs $\{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}$ or $\{9, 10\}$. In practice, $N_c$ is treated as a phenomenological parameter to include the non-factorized color-octet contributions.

If the amplitude is dominated by the tree amplitude, the BSW-classification can be applied. Class-I decays are color favored, whose matrix elements are proportional to $a_1 = C_1/N_c + C_2$. Class-II decays are color suppressed, whose matrix elements are proportional to $a_2 = C_1 + C_2/N_c$. If $N_c = 3$ as in QCD, then $a_2$ becomes a very small number, making the branching ratios of this class of decays also small. However from experiments we know that in this category of decays, the branching ratios are not so small. Therefore the non-factorizable contributions are found to be very important in order to explain the large branching ratios. Class-III decays are proportional to $a_1 + r a_2$. This class of decays will determine the relative sign between $a_1$ and $a_2$. The QCD and electroweak penguins are also present in the charmless decays of B mesons. For the penguin-dominant decays, we introduce two more classes: Class-IV decays involve one or more of the dominant penguin coefficients $a_4, a_6$ and $a_9$. Class-V decays are decays with strong $N_c$-dependent coefficients $a_3, a_5, a_7$ and $a_{10}$.

Class-I and Class-IV decays have relatively large branching ratios of the order of $10^{-5}$ and stable against variation of $N_c$. Most of the measured decay channels by experiments are belong to these classes. There is a good agreement between experiments and theory, which indicate the success of FA. Class-III decays are mostly stable, except for some $B \to PV$ decays. Class-II and Class-V decays are rather unstable against variation of $N_c$. Some measured decay channels of these classes indicate that we need an effective $N_c \simeq 2$ rather than $N_c = 3$ to explain the experimental data. This implies that large non-factorizable contribution exist in these decays. Many of them may receive significant contribution from the annihilation diagrams and/or soft final state interactions.

In FA, the strong phase comes only from the so called BSS mechanism. The inner charm quark loop produce a strong phase, when the charm quark on mass shell. This strong phase depends on the inner gluon line strongly. Therefore, in FA, the predicted CP violation of B decays are in a quite large range.

In the study of factorization approach, one finds that the non-factorizable contributions are important in some of the non-leptonic B decays, which can not be explained in the factorization approach itself. Therefore theoretical study of this effect is required.
3 QCD Factorization Approach

Recently, Beneke, Buchalla, Neubert, and Sachrajda (BBNS) proposed a formalism for two-body charmless $B$ meson decays\(^5\). In this approach, they expand the hadronic matrix element by the heavy $b$ quark mass

$$
\langle \pi\pi|Q|B\rangle = \langle \pi_1|j_1|B\rangle \langle \pi_2|j_2|0\rangle \left[ 1 + \sum r_n \alpha_s^n + \mathcal{O}(\Lambda_{QCD}/m_b) \right],
$$

where $Q$ is a local operator in the effective Hamiltonian and $j_{1,2}$ are bilinear quark currents. By neglecting the power corrections in $\Lambda_{QCD}$, one need only calculate the order $\alpha_s$ corrections including the vertex corrections for the four quark operators and the non-factorizable diagrams. These diagrams are shown in Fig.1. The first 6 diagrams have already been included in the generalized FA approach as next-to-leading order QCD corrections to local four quark operators. What new are the last two non-factorizable diagrams, which has a hard gluon line connecting the four quark operator and the spectator quark.

They claimed that factorizable contributions, for example, the form factor $F^{B\pi}$ in the $B \to \pi\pi$ decays, are not calculable in PQCD, but nonfactorizable contributions are in the heavy quark limit. Hence, the former are treated in the same way as FA, and expressed as products of Wilson coefficients and $F^{B\pi}$. The latter, calculated as in the PQCD approach, are written as the convolutions of hard amplitudes with three ($B, \pi, \pi$) meson wave functions. Annihilation diagrams are neglected as in FA. Hence, this formalism can be regarded as a mixture of the FA and PQCD approaches. Values of form factors at maximal recoil $q^2 = m_\pi^2$ and nonperturbative meson wave functions are all treated as input parameters. It is easy to see from eq.(3), that this equation is only applicable for those color enhanced decay modes, where the factorizable contribution dominates the final results. While for the color suppressed modes, where the non-factorizable contributions are not small, the expansion of eq.(3) is not right, since the large non-factorizable contribution is grouped into the second term of eq.(3).

When extending the BBNS formalism to the $B \to D^{(*)}\pi$ decays, difficulty occurs in the calculation of nonfactorizable amplitudes. The decay channel of $B^0 \to \pi^+D^-$ is calculable, because the end-point singularities in the non-factorizable diagrams (last diagrams in Fig.1) cancel each other. However, such a soft cancellation does not occur in the calculation of $B^+ \to \pi^+D^0$ and $B^0 \to \pi^0D^0$ decays. The $D$ meson is heavy on the upper side of the diagram, in which the light quark and the heavy $c$ quark do not move collinearly. Therefore, soft gluons can resolve their color structure, and interact with them. That is, not all the nonfactorizable amplitudes in the BBNS formalism are calculable.
There are many calculations for various decay channels in this approach. Group of people calculate $B$ meson decays to two light pseudoscalar mesons $^8$, $B$ meson decays to one pseudoscalar and one vector meson $^9$ and $B$ meson decays to final states with $\eta$ or $\eta'$ $^10$ in the QCD factorization approach. The numerical results show that the theory and experiments agree well for those class I and IV decays, which are color enhanced and dominated by the factorizable contribution. This also agrees with the FA result, since the dominant part in eq.(3) is the same as the FA. The success of QCD factorization is that one can calculate the sub-leading $O(\alpha_s)$ non-factorizable contribution (second term in eq.(3)) using perturbative QCD. While in the FA, one has to input a free parameter $N_{c eff}$ to accommodate the non-factorizable contribution.

Cheng and Yang did the calculation of $B$ meson decay to two vector mesons $^11$, including $B \rightarrow \phi K^*$. In the calculation of $B \rightarrow \phi K$ decay, they found that the annihilation contributions are important, but the relative strong phase can not be predicted exactly, due to the sensitivity of the cut-off introduced $^12$. In fact, they found that a real annihilation contribution is required to enhance the $B \rightarrow \phi K$ decay branching ratio. In the calculation of $B \rightarrow J/\psi K$ decay, they found that the leading twist contribution is too small to accommodate the experimental data $^13$. Problem remains for color suppressed decay modes in QCD factorization approach.

In the calculations above, those people found that there exist endpoint divergence in the annihilation diagram calculations of QCD factorization $^14$. Logarithm divergence occurred at twist 2 contribution, and linear divergence exists in twist 3 contribution. If not symmetric wave function, like $K^{(*)}$ meson,
there is also soft divergence in the non-factorizable diagrams. It is very difficult to treat these singularity in the BBNS approach. A cut-off is introduced to regulate the divergence, thus makes the QCD factorization approach prediction parameter dependent, especially for the strong phase. Recently an effort is made to introduce the $k_T$ dependence of the wave functions, and Sudakov form factors in the BBNS approach in order to remove the singularities. This makes the BBNS approach to go toward the direction of PQCD approach.

As for the strong phase in BBNS, like in FA, it may come from the BSS mechanism. Here the momentum of the inner gluon is well defined. However, it predicts too small strong phase, because of the small gluon momentum. There is also another source of strong phase from the annihilation diagrams, but strongly depends on the cut-off parameter. The strong phase in QCD factorization can be almost arbitrary large.

Finally, the QCD factorization approach is at least one step forward from Naive Factorization approach. It gives systematic prediction of sub-leading non-factorizable contribution for the class I and class IV decays, which are dominated by the factorizable contribution. Big problem is the endpoint singularity, but may be solved with Sudakov form factors like PQCD approach. The input parameters in QCD factorization approach are form factors, wave functions etc.

4 Perturbative QCD Approach

In this section, we will introduce the idea of PQCD approach. The three scale PQCD factorization theorem has been developed for non-leptonic heavy meson decays, based on the formalism by Brodsky and Lepage, and Botts and Sterman. In the non-leptonic two body $B$ decays, the $B$ meson is heavy, sitting at rest. It decays into two light mesons with large momenta. Therefore the light mesons are moving very fast in the rest frame of $B$ meson. In this case, the short distance hard process dominates the decay amplitude. The reasons can be ordered as: first, because there are not many resonance near the energy region of $B$ mass, so it is reasonable to assume that final state interaction is not important in two-body $B$ decays. Second, With the final light mesons moving very fast, there must be a hard gluon to kick the light spectator quark (almost at rest) in the $B$ meson to form a fast moving light meson. So the dominant diagram in this theoretical picture is that one hard gluon from the spectator quark connecting with the other quarks in the four quark operator of the weak interaction. Unlike the usual factorization approach, the hard part of the PQCD approach consists of six quarks rather than four. We thus call it six-quark operators or six-quark effective theory. There are also soft...
(soft and collinear) gluon exchanges between quarks. Summing over those leading soft contributions gives a Sudakov form factor, which suppresses the soft contribution to be dominant. Therefore, it makes the PQCD reliable in calculating the non-leptonic decays. With the Sudakov resummation, we can include the leading double logarithms for all loop diagrams, in association with the soft contribution.

There are three different scales in the B meson non-leptonic decay. The QCD corrections to the four quark operators are usually summed by the renormalization group equation. This has already been done to the leading logarithm and next-to-leading order for years. Since the b quark decay scale $m_b$ is much smaller than the electroweak scale $m_W$, the QCD corrections are non-negligible. The third scale $1/b$ involved in the $B$ meson exclusive decays is usually called the factorization scale, with $b$ the conjugate variable of parton transverse momenta. The dynamics below $1/b$ scale is regarded as being completely non-perturbative, and can be parameterized into meson wave functions. The meson wave functions are not calculable in PQCD. But they are universal, channel independent. We can determine them from experiments, and it is constrained by QCD sum rules and Lattice QCD calculations. Above the scale $1/b$, the physics is channel dependent. We can use perturbation theory to calculate channel by channel.

Besides the hard gluon exchange with the spectator quark, the soft gluon exchanges between quark lines give out the double logarithms $\ln^2(Pb)$ from the overlap of collinear and soft divergence, $P$ being the dominant light-cone component of a meson momentum. The resummation of these double logarithms leads to a Sudakov form factor $\exp[-s(P,b)]$, which suppresses the long distance contributions in the large $b$ region, and vanishes as $b > 1/\Lambda_{QCD}$. This form factor is given to sum the leading order soft gluon exchanges between the hard part and the wave functions of mesons. So this term includes the double infrared logarithms. It is shown in ref. that $e^{-s}$ falls off quickly in the large $b$, or long-distance, region, giving so-called Sudakov suppression. This Sudakov factor practically makes PQCD approach applicable. For the detailed derivation of the Sudakov form factors, see ref.

With all the large logarithms resummed, the remaining finite contributions are absorbed into a perturbative $b$ quark decay subamplitude $H(t)$. Therefore the three scale factorization formula is given by the typical expression,

$$C(t) \times H(t) \times \Phi(x) \times \exp \left[ -s(P,b) \right.$$

$$- 2 \int_{1/b}^{t} \frac{d \bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \left. \right], \quad \text{(4)}$$

where $C(t)$ are the corresponding Wilson coefficients, $\Phi(x)$ are the meson wave functions and the variable $t$ denotes the largest mass scale of hard process.
Figure 2: Diagrams for $B \to P_1 P_2$ decay in perturbative QCD approach. The factorizable diagrams (a),(b), non-factorizable (c), (d), factorizable annihilation diagrams (e),(f) and non-factorizable annihilation diagrams (g),(h).

$H$, that is, six-quark effective theory. The quark anomalous dimension $\gamma_q = -\alpha_s/\pi$ describes the evolution from scale $t$ to $1/b$. Since logarithm corrections have been summed by renormalization group equations, the above factorization formula does not depend on the renormalization scale $\mu$ explicitly.

The $\pi$ meson is treated as a light-light system. At the $B$ meson rest frame, pion is moving very fast. We define the momentum of the pion which contain the spectator light quark as $P_2 = (m_B/\sqrt{2})(1,0,0_T)$. The light spectator quark moving with the pion (with momentum $P_2$), has a momentum $(k_2^+,0,k_2^T)$. If we define the momentum fraction as $x_2 = k_2^+/P_2^+$, then the wave function of pion can be written as

$$\Phi_\pi = \frac{1}{\sqrt{2N_c}} \gamma_5 (\not p_\pi \phi_\pi (x_2, k_2T) + m_0 \phi^0_\pi (x_2, k_2T) + m_0 \phi^\sigma_\pi (x_2, k_2T)), \quad (5)$$

where $\phi_\pi (x_2, k_2T)$ is twist-2 wave function and $\phi^0_\pi (x_2, k_2T)$ and $\phi^\sigma_\pi$ are twist-3 wave functions. The $m_0$ in eq.(5) is given as

$$m_0 = \frac{m_\pi^2}{m_u + m_d}. \quad (6)$$

It is not the pion mass. Since this $m_0$ is a scale characterizing the Chiral symmetry breaking, it is estimated around $1 \sim 2$ GeV using the quark masses predicted from lattice simulations, one may guess contributions of $m_0$ term cannot be neglected because of $m_0 \ll m_B$. In the previous calculation of
$B \to \pi\pi^{23}$ and $B \to \pi K$ decays $^{22}$ we do not include the last term in eq.(5) for the pion wave function. However, by using a phenomenology twist 3 wave function for $\phi_{\pi}^{p}$, we get the right result for those branching ratios. The reason is that this choice of twist 3 wave function $\phi_{\pi}^{p}$, accommodate the full twist 3 contribution effectively. This phenomenological model also accommodate effectively the threshold resummation effect discussed below.

In the PQCD approach, we can calculate not only the factorizable diagrams (Fig.2(a),(b)) and non-factorizable diagrams (Fig.2(c),(d)) contribution but also the annihilation type diagrams (Fig.2(e,f,g,h)). Unlike the QCD factorization approach, there is no logarithm or linear divergence in these calculations due to the reason of Sudakov suppression with $k_T$ resummation and the threshold resummation discussed above.

As shown above, in the PQCD approach, we keep the $k_T$ dependence of the wave function. In fact, the approximation of neglecting the transverse momentum can only be done at the non-endpoint region, since $k_T \ll k^+$ is qualified at that region. At the endpoint, $k^+ \to 0$, $k_T$ is not small any longer, neglecting $k_T$ is a very bad approximation. By, keeping the $k_T$ dependence, there is no endpoint divergence as occurred in the QCD factorization approach, while the numerical result does not change at other region. Furthermore, the Sudakov form factors suppress the endpoint region of the wave functions. Recently another type of resummation has been observed. The loop correction to the weak decay vertex produces the double logarithms $\alpha_s \ln^2 x_2^{25}$. Using the wave functions from light-cone sum rules, at the endpoint region, these large logarithms are important, they must be resummed. The threshold resummation for the jet function results in Sudakov suppression, which decreases the contribution of endpoint region of wave functions. Therefore, the main contributions to the decay amplitude in PQCD approach comes not from the endpoint region. The perturbative QCD is applied safely.

By including $k_T$ to regulate the divergence, large logarithmic corrections $\alpha_s \ln k_T$ appear, and Sudakov resummation is demanded. With the resultant Sudakov suppression, we have explicitly shown that almost 100% of the full contribution to the $B \to \pi$ transition form factor arises from the region with the coupling constant $\alpha_s/\pi < 0.3^{22}$. It indicates that dynamics from hard gluon exchanges indeed dominate in the PQCD calculation. In $^{5}$ Sudakov resummation is irrelevant, since all QCD dynamics has been parameterized into models of form factors.

We emphasize that nonfactorizable and annihilation diagrams are indeed subleading in the PQCD formalism as $M_B \to \infty$. This can be easily observed from the hard functions in appendices of ref.$^{22,23}$. When $M_B$ increases, the $B$ meson wave function enhances contributions to factorizable diagrams. How-
ever, annihilation amplitudes, being independent of B meson wave function, are relatively insensitive to the variation of $M_B$. Hence, factorizable contributions become dominant and annihilation contributions are subleading in the $M_B \to \infty$ limit \textsuperscript{26}. Although the non-factorizable and annihilation diagrams are subleading for the branching ratio in color enhanced decays, they provide the main source of strong phase, by inner quark or gluon on mass shell. The BSS mechanism strong phase is negligible in the PQCD approach. In fact, the factorizable annihilation diagrams are Chirally enhanced. They are not negligible in PQCD approach \textsuperscript{22,23}. In the decays of B meson to two light mesons, we collect terms up to chirally enhanced terms ($O(m_0/m_B)$), but still drop the terms suppressed by $\Lambda/m_B$ \textsuperscript{26}.

The main input parameters in PQCD are the meson wave functions. It is not a surprise that the final results are sensitive to the meson wave functions. Fortunately, there are many channels involve the same meson, and the meson wave functions should be process independent. In all the calculations of PQCD approach, we follow the rule, and we find that they can explain most of the measured branching ratios of B decays. For example: $B \to \pi\pi$ decays \textsuperscript{23}, $B \to K\pi$ decays \textsuperscript{22}, $B \to \pi\rho$, $B \to \pi\omega$ decays \textsuperscript{24}, $B \to KK$ decays \textsuperscript{27}, the form factor calculations of $B \to \pi$, $B \to \rho$ \textsuperscript{28}, $B \to K\eta/\eta'$ decays \textsuperscript{29}, $B \to K\phi$ decays \textsuperscript{26} etc.

5 Summary

For a comparison, in the QCD factorization: Form factors are input parameters, which are claimed to be dominated by soft contribution and not calculable. The endpoint singularity is a crucial point in this approach, a cut off is needed to regulate the divergence. The QCD factorization follows FA, in which it has been assumed that factorizable contributions, being the dominant contribution. And all other contributions such as non-factorizable and annihilation diagrams, being $\alpha_s$ corrections, are sub-leading. In the limit of $\alpha_s \to 0$, it goes back to FA. Therefore the current BBNS approach can not be applied to the non-factorizable dominant process such as $B^0 \to D^0\pi^0$, and also those annihilation diagram dominant processes.

In the PQCD approach, the form factors are calculable, which are dominant by short distance contribution. By including the $k_T$ dependence and Sudakov suppression, there is no endpoint divergence. In the PQCD formalism non-factorizable amplitudes are of the same order as factorizable ones in powers of $1/M_B$, which are both $O(1/(M_B\Lambda_{QCD}))$. The smaller magnitude of nonfactorizable amplitudes in color enhanced decays are due to the cancellation of the two non-factorizable diagrams. From the viewpoint of power
counting, they are of the same order. In case of $B \to D\pi$ decays, the cancellation is absent. The power counting changes, so that we can also calculate the non-factorizable contribution dominant processes.

For a conclusion, non-Leptonic B decays are important in B physics. The theory of non-leptonic B decays is a challenging work. There are still problems to be solved. The QCD factorization and PQCD approach are going toward the same object, although there are still some differences. They will be tested by experiments soon.

Acknowledgments

We thank Y.Y. Keum, E. Kou, T. Kurimoto, H.N. Li, T. Morozumi, A.I. Sanda, K. Ukai and M.Z. Yang for helpful discussions.

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