Feasibility of Probing Dark Energy with Strong Gravitational Lensing Systems

Fisher-Matrix Approach

Kazuhiro Yamamoto\textsuperscript{1}, Yasufumi Kadoya\textsuperscript{1}, Tsukasa Murata\textsuperscript{1} and Toshifumi Futamase\textsuperscript{2}

\textsuperscript{1}Department of Physical Science, Hiroshima University, Higashi-Hiroshima 739-8526, Japan
\textsuperscript{2}Astronomical Institute, Graduate School of Science, Tohoku University, Sendai 980-8578, Japan

(Received June 8, 2001)

We assess the feasibility of probing dark energy with strong gravitational lensing systems. The capability of the method, which depends on the accuracy with which the lensing systems are modeled, is quantitatively investigated using the Fisher-matrix formalism. We show that this method might place useful constraints on the density parameter and the redshift evolution of the dark energy by combining it with a constraint from supernova measurements. For this purpose, the lens potential needs to be precisely reconstructed. We determine the required quality of data. We also briefly discuss the optimal strategy to constrain the cosmological parameters using gravitational lensing systems.

§1. Introduction

Recent observations of distant Type Ia supernovae have provided strong evidence for the acceleration of the universe.\textsuperscript{1,2} Recent measurements of cosmic microwave background anisotropies favor a spatially flat universe with cold dark matter,\textsuperscript{3,4} while measurements of the large-scale structure in the distribution of galaxies favor a low-density universe.\textsuperscript{5} These observations can be explained by the hypothesis that our universe is dominated by dark energy in addition to cold dark matter. Motivated by these findings, many authors have recently investigated the origin of dark energy\textsuperscript{6-8} An attractive feature of these theoretical models is that the coincidence problem, the near coincidence of the density of matter and the dark energy component may be explained,\textsuperscript{7,9,10} though the plausibility is under debate.\textsuperscript{11}

An evolving scalar field has been investigated as a model of a decaying cosmological constant (dark energy).\textsuperscript{12-15} Dark energy can be characterized by the effective equation of state $w = \rho_D / P_D$, where $\rho_D$ and $P_D$ are the energy density and the pressure of the dark energy, respectively. Different theoretical scenarios of dark energy may be distinguished by the measurement of the effective cosmic equation of state. For example, the quintessence model generally predicts that $w$ can be a function of the cosmic scale factor.
of redshift and satisfies $w \geq -1$.\(^6\) If dark energy originated from the cosmological constant or a false vacuum energy, $w = -1$.\(^7\) Parker and Raval have explained dark energy in terms of a quantum vacuum effect produced by the curved spacetime of general relativistic cosmology. In their model, $w \leq -1$ is allowed.\(^8\) Models which predict $w \leq -1$ have also been proposed within the framework of the evolving scalar field.\(^16,17\) Therefore, the measurement of $w$ may provide information regarding the origin of dark energy. For this reason various observational strategies have been proposed to probe characteristics of dark energy.\(^18\)-\(^27\) One of the most promising approaches for determining $w$ employs type Ia supernovae, while other approaches using CMB anisotropies and number-counts should be useful too.\(^28\)-\(^30\) However, it has been pointed out that the method using supernovae is fundamentally limited because the luminosity distance depends on $w$ through a multiple integration, which smears out information concerning $w$ and its time variation.\(^31\)

With the situation as described above, it is very useful to have an independent method to determine $w$. One possible such method is to use gravitational lensing. In fact, it is known that lensing statistics depend strongly on the value of the cosmological constant, as well as the characteristics of dark energy.\(^32\)-\(^36\) Another possible method to probe dark energy has been proposed.\(^37\)-\(^40\) This method relies on precise measurements of ‘clean’ gravitational lensing systems. Here ‘clean’ means suitable for modeling a gravitational lensing system, as is an Einstein ring or Einstein cross. Motivated by this recent proposition, we assess the feasibility of the method to probe dark energy that employs the Fisher-matrix formalism, which is useful to demonstrate how accurately one can estimate model parameters from a given data set. (For a review see, e.g., Ref.41.) The primary goal of the present paper is to determine the quality necessary for a data set from gravitational lensing systems in order to probe dark energy.

A gravitational lensing system can be used to measure the ratio of (angular diameter) distances, while the supernovae can be used to determine the luminosity distance itself. One might ask whether the method of the gravitational lens provides new independent information. The second goal of the present paper is to answer this question. We show that the gravitational lens method can be a useful tool that complements the supernova method as a probe of dark energy.

This paper is organized as follows. In §2, we briefly review basic formulas. Then, we investigate the capability of the method employing strong gravitational lensing systems, focusing on errors in determining cosmological parameters, in §3. Uncertainties involved in modeling a lens galaxy are discussed in §4. In §5, we discuss the redshift distribution of a lensing system for the purpose of optimizing the method. Section 6 is devoted to a summary and conclusions. Throughout this paper we use units in which the velocity of light, $c$, equals 1.

### §2. Basic formulas

In this section we give a brief review of basic formulas for the topics considered in the present paper. We restrict ourselves to a spatially flat FRW universe, in which case the Friedmann equation is written $H(z)^2 = H_0^2[\Omega_0(1+z)^3 + (1-\Omega_0)f(z)]$, where
\( H_0 = 100 \text{h} \text{km/s/Mpc} \) is the Hubble constant, \( \Omega_0 \) is the cosmic density parameter of the matter component, and \( f(z) \) describes the redshift-evolution of the dark energy. When the cosmic equation of state is parameterized as \( w(z) = w_0(1 + z)^\nu \), we have

\[
    f(z) = (1 + z)^3 \exp \left[ 3w_0 \left( \frac{(1 + z)^\nu - 1}{\nu} \right) \right],
\]

which reduces to \( f(z) = (1 + z)^{3(1 + w_0)} \) in the limit \( \nu \to 0 \). In a spatially flat universe, the angular diameter distance between \( z_1 \) and \( z_2 \) is

\[
    D_A(z_1, z_2) = \frac{1}{H_0(1 + z_2)} \int_{z_1}^{z_2} \frac{dz'}{[\Omega_0(1 + z')^3 + (1 - \Omega_0)f(z')]^{1/2}},
\]

where we have assumed \( z_1 < z_2 \).

In order to estimate the accuracy to which we can constrain \( \Omega_0 \), \( w_0 \), and \( \nu \) with measurements using gravitational lensing systems, we employ the Fisher-matrix approach. With the Fisher-matrix analysis, one can estimate the best statistical errors on parameters from a given data set. For this reason, this approach is widely used to estimate how accurately cosmological parameters are determined from the large scale structure of galaxies or the cosmic microwave background anisotropies.\(^{41-43}\) Tegmark et al. applied the Fisher-matrix analysis to supernova data sets.\(^{44}\) Their analysis is useful for the present study. In general, the Fisher-matrix is defined by

\[
    F_{ij} = \left\langle -\frac{\partial^2 \ln L}{\partial \theta_i \partial \theta_j} \right\rangle,
\]

where \( L \) is the probability distribution function of a data set, given model parameters \( \theta_i \).

To compute the Fisher matrix, we need to make an assumption concerning the probability function \( L \), and more specifically concerning the statistics of errors of measurements. The method employing a strong gravitational lensing system relies on the assumption that the ratio of the angular diameter distances, \( D_{LS}/D_S \), is precisely determined for each system,\(^{37,38}\) where \( D_{LS} \) is the angular diameter distances between the lens and a source object, and \( D_S \) is the distance between the source and the observer. A simple example in which \( D_{LS}/D_S \) is determined with a lensing system is considered in §4. Here, we assume that the lens model can be determined by measurements of images of a lensed object and the velocity dispersion of the lens galaxy, as well as the redshifts of the source and the lens objects. In a realistic situation, observational data is contaminated with errors in modeling the lensing potential and measurement of the velocity dispersion, as we discuss in §4. Hence, an observed value of \( D_{LS}/D_S \) contains error. Throughout this paper, we also assume that this error can be described by a Gaussian random variable, for simplicity. For definiteness we assume that a data set consists of \( N \) gravitational lensing systems and that the observed ratio of the angular diameter distances of the \( n \)-th lens system can be expressed as \( R_n = D_{LSn}/D_{Sn} + \Delta_n \), where \( D_{LSn} \) and \( D_{Sn} \) are the angular diameter distances \( D_{LS} \) and \( D_S \) of the \( n \)-th lensing system, and the error \( \Delta_n \) behaves
as a Gaussian variable with zero mean, $\langle \Delta_n \rangle = 0$. In this case the Fisher matrix (2.3) can be written

$$F_{ij} = \sum_{n=1}^{N} \left( 2 + \frac{1}{\varepsilon_n^2} \right) W_{ni} W_{nj} \approx \frac{1}{\varepsilon^2} \sum_{n=1}^{N} W_{ni} W_{nj},$$

(2.4)

where $W_{ni} \equiv \partial \ln \left( \frac{D_{LSn}}{D_{Sn}} \right) / \partial \theta_i$, $\varepsilon_n$ describes the variance of the Gaussian variable with $\langle \Delta_n^2 \rangle = \left( \frac{D_{LSn}}{D_{Sn}} \right)^2 \varepsilon_n^2$, and the last approximate equality holds in the case that $\varepsilon_n (= \varepsilon) \ll 1$ for all $n$. In contrast to the case of supernovae, the factor 2 appears on the right-hand side of Eq. (2.4). The corresponding term can be ignored in the case $\varepsilon \ll 1$. This factor is traced back to the assumption that the error $\Delta_n$ is proportional to $D_{LSn}/D_{Sn}$. By using the Bayse theorem, the probability distribution in the parameter space can be written

$$P(\theta) \propto \exp \left[ -\frac{1}{2} \sum_{ij} \left( \theta_i - \theta_i^{tr} \right) F_{ij} \left( \theta_j - \theta_j^{tr} \right) \right],$$

(2.5)

where we have assumed that errors in the target model parameters $\theta_i^{tr}$ are small.

Fig. 1. Confidence regions from various data sets for gravitational lensing systems. Here we have chosen the target parameters as $\Omega_0 = 0.3$ and $w_0 = -1$, with the number and the individual uncertainties as follows: (a) $N/\varepsilon^2 = 10^3$; (b) $N/\varepsilon^2 = 3 \times 10^3$; (c) $N/\varepsilon^2 = 10^4$; (d) $N/\varepsilon = 3 \times 10^4$, where we have set $\varepsilon_n$ equal for all systems, $\varepsilon_n = \varepsilon$. The curves are the contours of the 68%, 95%, and 99.7% confidence regions. We chose the redshift distribution of lensing systems $z_S$ and $z_L$ through a homogeneous random process in the ranges $1 \leq z_S \leq 3$ and $0.3 \leq z_L \leq z_S - 0.5$, respectively.
Thus the Fisher matrix gives the uncertainties in the parameter spaces, which are described by a Gaussian distribution function around $\theta^*_t$.

§3. Estimating errors in determining cosmological model parameters

We now demonstrate how accurately cosmological model parameters can be determined by using the Fisher-matrix formalism. We first consider the model in which the cosmic equation of state is constant ($w(z) = w_0$), which is equivalent to the assumption $f(z) = (1 + z)^3(1 + w_0)$. Considering the Fisher-matrix to be the $2 \times 2$ matrix corresponding to $\Omega_0$ and $w_0$, we show how accurately these two parameters can be determined. Figure 1 displays the confidence regions in the $\Omega_0$-$w_0$ plane described by the probability function (2.5), where we used the cosmological parameters $\Omega_0 = 0.3$ and $w_0 = -1$ as the target model parameters. We considered various data sets for the number and the variance of the errors in the gravitational lensing systems. The quality of the data is characterized by $N/\varepsilon^2$, because the Fisher-matrix scales in proportion to $N/\varepsilon^2$, whose value is shown on each panel of the figure. For the redshift distribution of the lensing systems, we chose $z_S$ and $z_L$ through a homogeneous random process in the ranges $1 \leq z_S \leq 3$ and $0.3 \leq z_L \leq z_S - 0.5$. The three curves in each panel indicate the 68%, 95%, and 99.7% confidence regions. Figure 1 shows that the method of the gravitational lens is sensitive to $\Omega_0$, but rather insensitive to $w_0$ in the case that $N/\varepsilon^2$ is small. It also shows that a useful constraint on $\Omega_0$ and $w_0$ can be obtained if many lensing systems are measured precisely and $N/\varepsilon^2$ is sufficiently large.

One might think that the gravitational lens method can be no better than the supernova method because the latter supernova measures the luminosity distance itself, while the former measures only the ratio of the (angular diameter) distances.

![Fig. 2. Confidence regions from a data set from supernovae with individual uncertainties of 0.15 magnitude. Here, 100 supernovae randomly distributed in the range $0.5 \leq z \leq 1.5$ are considered. The curves are the contours of 68%, 95%, and 99.7% confidence regions, and the same target model as Fig. 1 is used.](image-url)
However, in principle, these two methods give independent information and can be complementary. Figure 2 demonstrates how accurately we can put a constraint on model parameters using a data set from 100 supernovae randomly chosen in the range $0.5 \leq z \leq 1.5$ with individual statistical uncertainties of 0.15 mag. By comparing Figs. 1 and 2, it is clear that the two methods provide independent constraints. Thus, using both methods, we can eliminate ambiguities in determining model parameters that arise when using only one. Figure 3 displays the confidence regions in the $\Omega_0$-$w_0$ plane, which is the same as Figure 3, but with the addition of the constraints obtained from the data set of 100 supernovae presented in Fig. 2. It is clear from Fig. 3 that the ability to constrain $\Omega_0$ and $w_0$ is significantly improved.

We next consider the case that the density parameter $\Omega_0$ is fixed from other experiments. We assume $\Omega_0 = 0.3$. By considering the Fisher matrix to be the $2 \times 2$ matrix corresponding to the parameters $w_0$ and $\nu$, Fig. 4 displays the confidence regions on the $w_0$-$\nu$ plane, where we used $w_0 = -1$ and $\nu = 0$ as the target model parameters. This figure displays the results using both the gravitational lensing systems and the data set of 100 supernovae, for which we assumed the same range of distribution and uncertainty of sources as in Fig. 2. The constraint on $\nu$ might not be strict when $N/\varepsilon^2$ is of order $10^3$, but the useful constraint is obtained when $N/\varepsilon^2 \sim O(10^4)$. Hence precise measurements of many gravitational lensing systems might provide a useful probe, even for the parameter $\nu$. 

Fig. 3. Same as Fig. 1, but used combining the constraint from the data set of 100 supernovae displayed in Fig. 2.
Fig. 4. Confidence regions in the $w_0-\nu$ plane, with density parameter fixed as $\Omega_0 = 0.3$. The target model parameters $w_0 = -1$ and $\nu = 0$ are used. The situation regarding the redshift distribution and $N/\varepsilon^2$ are the same as in Fig. 1.

§4. Errors in modeling lens galaxies

In this section we discuss $\varepsilon$, which characterizes systematic errors in the ratio $D_{LS}/D_S$. For this purpose, we start by demonstrating one of the simplest ways to determine the ratio, adopting a lens potential based on an isothermal ellipsoid model.\textsuperscript{38,45} In this model, the lens equation gives an elliptical image of the Einstein ring with minor and major axes

$$\theta_\pm = \theta_E \sqrt{1 \pm \epsilon},$$

with

$$\theta_E = \frac{4\pi \sigma_v^2 D_{LS}}{D_S},$$

where $\sigma_v$ is the one-dimensional velocity dispersion, and the ratio $e$ of the minor axis to the major axis is related to the ellipticity $\epsilon$ by $e = \sqrt{(1 + \epsilon)/(1 - \epsilon)}$. Thus the ratio $D_{LS}/D_S$ can be determined by measurements of $e$, $\theta_E$ and $\sigma_v$.

We note that several authors have recently pointed out possible errors involved in modeling the lensing potential.\textsuperscript{46-49} With regard to this matter, we first consider a possible density profile that differs from the singular isothermal profile, though we restrict ourselves to a spherically symmetric density profile. We assume a generalized Navarro-Frenk-White\textsuperscript{50,51} or Zhao density profile,\textsuperscript{52}

$$\rho(r) = \frac{\rho_s}{g(r/r_s)},$$

\textsuperscript{38) \textsuperscript{45) \textsuperscript{46-49) \textsuperscript{50,51) \textsuperscript{52)}}}
\( g(x) = x^\alpha (1 + x)^{\zeta - \alpha} \) \hspace{1cm} (4.4)

where we have defined

\[ \theta_E - F(\theta_E, D_L/r_s, \alpha)4\pi \sigma_v^2 \frac{D_{LS}}{D_S} = 0, \]

where we have defined

\[ F(\theta_E, D_L/r_s, \alpha) = \frac{1}{\theta_E} \int_0^{\theta_E} d\theta' \theta' \frac{1}{\pi} \int_{-\infty}^{\infty} dt \frac{D_L/r_s}{g \left( \sqrt{\theta'^2 + t^2} D_L/r_s \right)}. \] 

(4.6)

Here we have considered the case \( \zeta = 2 \), so that the lens model possesses the same velocity dispersion as the singular isothermal sphere at large distances. In the limit \( \alpha \to 2 \), we have \( F(\theta_E, D_L/r_s, \alpha) = 1 \), and the model reduces to the singular isothermal model. This model indicates how the ratio \( D_{LS}/D_S \), which is determined by solving the lens equation (4.5), depends on the density profile parameterized by \( r_s \) and \( \alpha \) with the same observable quantities \( \theta_E \) and \( \sigma_v^2 \). Figure 5 plots the normalized difference between \( D_{LS}/D_S \) and the corresponding value in the singular isothermal model. Explicitly, this figure displays contours of the quantity

\[ \frac{D_{LS}/D_S - D_{LS}/D_S|_{SIS}}{D_{LS}/D_S|_{SIS}} = \frac{1}{F(\theta_E, D_L/r_s, \alpha)} - 1, \]

(4.7)

in the \( r_s-\alpha \) plane, where \( D_{LS}/D_S|_{SIS} = \theta_E/(4\pi \sigma_v^2) \) is the value of this ratio in the singular isothermal model. In this figure we fixed \( \theta_E = 1'' \) and used the cosmological model with density parameters \( \Omega_0 = 0.3 \) and \( w(z) = -1 \). The four panels correspond to the cases in which the redshift of the lens is \( z_L = 0.5, 1.0, 1.5 \) and \( 2.0 \). For example, the normalized difference is of order 5% and 10%, respectively, for \( \alpha = 1 \) and \( \alpha = 0 \) at \( r_s = 0.1 \) \( h^{-1}\text{kpc} \), which depends weakly on the redshift of the lens objects. The difference between the ratios becomes larger as \( r_s \) increases or \( \alpha \) decreases. We thus find that the ambiguity in the density profile can be an important factor in modeling the lensing potential.

We next consider the errors involved in the observation of the velocity dispersion, which we have not yet discussed. Mortlock and Webster systematically investigated the errors involved in gravitational lensing statistics and pointed out that the greatest uncertainty comes from the dynamical normalization of lens galaxies.\(^{54}\) According to their results, the velocity dispersion \( \sigma_v \) in the isothermal sphere model (the line-of-sight velocity dispersion away from core radius) can be related with the observed line-of-sight velocity dispersion of stars \( \sigma_v(R_f) \), assuming a circular aperture of radius \( R_f \) as

\[ \sigma_v = \sigma_v(R_f) \left( A + B \frac{r_c}{R_g} \right), \]

(4.8)
Fig. 5. Contours of the normalized difference defined by Eq. (4.7) in the $r$-$\alpha$ plane. This figure displays the normalized difference between the ratio $D_{LS}/D_S$ and the corresponding value for the singular isothermal model. Note that $\alpha = 2$ represents the case of the singular isothermal model. We fixed $\theta_E = 1''$ and adopted the cosmological model with $\Omega_0 = 0.3$ and $w(z) = -1$. In each panel, the redshift of the lens object is adopted as $z_L = 0.5$, 1.0, 1.5 and 2.0, respectively. The curves are the contours of the 1%, 2%, 5% and 10% levels from left to right for all panels.

where $A$ and $B$ are constants independent of the core radius $r_c$, and $R_g$ is the effective radius of the lens galaxy. Kochanek used $A = 1$ and $B = 2$.\textsuperscript{55)} Then, the non-dimensional uncertainty of $\sigma_v$ may be expressed as

$$\frac{\delta \sigma_v}{\sigma_v} \simeq \frac{\delta \sigma_{\|}(R_f)}{\sigma_{\|}(R_f)} + \frac{B \delta r_c/R_g}{A + B r_c/R_g},$$

(4.9)

where $\delta \sigma_{\|}(R_f)$ and $\delta r_c$ represent uncertainties on $\sigma_{\|}(R_f)$ and $r_c$. Because the ratio of the distance $D_{LS}/D_S$ is proportional to $\sigma_v^2$, the error that contributes to $\varepsilon$ is estimated as $\varepsilon \simeq 2 \delta \sigma_v/\sigma_v$, where we have assumed $\delta \sigma_v/\sigma_v \ll 1$. Concerning the uncertainty of the stellar velocity dispersion, measurements of the velocity dispersion of lensing galaxies with accuracy to within approximately 3% have been made with the Keck-II 10m telescope,\textsuperscript{56,57)} while the uncertainty on the core radius $r_c$ might be very important. For example, the second term on the right-hand side of (4.9) is of order 0.1 for $r_c = 0.1$ kpc and $R_g = 2$ kpc. Thus the investigation of this section demonstrates the importance of precisely modeling the lensing galaxy, taking the possible finite core radius and the density profile into account in a realistic analysis.
§5. Optimizing constraint on model parameters

In this section we address the problem of determining the optimal distribution in redshift of gravitational lensing systems in order to best constrain the cosmological parameters. This problem might be only of theoretical interest, but having such information could be useful in planning observation of gravitational lensing systems. The same problem has been considered by Huterer and Turner with regard to supernova data.\(^{28}\) Our discussion here is based on the following observation: By a theorem concerning the Fisher matrix, it can be shown that the minimum error attainable on \(\theta_i\) is expressed by the diagonal part of the inverse Fisher-matrix if the other parameters are known.\(^{41}\) To address the above stated problem, we investigate what the pair of redshifts of the source and the lens objects maximize the diagonal part

\[ \varepsilon^2 F_{ii} \approx \frac{1}{D_S^2 D_{LS}^2} \left( D_S \frac{\partial D_{LS}}{\partial \theta_i} - D_{LS} \frac{\partial D_S}{\partial \theta_i} \right)^2, \]  

(5.1)

where the case of one lensing system is assumed for simplicity. It is useful to note how the normalized variation of the ratio \(D_{LS}/D_S\) is related with the variances of the cosmological parameters and the Fisher-matrix:

\[ \frac{\Delta (D_{LS}/D_S)}{D_{LS}/D_S} = \sum_i W_i \Delta \theta_i = \varepsilon F_{i\theta_0 \Omega_0}^{1/2} \Delta \Omega_0 + \varepsilon F_{i\theta_0 \omega_0}^{1/2} \Delta \omega_0 + \varepsilon F_{i\theta_0 \nu}^{1/2} \Delta \nu. \]  

(5.2)

Figure 6 displays the amplitude of the diagonal part given in \(\varepsilon F_{ii}^{1/2}\) as a function of \(z_L\) for \(z_S = 5, 4, 3, 2, 1\) and 0.5. The left and right panels depict the cases of the target model with \(\Omega_0 = 0.3, \omega_0 = -1\) and \(\nu = 0\) and with \(\Omega_0 = 0.3, \omega_0 = -1\) and \(\nu = -1/2\), respectively. The top, middle, and bottom panels, respectively, display the components \(\theta_i = \Omega_0, \omega_0,\) and \(\nu\). The amplitude shown in each panel confirms that \(\Omega_0\) can be most easily determined, while \(\omega_0\) and \(\nu\) are relatively difficult to determine than \(\Omega_0\). From the top panels, it is clear that \(\Omega_0\) is more precisely determined as \(z_L\) becomes larger and that the precision with which \(\Omega_0\) can be determined is almost the same for all \(z_L\) satisfying \(z_S > z_L > 1\). Contrastingly, the situation for \(\omega_0\) and \(\nu\) is complicated. We see that as the redshifts of the source and the lens objects become larger (e.g., for \(z_S > z_L \gtrsim 2\), the precision with which \(\omega_0\) and \(\nu\) can be determined becomes better. However, this is not always the case, because \(F_{ii}\) becomes zero for certain combinations of \(z_L\) and \(z_S\), which depend on the target model parameters. From the middle and lower panels, we can find another peak for \(z_S \lesssim 1\) at which \(\omega_0\) and \(\nu\) can be determined precisely. It is thus seen that low redshift lensing systems are also useful to determine the redshift evolution of dark energy. This is consistent with the result of Ref. 39, and it is also consistent with the result obtained from supernova analysis.\(^{28}\) This feature can be understood from the fact that the dark energy component increases relative to the matter component as the redshift decreases.
Fig. 6. Amplitude of $\varepsilon F_{1/2}^{1/2}$ as a function of the redshift of the lens object $z_L$ with various redshifts of the source object $z_S$. The top, middle and bottom panels, respectively, display the components $\Omega_0$, $w_0$ and $\nu$. The left and right panels correspond to the case of the target model with $\Omega_0 = 0.3$, $w_0 = -1$ and $\nu = 0$ and with $\Omega_0 = 0.3$, $w_0 = -1$ and $\nu = -0.5$, respectively. In each panel, the curves correspond to $z_S = 0.5$, 1, 2, 3, 4 and 5, from top to bottom near $z_L = 0$.

§6. Summary and conclusions

In summary, we have assessed the feasibility of probing dark energy using gravitational lensing systems. We have demonstrated how precisely the model parameters can be constrained using the Fisher-matrix formalism. Our results show that the method might place useful constraints on the density parameters and the cosmic equation of state if many lensing systems are measured precisely. Our results also
demonstrate the toughness of the method. It was found that \( N/\varepsilon^2 > \sim 10^3 \) is required to probe the cosmic equation of state \( w_0 \) even if combined with the supernova method employing data from 100 supernovae, and \( N/\varepsilon^2 \gtrsim 10^4 \) is required to determine \( w_0 \) using only gravitational lensing systems.

In order to realize the condition \( N/\varepsilon^2 = 10^3 - 10^4 \) we need \( \varepsilon \lesssim 0.1 - 0.3 \) for \( N = 100 \). This required accuracy of measurement and modeling the lensing system might be difficult to realize, due to theoretical and observational ambiguities.\(^{58}\) As discussed in \( \S 4 \), uncertainties involving the density profile and the velocity dispersion are problematic for precise modeling of a lensing system. Nevertheless, the possibility of precisely modeling lens potentials has been studied. For example, the ability of observed Einstein rings to precisely model lens potentials has been discussed by several authors.\(^{59,60}\) The advantage of Einstein rings is the richness of the information they provide. The authors of those works have claimed that the distribution of the magnifications of Einstein rings makes it possible to eliminate lens model ambiguities and to model the mass distribution of a lens precisely.\(^{59}\) Furthermore, they have pointed out that Einstein rings must be found frequently in multiply imaged quasars with sufficiently long observations, because it is likely that all quasars and AGN have host galaxies. Thus we might expect that studies of modeling lensing galaxies will provide more accurate results in the near future, employing many sample galaxies in SDSS, and that future progress might allow us to solve the problems involved in modeling the lensing potential.\(^{61}\)

In \( \S 5 \), we considered optimizing the analysis. Though our consideration was restricted to the case of one gravitational lensing system, it is worth comment. We found that the gravitational lensing system method is most sensitive to \( \Omega_0 \), and less sensitive to \( w_0 \) and \( \nu \). To determine \( \Omega_0 \), it is most advantageous to have larger redshifts of the source and the lens objects, while this is not always true for \( w_0 \) and \( \nu \). This feature is due to the fact that the dark energy component decreases relative to the matter component as the redshift increases. Lensing systems with smaller redshifts can be useful to probe dark energy.

Acknowledgements

We are grateful to Y. Kojima and S. Yoshida for useful conversations and comments. We also thank T. Hamana for his important comments and discussions. Parts of \( \S 4 \) are based on the valuable communications with him. We also thank R. R. Caldwell for useful conversations and important comments on a preliminary manuscript, which helped improve the paper. We are also grateful to referees for helpful comments. One of the authors (K. Y.) thanks Professor S. D. M. White and the people at the Max-Planck Institute for Astrophysics (MPA) for their hospitality. Some parts of this work were done there. He acknowledges financial support from the Deutscher Akademischer Austauschdienst (DAAD) and the Japan Society for the Promotion Science (JSPS) for a visiting program to MPA. This research was also supported by the Inamori Foundation.

References